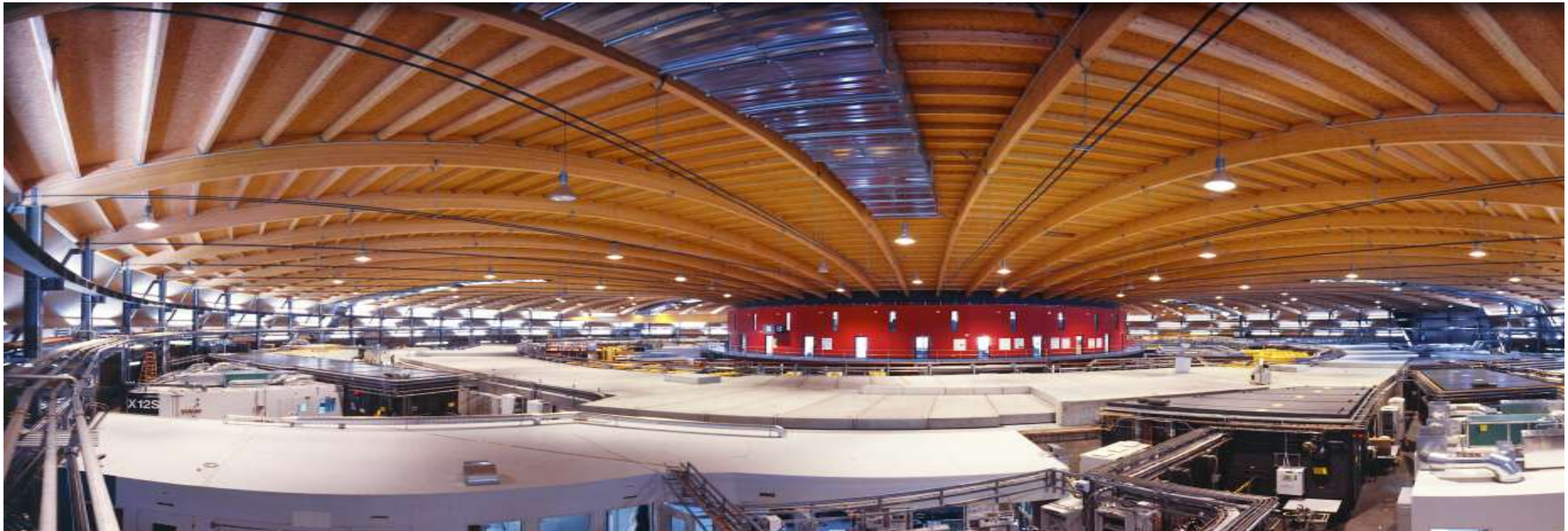
uzh | eth | zürich

Radiography & Tomography

Federica Marone

Swiss Light Source, Paul Scherrer Institut, Villigen, Switzerland



Contents

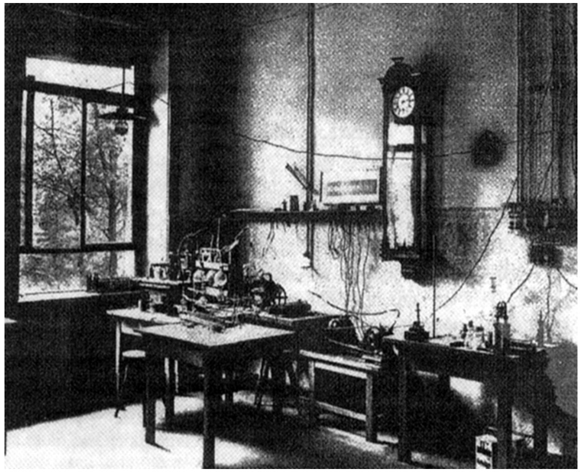
- **Mini-tutorial on X-ray tomography**
  - Beer-Lambert law
  - Tomographic reconstruction algorithms
    - Analytical algorithms
    - Iterative algorithms
- **Examples**
- **Practical issues**
  - Flat field correction
  - Local tomography

X-ray imaging yesterday…

First X-ray image, 1896

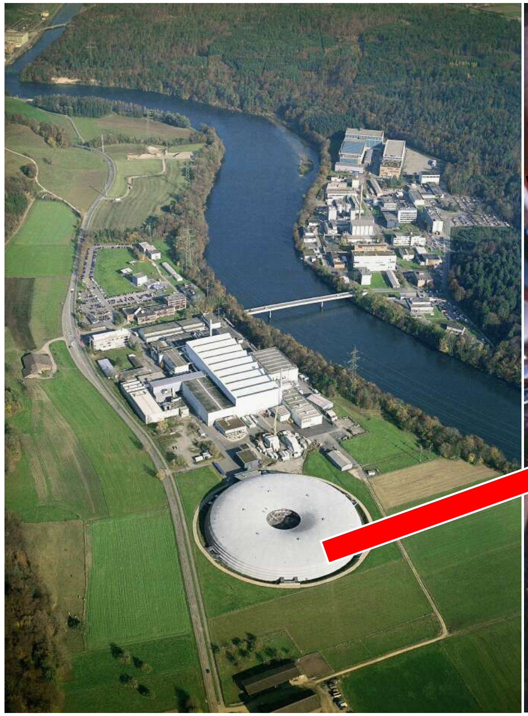
X-ray micro-radiography of a fruit fly

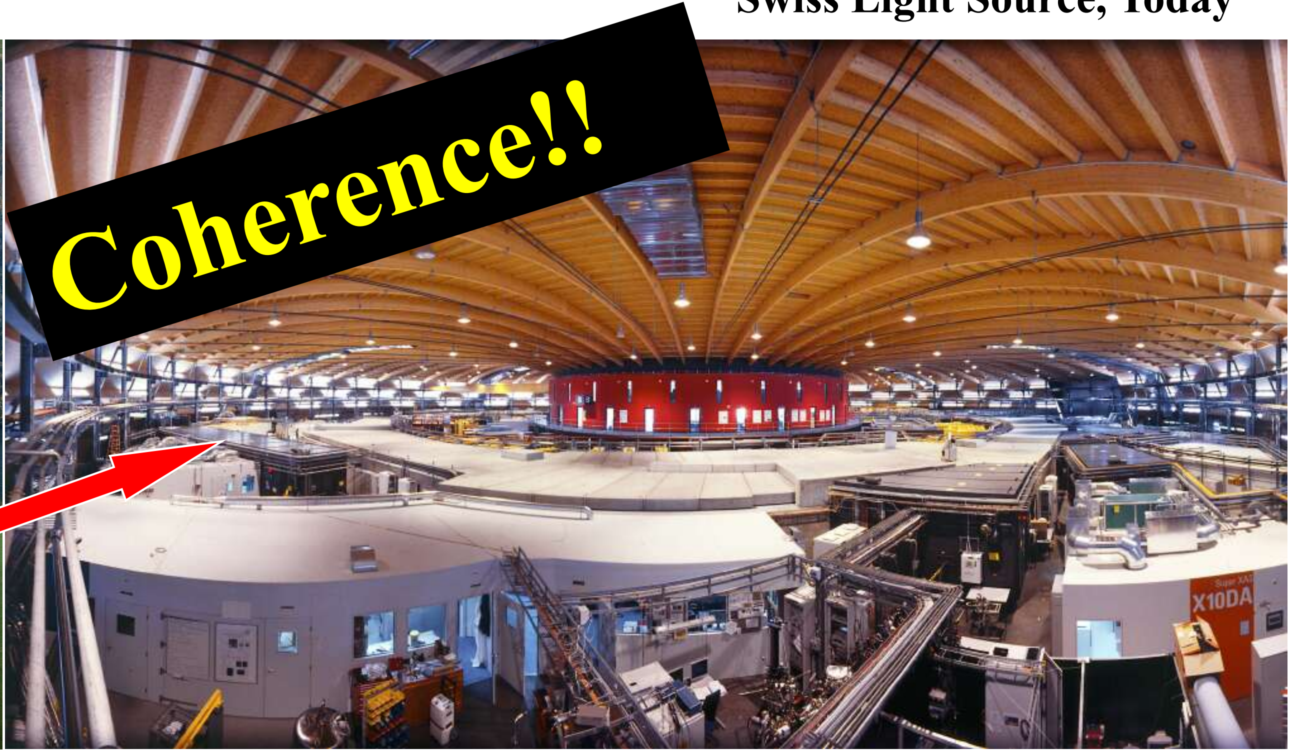
X-ray sources of the 21st century



Röntgen's Lab, Late 19th century

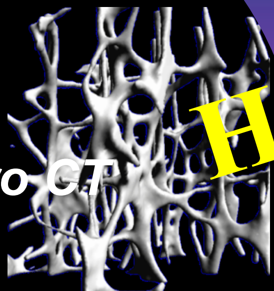
Swiss Light Source, Today



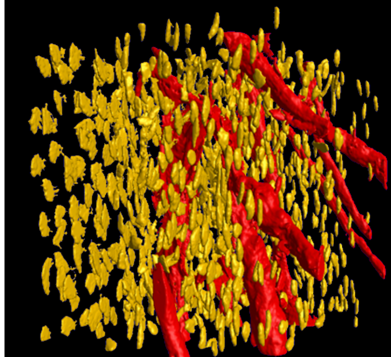


Why a synchrotron for imaging ?

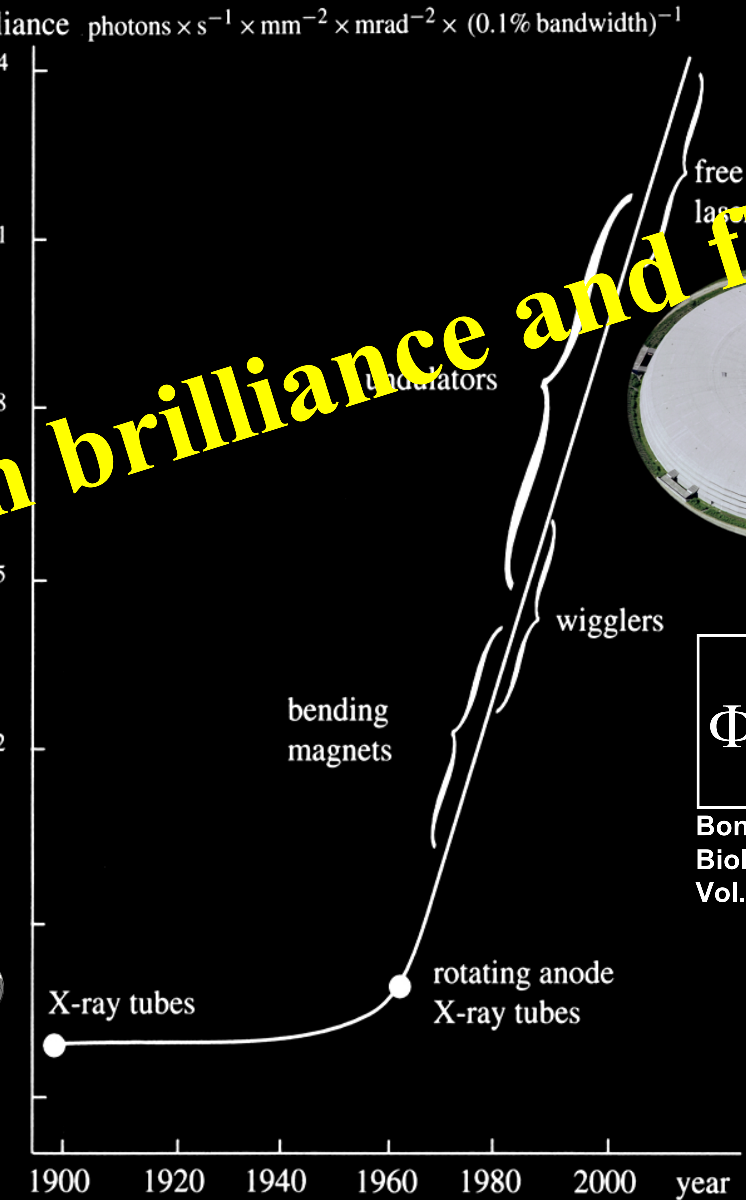
1000-500 µm
Hospital



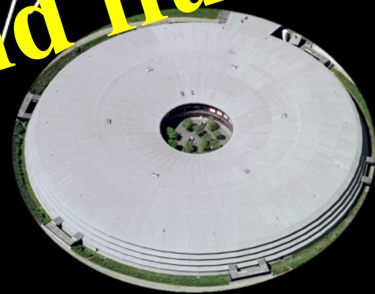
50-10 µm
Table Top micro CT



1-0.1 µm
SR - Tomographic Microscopy



High brilliance and flux!!



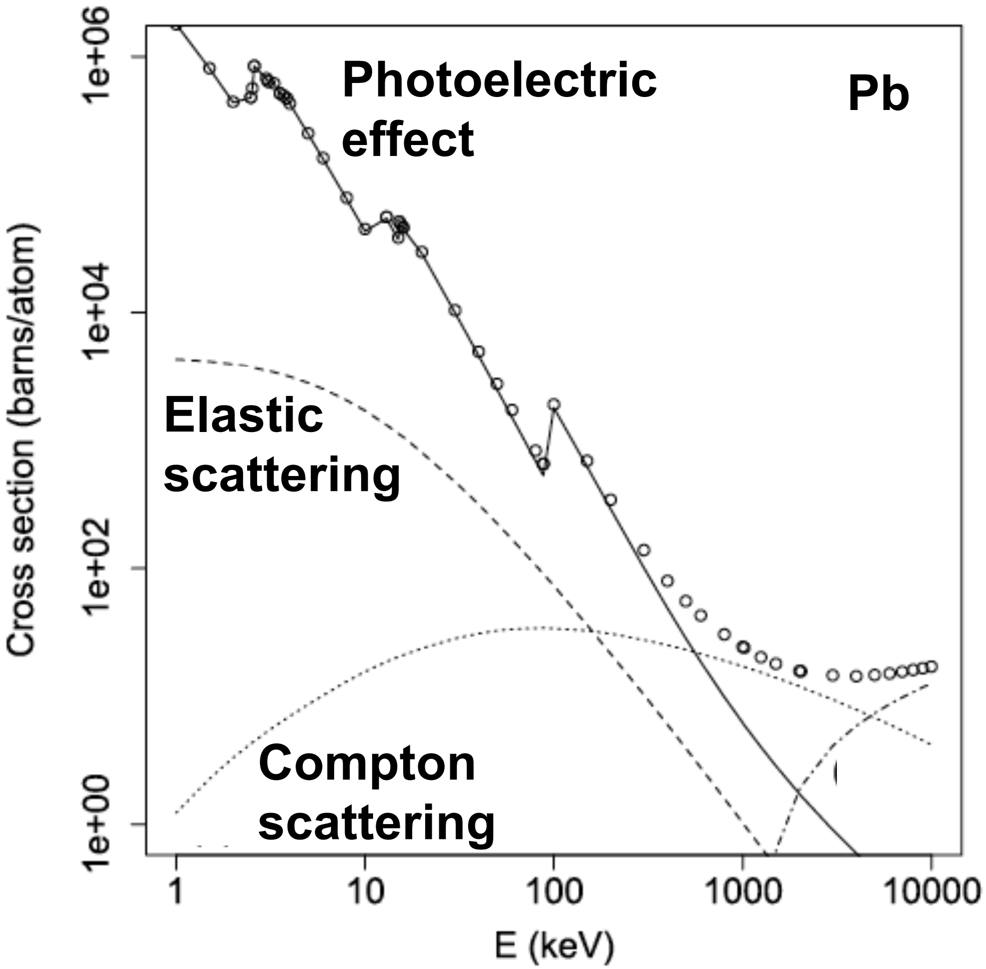
$$\Phi \propto \frac{quality^2}{res^4 \cdot contr^2}$$

Bonse et al., Prog. Biophys. Molec. Biol., Vol. 65, No.1, pp. 133-169, 1996

Mini-tutorial on X-ray tomography

X-ray interaction with matter

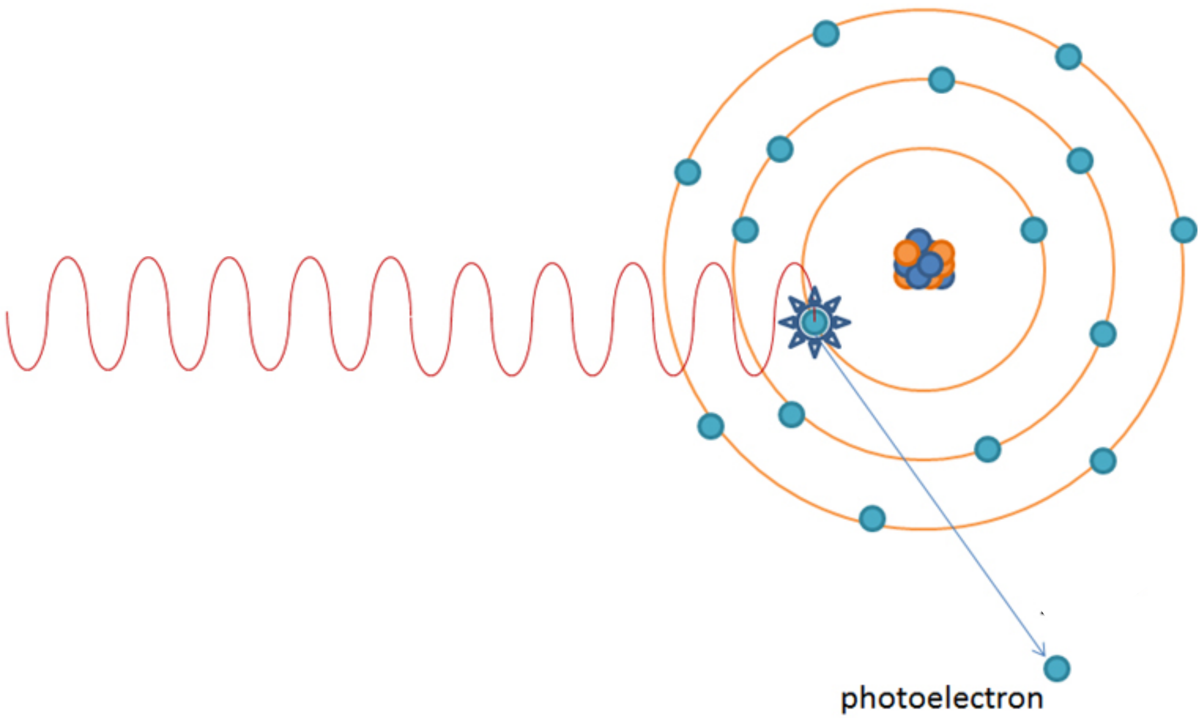
- Photoelectric effect
- Elastic scattering
- Compton scattering



Source: NIST, Bertrand et al, 2012

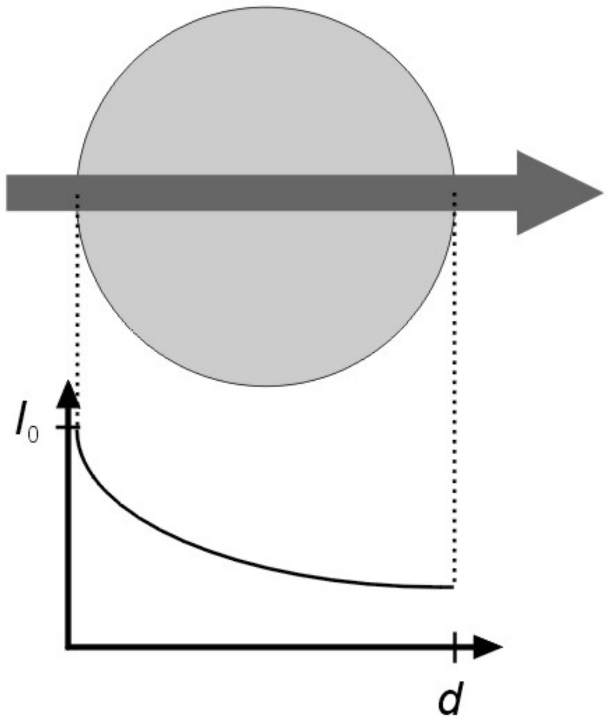
X-ray interaction with matter

- Photoelectric effect



Cross section $\sigma_{ph} \propto \frac{Z^4}{E^3}$

Beer-Lambert law



$$I = I_0 \cdot e^{-\mu d}$$

$$P = ln\frac{I_0}{I} = \mu d$$

$$\mu = \frac{P}{d} = \frac{1}{d} ln\frac{I_0}{I}$$

- Monochromatic radiation
- Homogeneous object

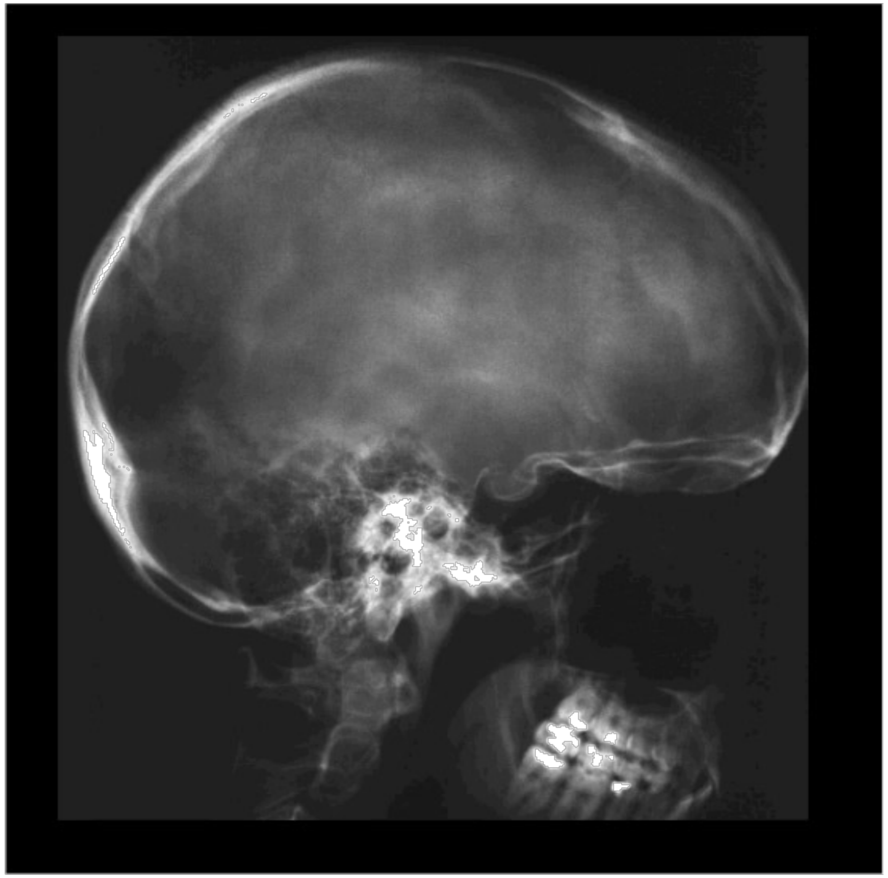
μ = linear attenuation coefficient

$$\mu \propto \sigma = \sigma_{ph} + \sigma_{cs} + \sigma_{rs} + \ \ldots$$

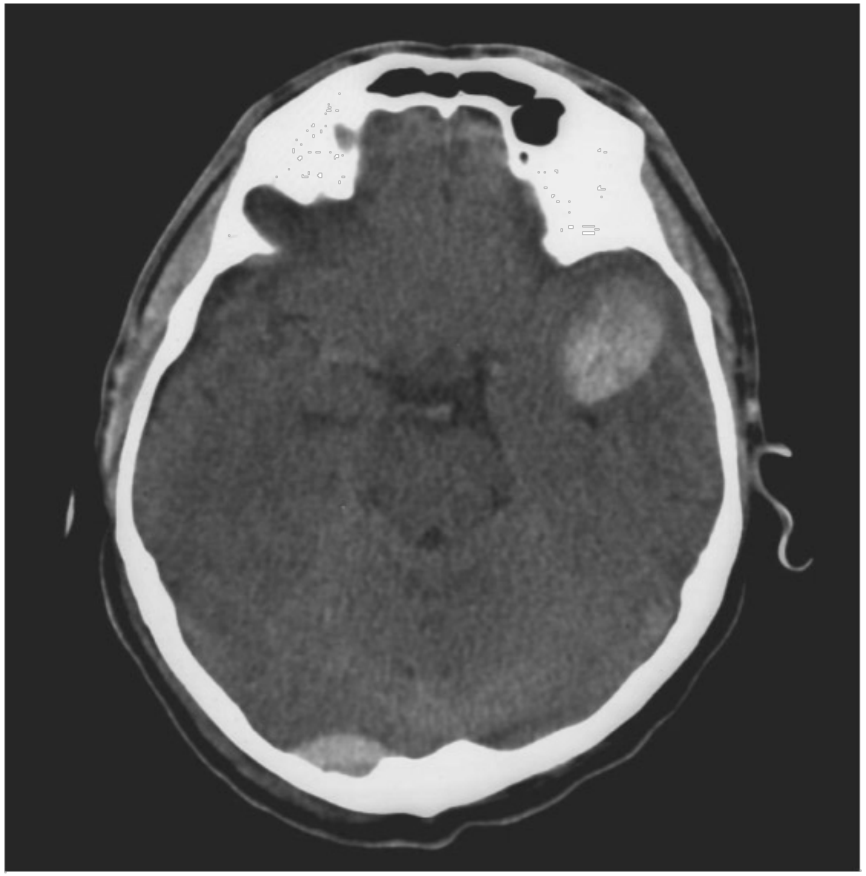
Slicing Imaging – Why ?

- **Tomography** means imaging by sections or slices.

  (From the Greek word *tomos*, meaning "a section" or "a cut”)

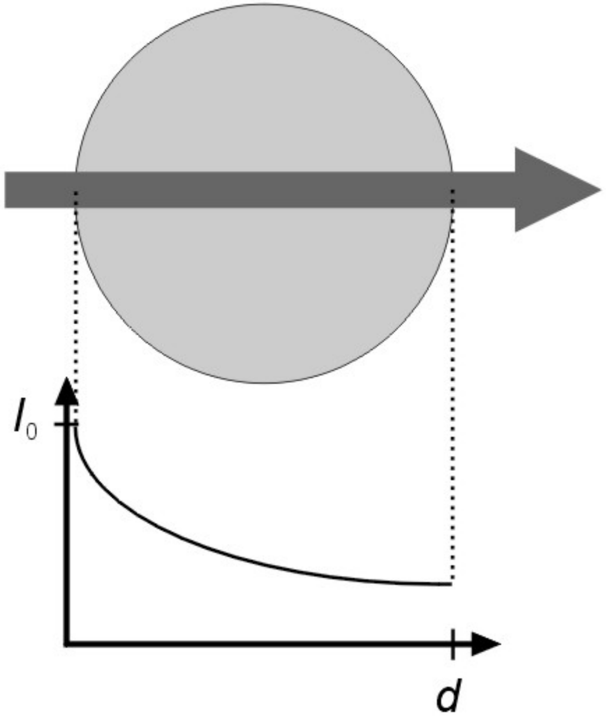


Radiographic projection



Tomographic slice

Beer-Lambert law



$$I = I_0 \cdot e^{-\mu d}$$

$$P = ln \frac{I_0}{I} = \mu d$$

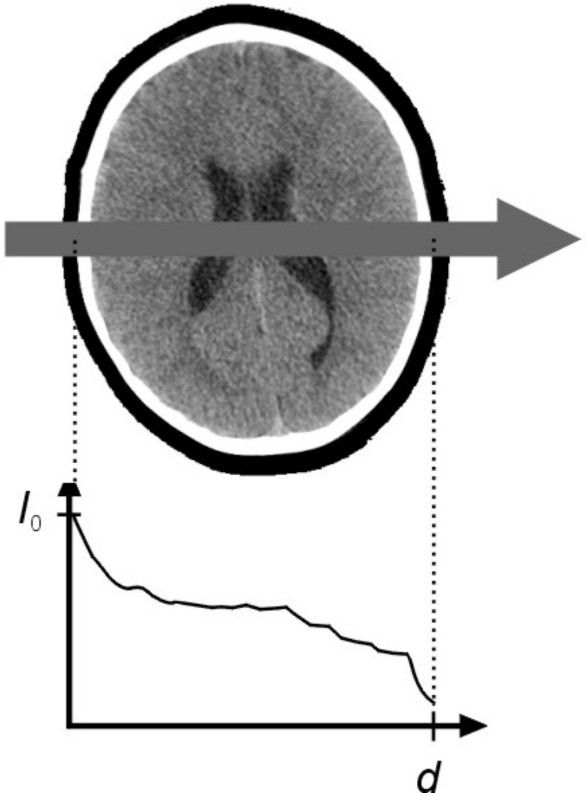
$$\mu = \frac{P}{d} = \frac{1}{d} ln \frac{I_0}{I}$$

- Monochromatic radiation
- Homogeneous object

μ = linear attenuation coefficient

$$\mu = \sigma_{ph} + \sigma_{cs} + \sigma_{rs} + \ ...$$

Beer-Lambert law



$$I = \int_0^{E_{max}} I_0(E) \cdot e^{-\int_0^d \mu(E,z)dz} dE$$

$$\mu(z) = ?$$

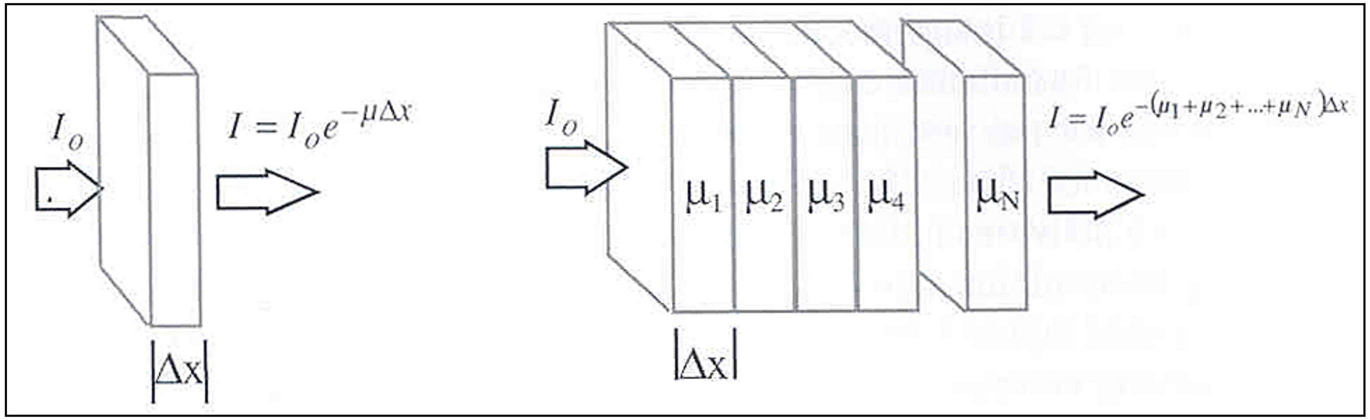
$$I = I_0 \cdot e^{-\int_0^d \mu(z)dz}$$

$$P = ln\frac{I_0}{I} = \int_0^d \mu(z)dz$$

Radon Transform

- Polychromatic radiation
- Inhomogeneous object

Beer-Lambert law



$$I = I_0 \cdot e^{-\mu_1 \Delta x} \cdot e^{-\mu_2 \Delta x} \dots \; e^{-\mu_n \Delta x} = I_0 \cdot e^{-\sum_{n=1}^{N} \mu_n \Delta x}$$

$$P = ln \frac{I_0}{I} = \sum_{n=1}^{N} \mu_n \Delta x$$

?

Tomographic reconstruction algorithms

- Radon Transform
- Fourier Slice Theorem

- Analytical algorithms
  - Direct Fourier Methods
  - Filtered Backprojection

- Iterative algorithms
  - Algebraic
  - Statistical

Radon transform

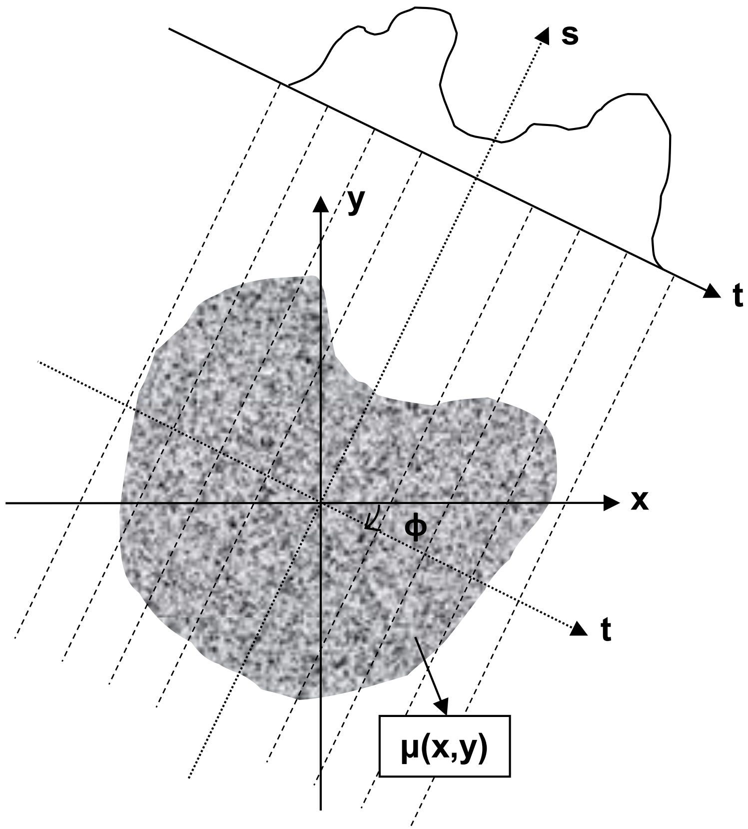
- The Radon transform in 2D is the integral of a function over straight lines and therefore represents the projections data as obtained in a tomographic scan

$$R(t,\phi) = \int_l \mu(\mathrm{x,y})\,dl$$

$$R(t,\phi) = P_\phi(t) := -\ln\left(\frac{I_\phi(t)}{I_0}\right)$$

$$R(t,\phi) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mu(\mathrm{x,y})\,\delta(x\cos\phi + y\sin\phi - t)\,dx\,dy$$

Wanted !

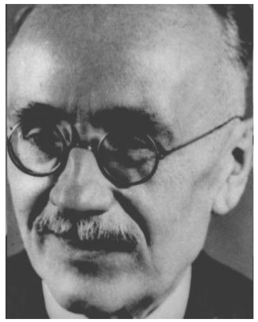


Radon transform

- The Radon transform was introduced by Radon, who also provided a formula for the inverse transform
- The inverse of the Radon transform can be used to reconstruct the original density from the projection data, and thus it forms the mathematical underpinning for tomographic reconstruction
- Radon found the solution to the tomographic problem already in 1917
  - but assumes an **infinite** number of projections and **continuous** projection functions
  - while we only have a **finite** number of projections and a **finite** number of detector points.

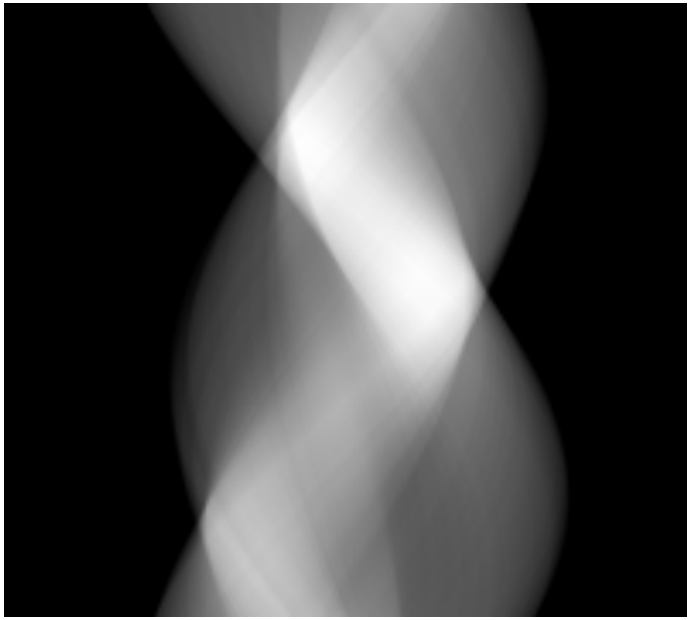
$$R(t,\phi)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mu(x,y)\,\delta(x\cos\phi+y\sin\phi-t)\,dx\,dy$$

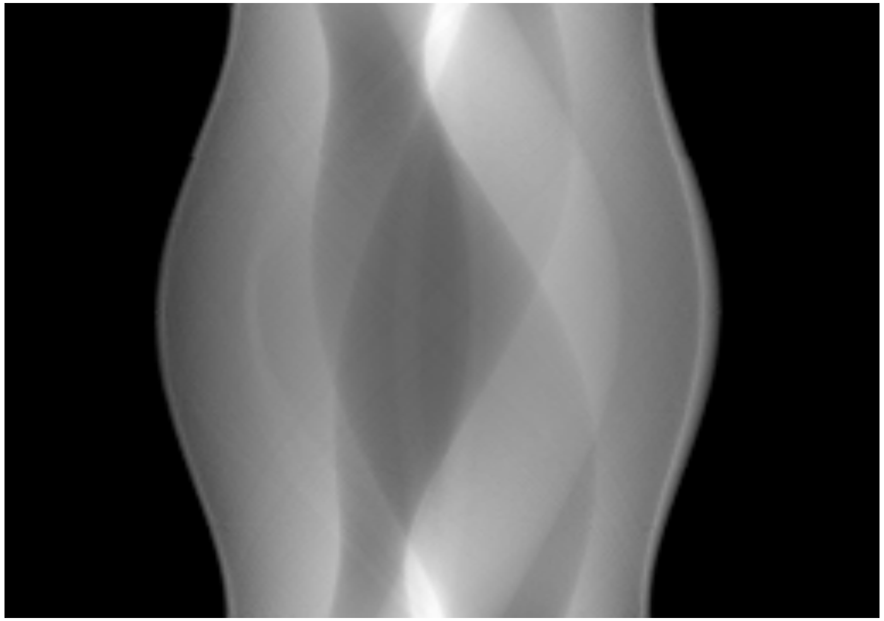
Wanted !



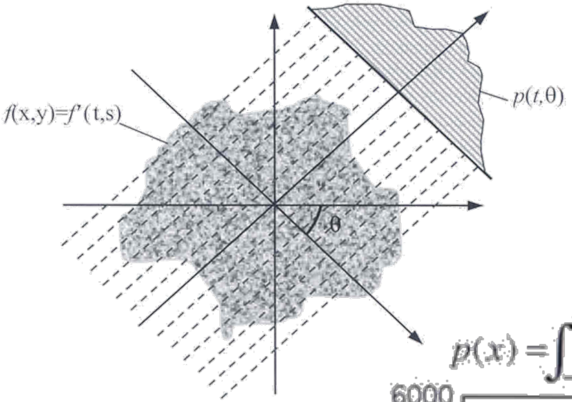
Radon transform

- The Radon transform is often called a **sinogram** because the Radon transform of a Dirac delta function is a distribution supported on the graph of a sine wave

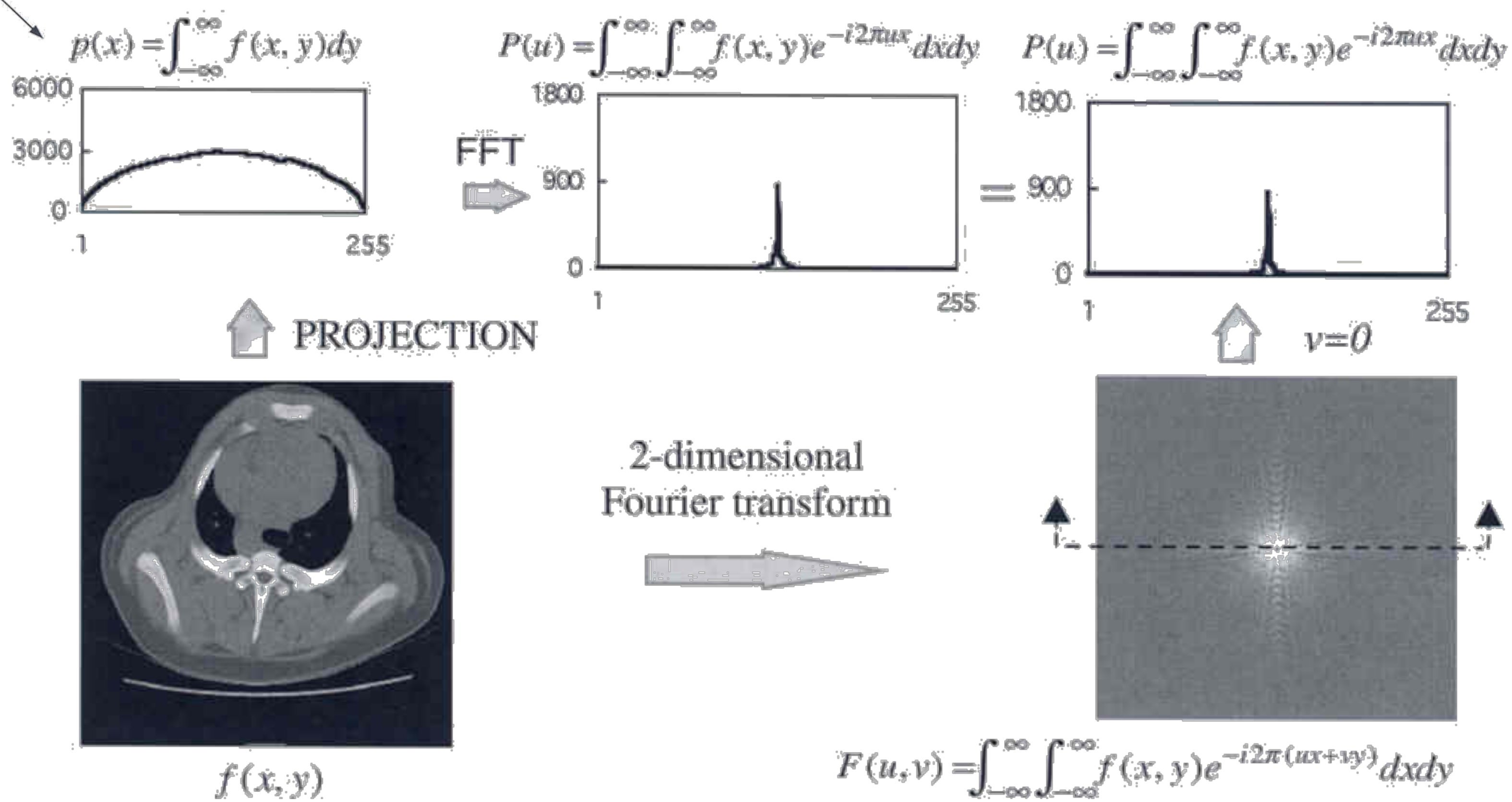




Fourier Slice Theorem

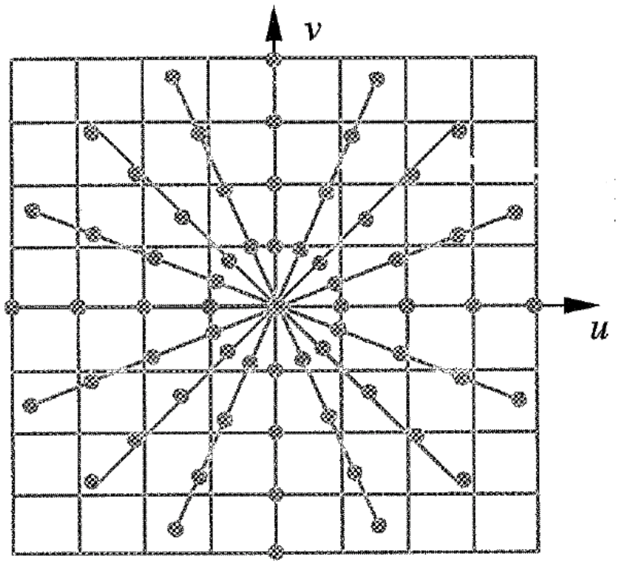


The Fourier transform of a parallel projection of an object f(x,y) obtained at an angle θ equals a line of the 2D Fourier transform of f(x,y) taken at the same angle.



Direct Fourier Methods

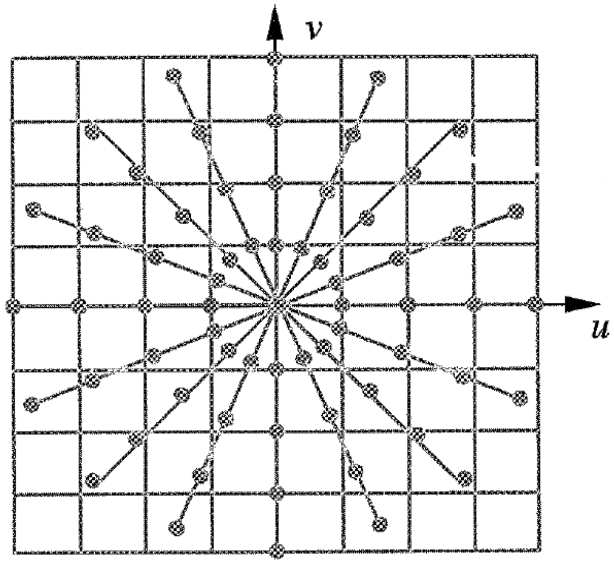
- 1D FFT of the projections
- Filtering
- Resampling
- 2D inverse FFT



Direct Fourier Methods - resampling

- Interpolation
  - Limited accuracy

- Gridding
  - Most accurate Fourier reconstruction method
  - Mapping by convolution with FFT of *w(x,y)*



- Gridrec
  - W(x): 1D Prolate Spheroidal Wave Functions (PSWF) of zeroth order
  - W(x,y): maximally concentrated in a square region of interest
  - FFT(w(x,y)): concentrated as much as possible around 0
  - PSWF: calculated using rapidly converging expansion in terms of Legendre polynomials
  - PSWF and FFT (PSWF) can be efficiently computed and stored at run time

Dowd et al., 1999; Marone et al., 2010

Filtered backprojection (mathematical background)

$$-\ln\left(\frac{I_\phi(r)}{I_0}\right) := P_\phi(r) = R(t,\phi) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mu(\mathrm{x},\mathrm{y})\,\delta(x\cos\phi + y\sin\phi - t)\,dx\,dy$$

Image function:

$$\mu(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{M}(u,v)\,e^{j2\pi(ux+vy)}\,du\,dv$$

Coord. transform:

Cartesian to Polar

$$\mu(x,y) = \int_0^{2\pi} d\theta \int_0^{\infty} \tilde{M}(\omega\cos\theta,\omega\sin\theta)\,e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\,|g|\;d\omega$$

with
$$\begin{cases} u = \omega\cos\theta \\ v = \omega\sin\theta \end{cases}$$
and
$$g = \begin{pmatrix} \frac{\partial u}{\partial \omega} & \frac{\partial u}{\partial \theta} \\ \frac{\partial v}{\partial \omega} & \frac{\partial v}{\partial \theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\omega\sin\theta \\ \sin\theta & \omega\cos\theta \end{pmatrix}$$

Fourier Slice Theorem:

$$\mu(x,y) = \int_0^{2\pi} d\theta \int_0^{\infty} \tilde{P}(\omega,\theta)\,e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\,\omega\,d\omega$$

Symmetry properties:

$$\tilde{P}(\omega,\theta+\pi) = \tilde{P}(-\omega,\theta)$$

Image function:

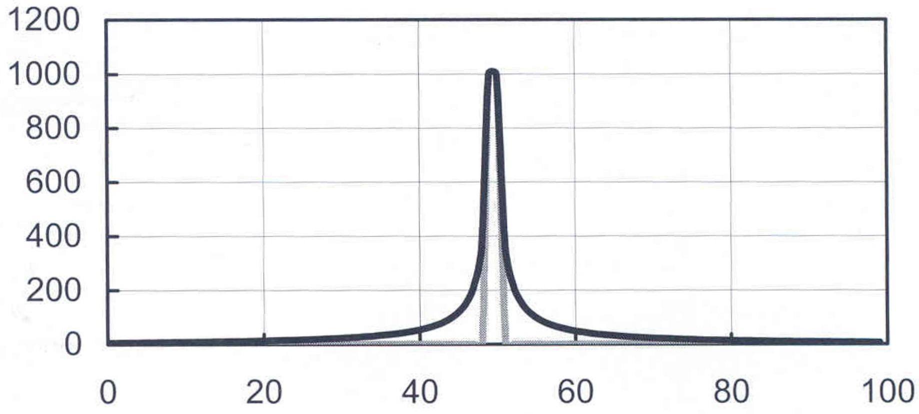
$$\mu(x,y) = \int_0^{\pi} d\theta \int_{-\infty}^{\infty} \tilde{P}(\omega,\theta)\,|\omega|\,e^{j2\pi\omega t}\,d\omega$$

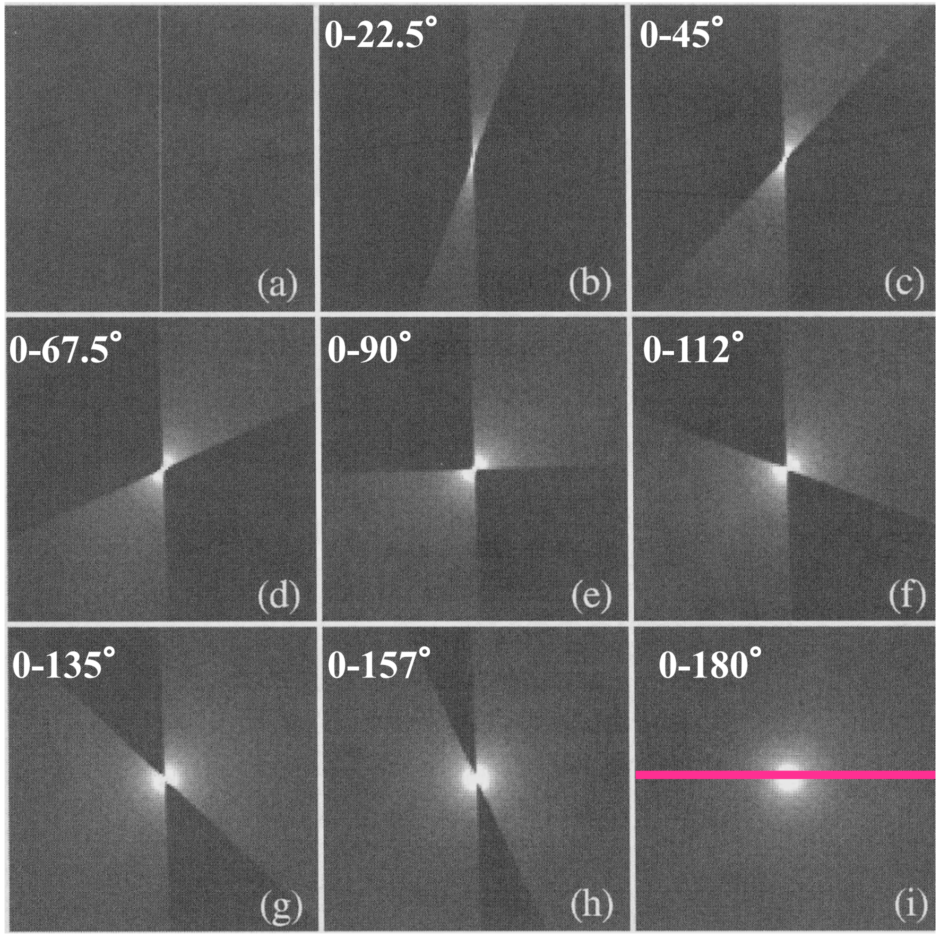
A. C. Kak, M. Slaney, Principle of Computerized Tomographic Imaging", SIAM Classics in Applied Mathematics 33, New York, 2001, ISBN 0-89871-494-X

Reconstruction using backprojection

- If **no a priori** information is known, the intensity of the object is assumed to be uniform along the beam path.
- The projection intensity is evenly distributed among all pixels along the ray path

→ **Concept of backprojection!**



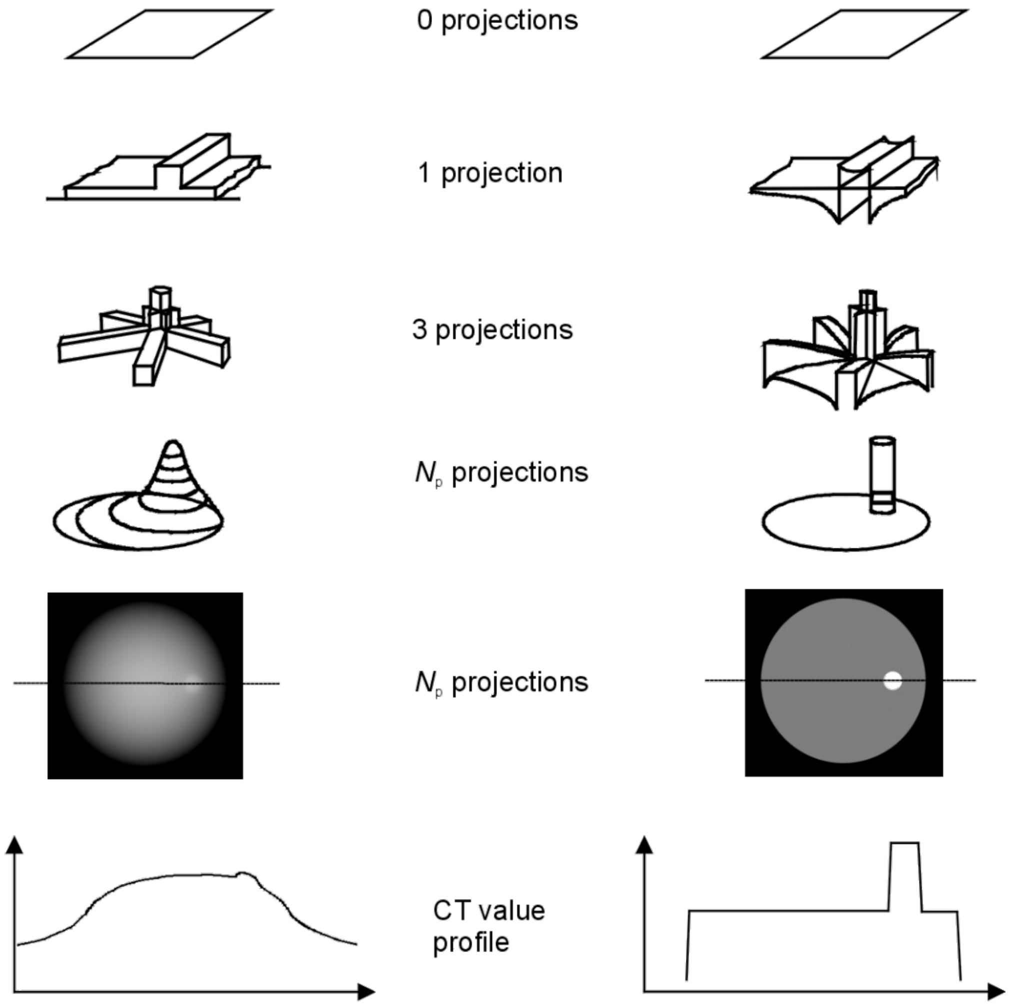


Backprojection of a point …

Image reconstruction

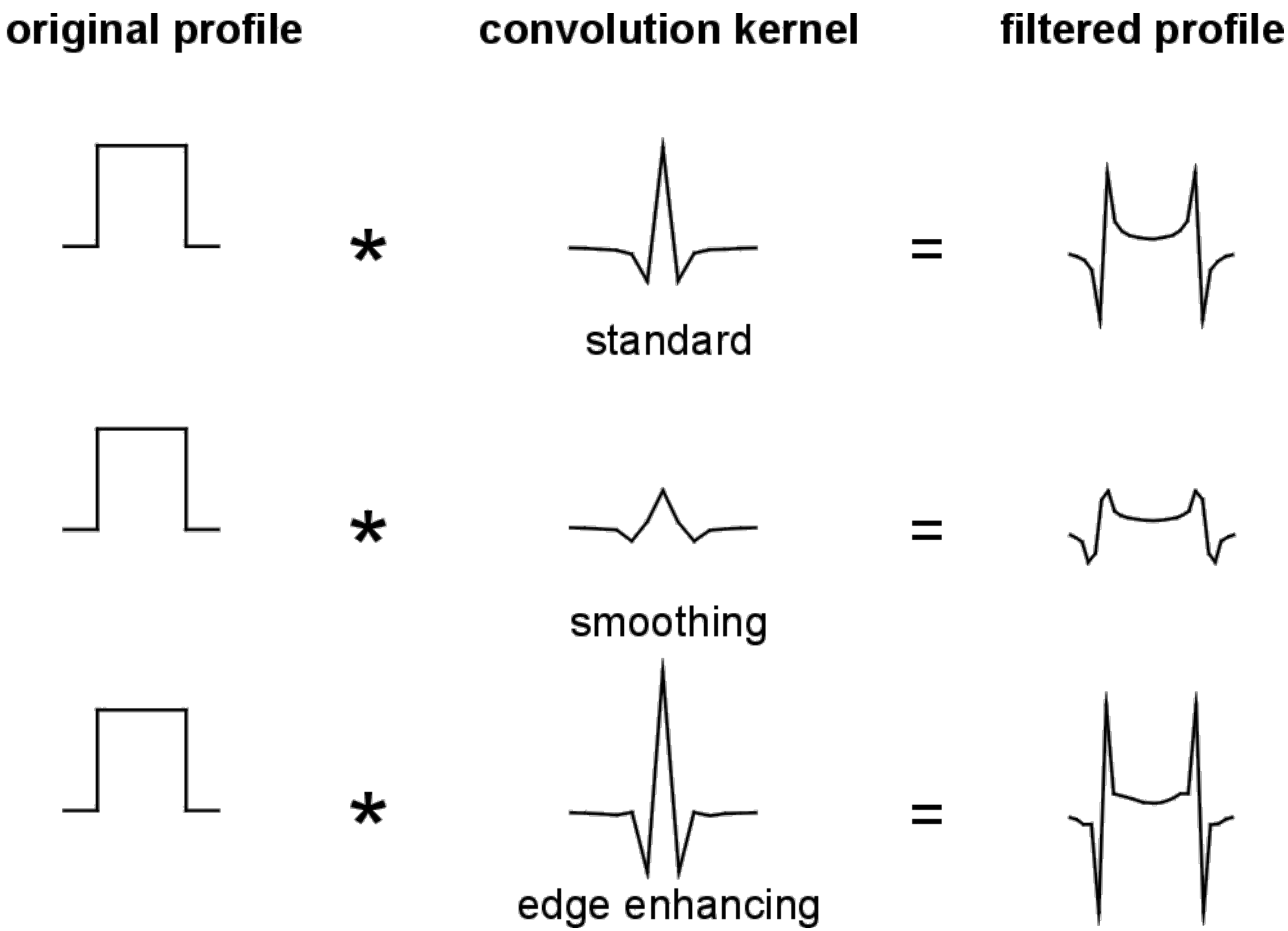
$$\mu(x,y)=\int_0^{\pi} d\theta \int_{-\infty}^{\infty} \tilde{P}(\omega,\theta)\left|\omega\right| e^{j2\pi\omega t}\, d\omega$$

without pre-filtering | **with filtering**



Filtered backprojection

- Image characteristic can be influenced by the choice of a convolution kernel, whereby increasing spatial resolution or edge enhancement also means increasing image noise !



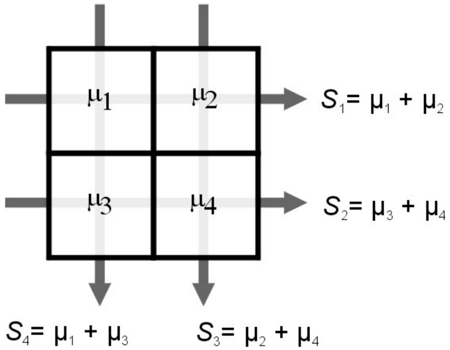
Analytical algorithms - Summary

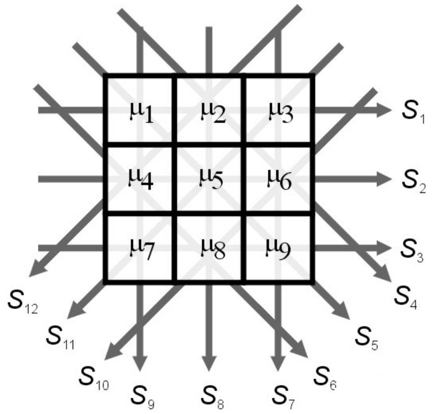
- ✓ Simple and fast

- ✖ Need large number of projections
  - In the order of the number of detector rows
- ✖ Cannot include a-priori information
- ➢ Not suitable for under-sampled datasets

Direct inversion (historic)

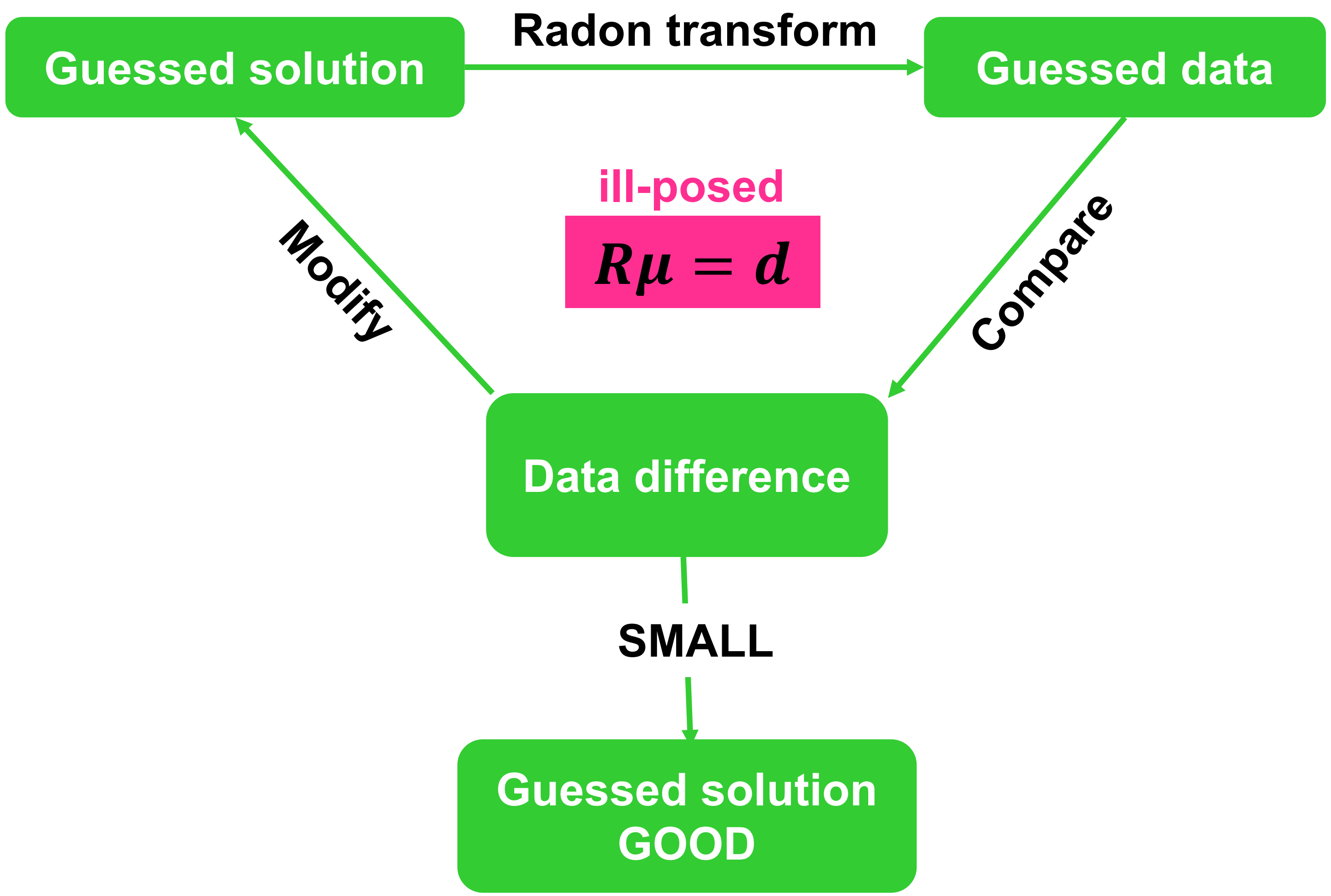
- Brute force approach
  - N equations with N unknowns
  - Need more than N^2 equations to ensure linear independence



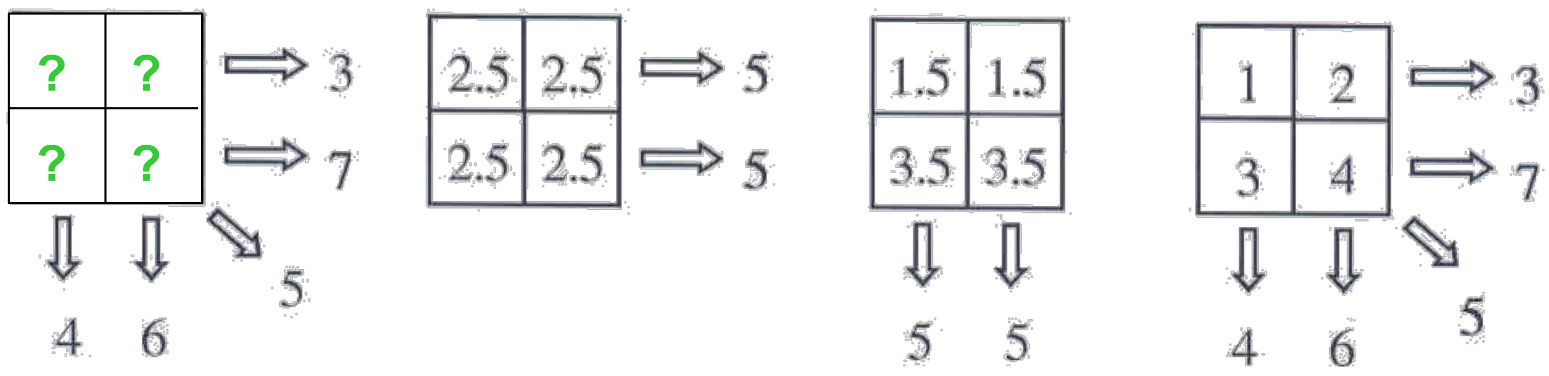


- Sir Godfrey Hounsfield (in the late 60s)
  - Reconstructed the first human head
  - Solved 28000 equations simultaneously
  - Nobel Prize in 1979 together with A. Cormack

Iterative algorithms



Iterative algorithms



Algebraic ReconstructionTechnique (ART)

Iterative algorithms

ill-posed

$$\boldsymbol{R\mu = d}$$

Optimization problem

Cost function

$$\underset{\mu}{\operatorname{argmin}}\, f(\mu, d) + g(\mu)$$

Fidelity term (→ $f(\mu, d)$)

Regularization term (→ $g(\mu)$)

Iterative algorithms

Optimization problem

Cost function

$$\underset{\mu}{\operatorname{argmin}}\, f(\mu, d) + g(\mu)$$

Fidelity term (→ $f(\mu, d)$)

Regularization term (→ $g(\mu)$)

- Stability
- A priori knowledge

Algebraic methods (ART,SIRT,…) $f(\mu, d) = \frac{1}{2}||R\mu - d||_2^2$ $\quad g(\mu) = 0$

Tikhonov – small norm $\quad g(\mu) = \lambda||\mu||_2^2$

Lasso – sparsity $\quad g(\mu) = \lambda||\mu||_1$

Total Variation (TV) – piecewise constant
Preserve edges $\quad g(\mu) = \lambda||\nabla\mu||_1$

Dictionary, Nonlocal means, Nonlocal TV, …

Iterative algorithms

$$\bar{N}_i = N_0^i \cdot e^{-[R\mu]_i}$$

N – expected photon counts
i – each pixel in each view

Optimization problem

Cost function

$$\underset{\mu}{\operatorname{argmin}} f(\mu, d) + g(\mu)$$

Fidelity term

Regularization term

Iterative algorithms

Optimization problem

Cost function

$$\underset{\mu}{\operatorname{argmin}}\, f(\mu, d) + g(\mu)$$

Fidelity term (→ $f(\mu, d)$)

Regularization term (→ $g(\mu)$)

- Stability
- A priori knowledge

Statistical methods

Negative log-likelihood

$$L(\mu|N) = \sum_i N_i \log \bar{N}_i - \bar{N}_i$$

Many cost functions (model geometries, artifacts, …)

↓

Many possibilities

Iterative algorithms

- **FIRST STEP** - Build the cost function

- **SECOND STEP** – Find minimum/maximum
  - Optimization techniques
    - Linear/non-linear
    - Least-squares
    - Convex/non-convex
    - Constrained/unconstrained

  - Gradient methods
  - (Gauss-)Newton methods
  - Lagrangian methods
  - Expectation-maximization algorithms

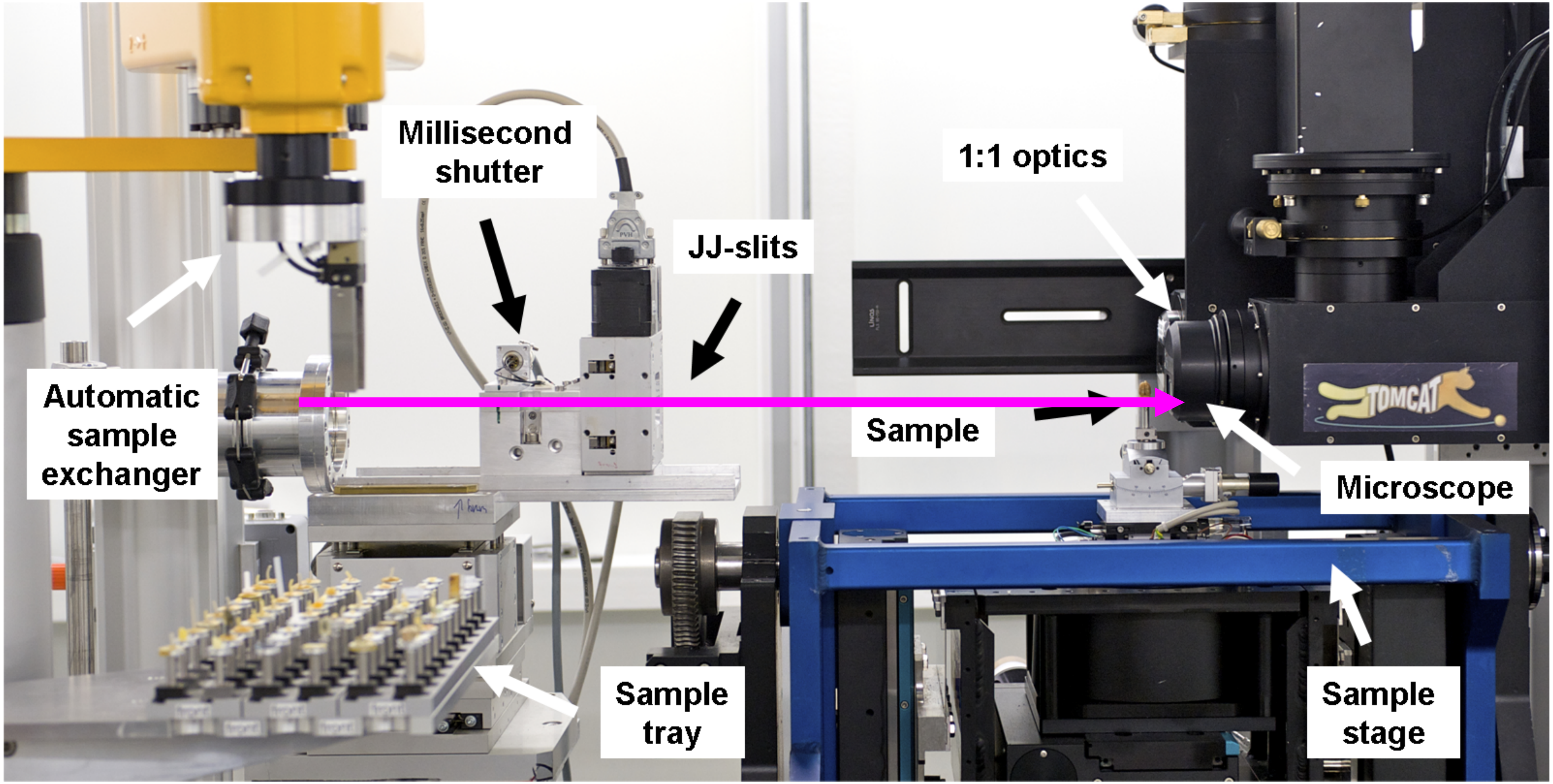
Iterative algorithms - Summary

- ✓ Can include a-priori information
- ✓ Flexible – can model almost anything
- ✓ Suitable for under-sampled datasets

- ✖ Computational intensity
- ✖ Highly dataset specific
- ✖ Parameters to tune

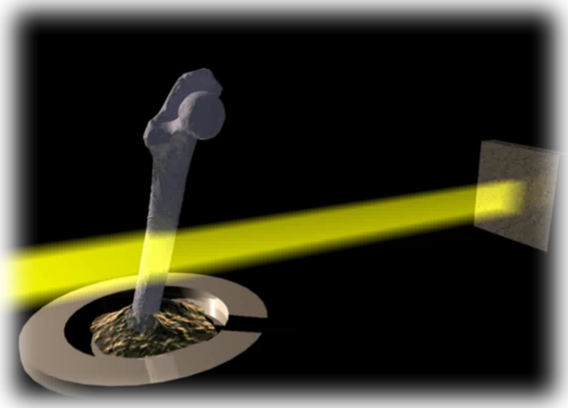
Examples

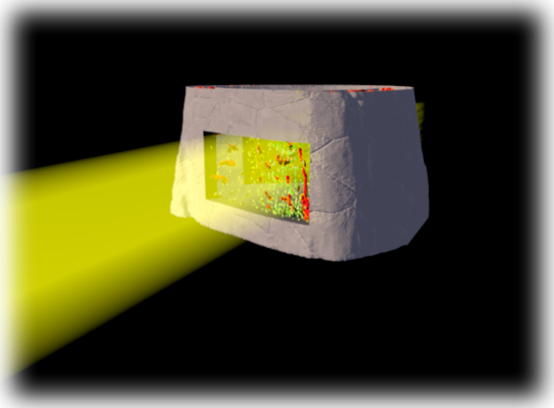
State-of-the-art SRXTM (1-50 um)

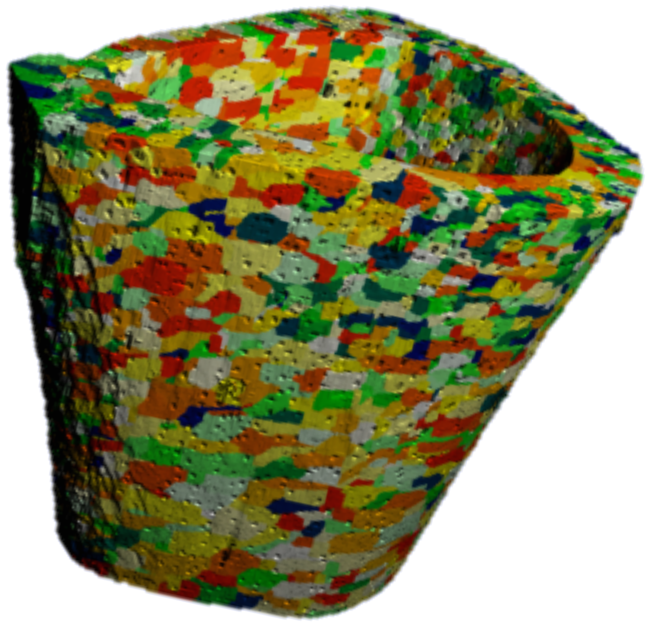


1 micron resolution routinely achieved at 10% MTF

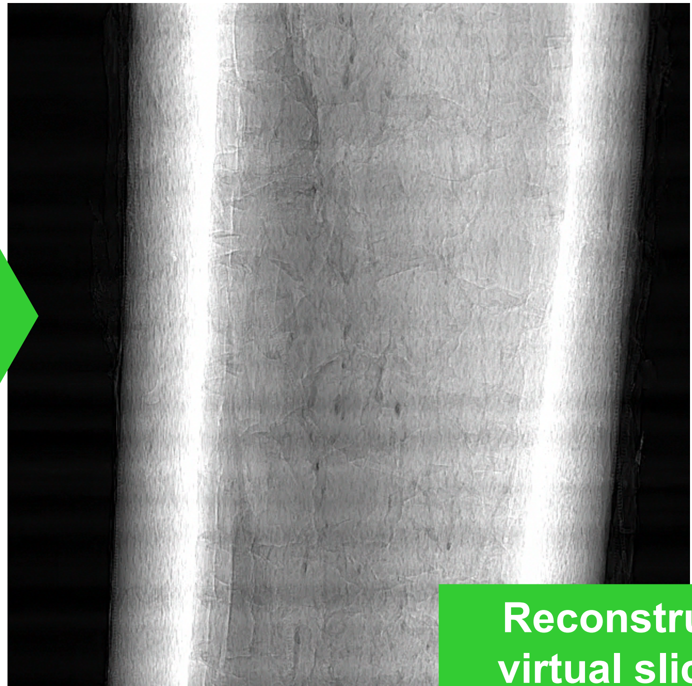
Gathering complex information



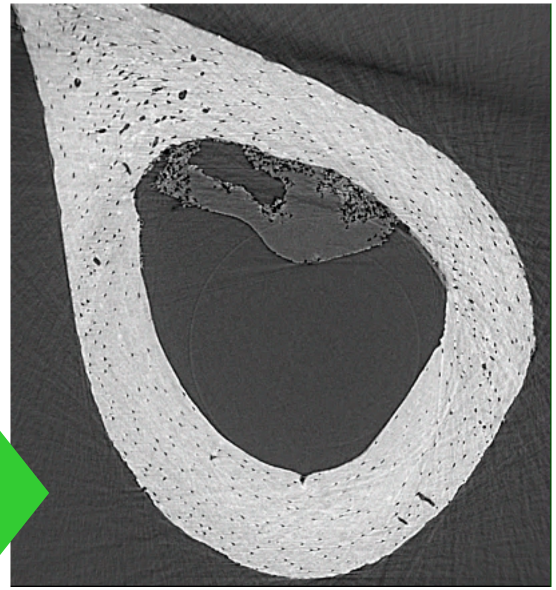




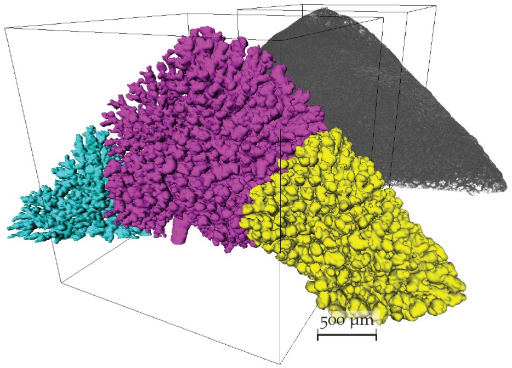
Measure
X-ray projections

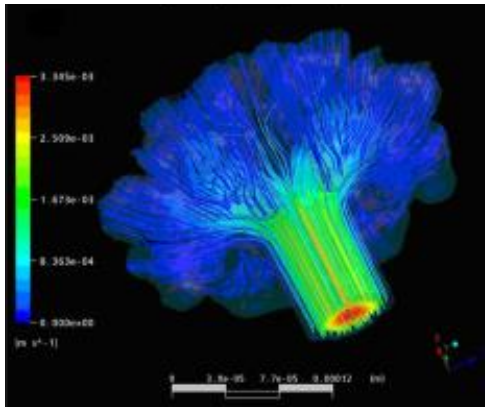


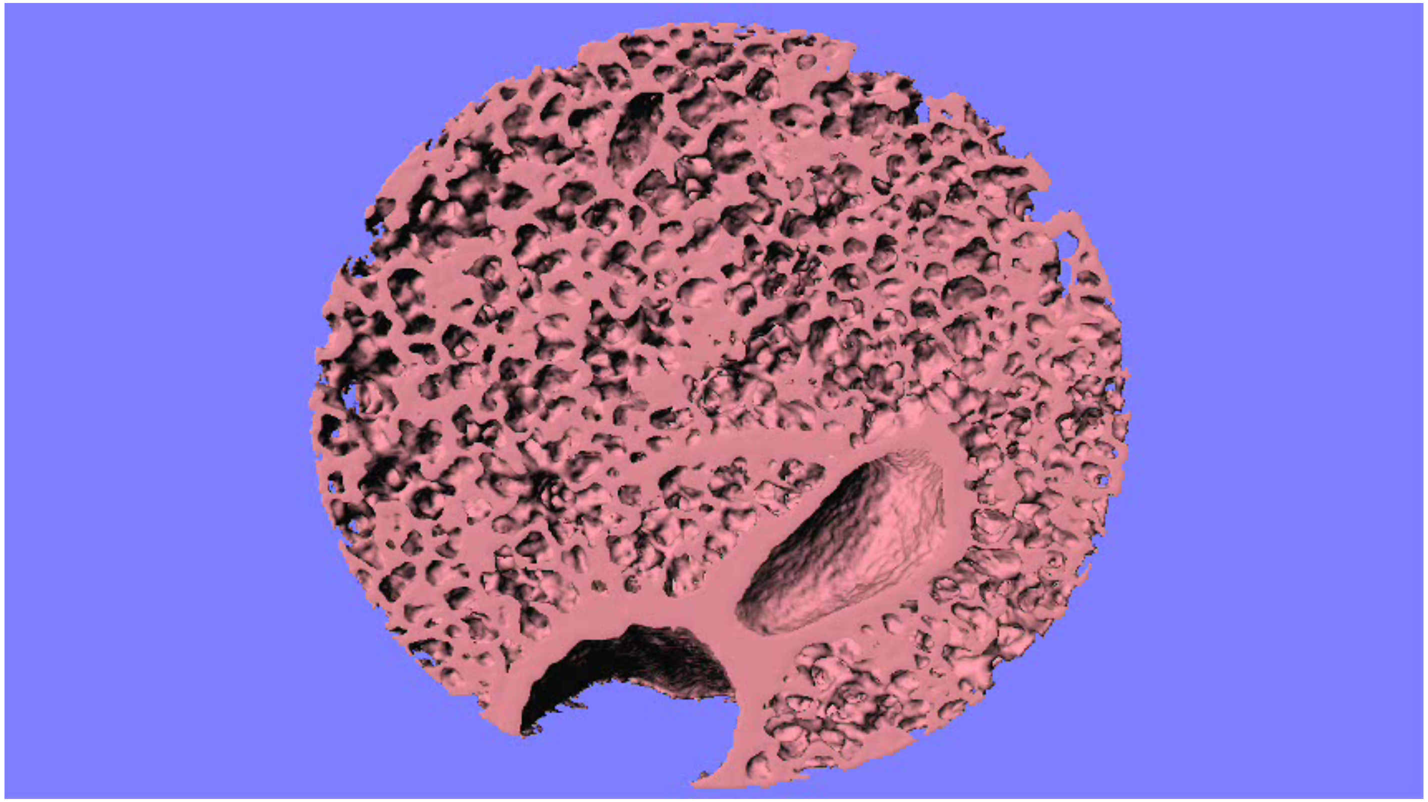
Reconstruct
virtual slices



Looking into a very small region of the lung



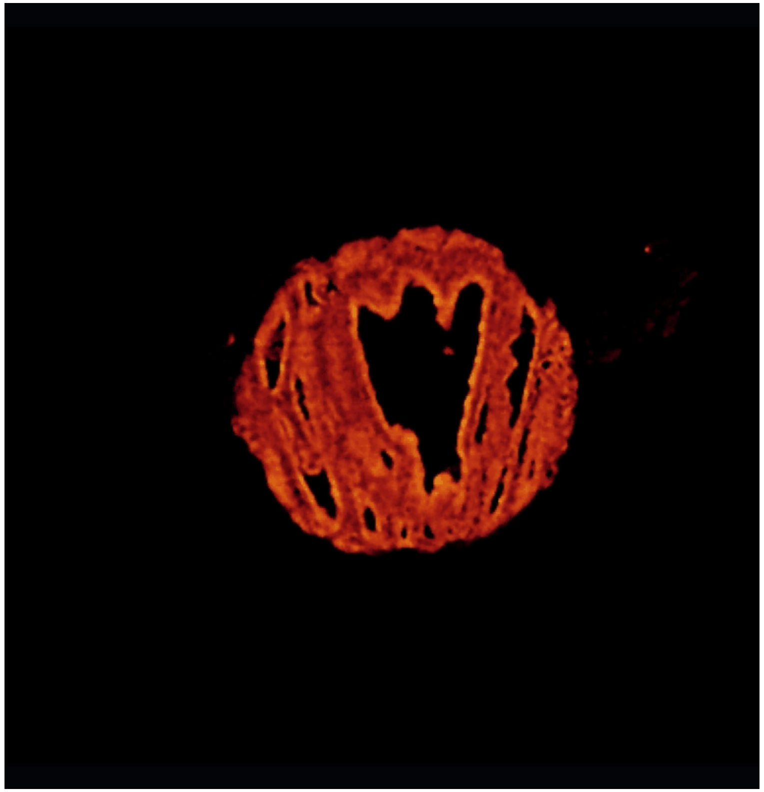


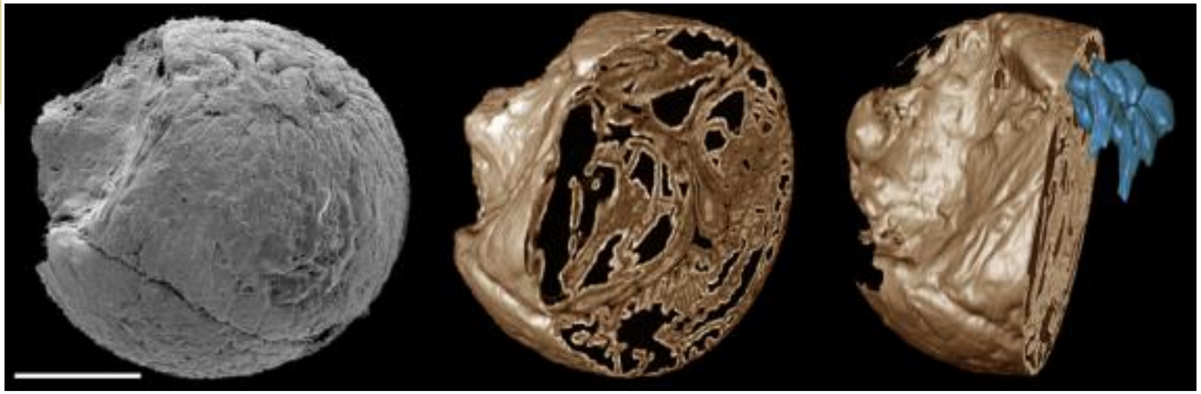


Morphology of lung acini

Haberthür et al., Journal of Synchrotron Radiation, 17(5), 2010
Schittny et al., American Journal of Physiolgy 294 (L246), 2008

The first predator on Earth: 500 Myears ago

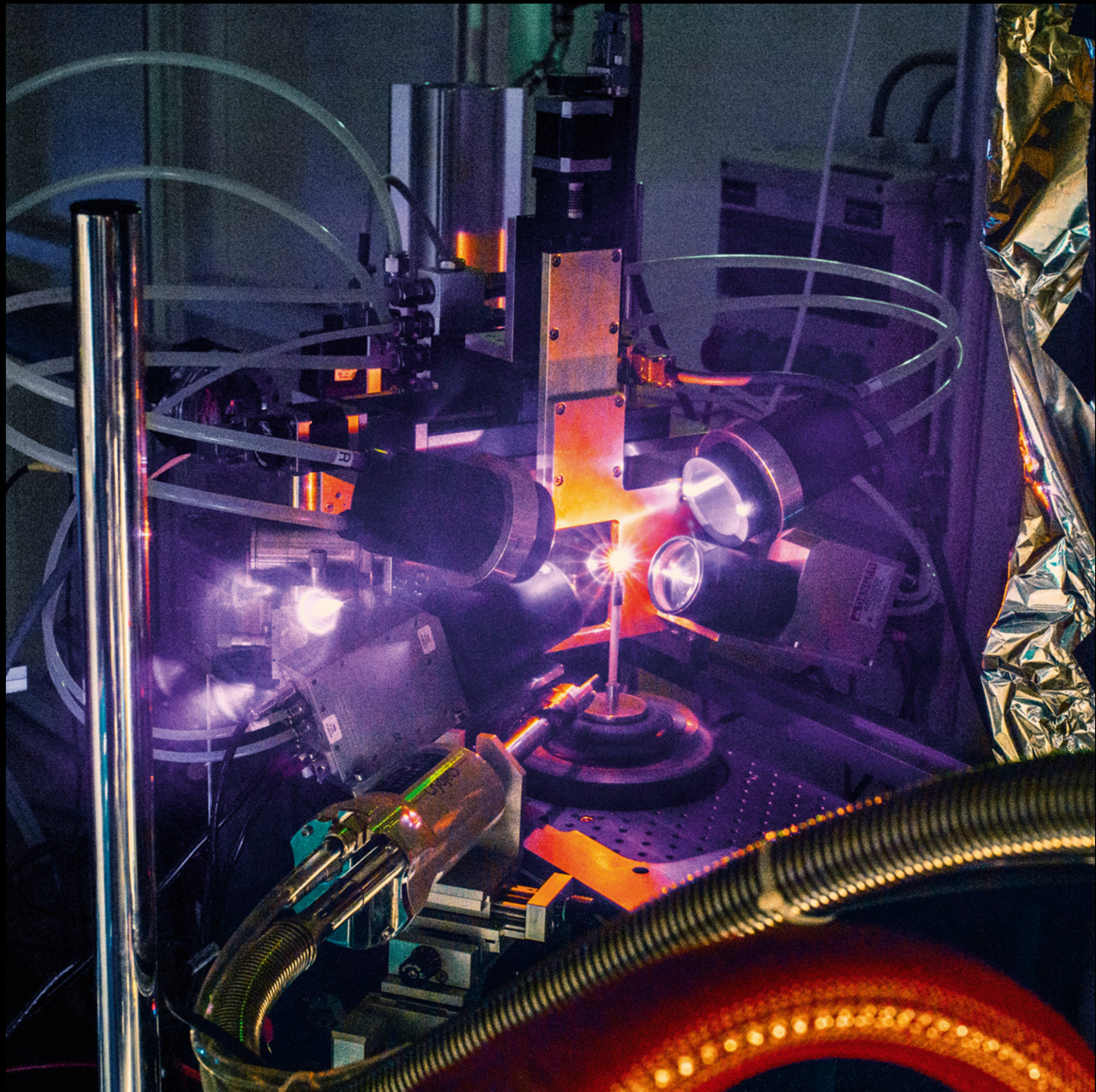




Tomographic microscopy of fossil materials

P. Donoghue et al., Nature 442, Aug. 2006

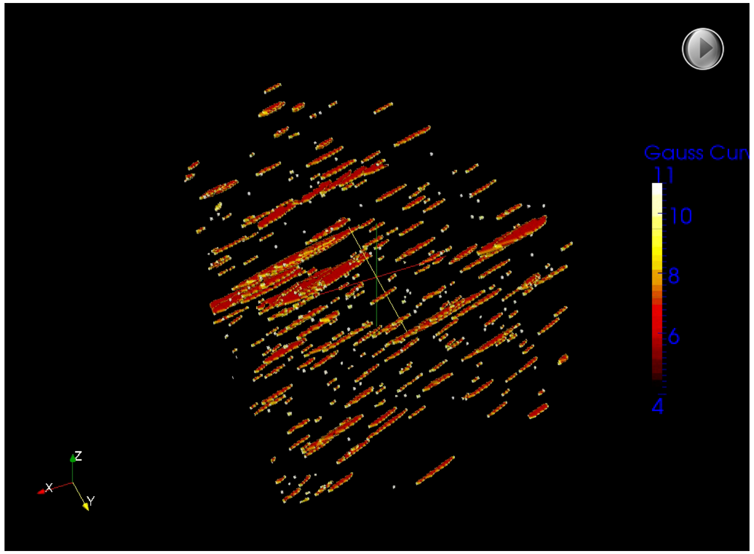
Mimicking volcano eruptions



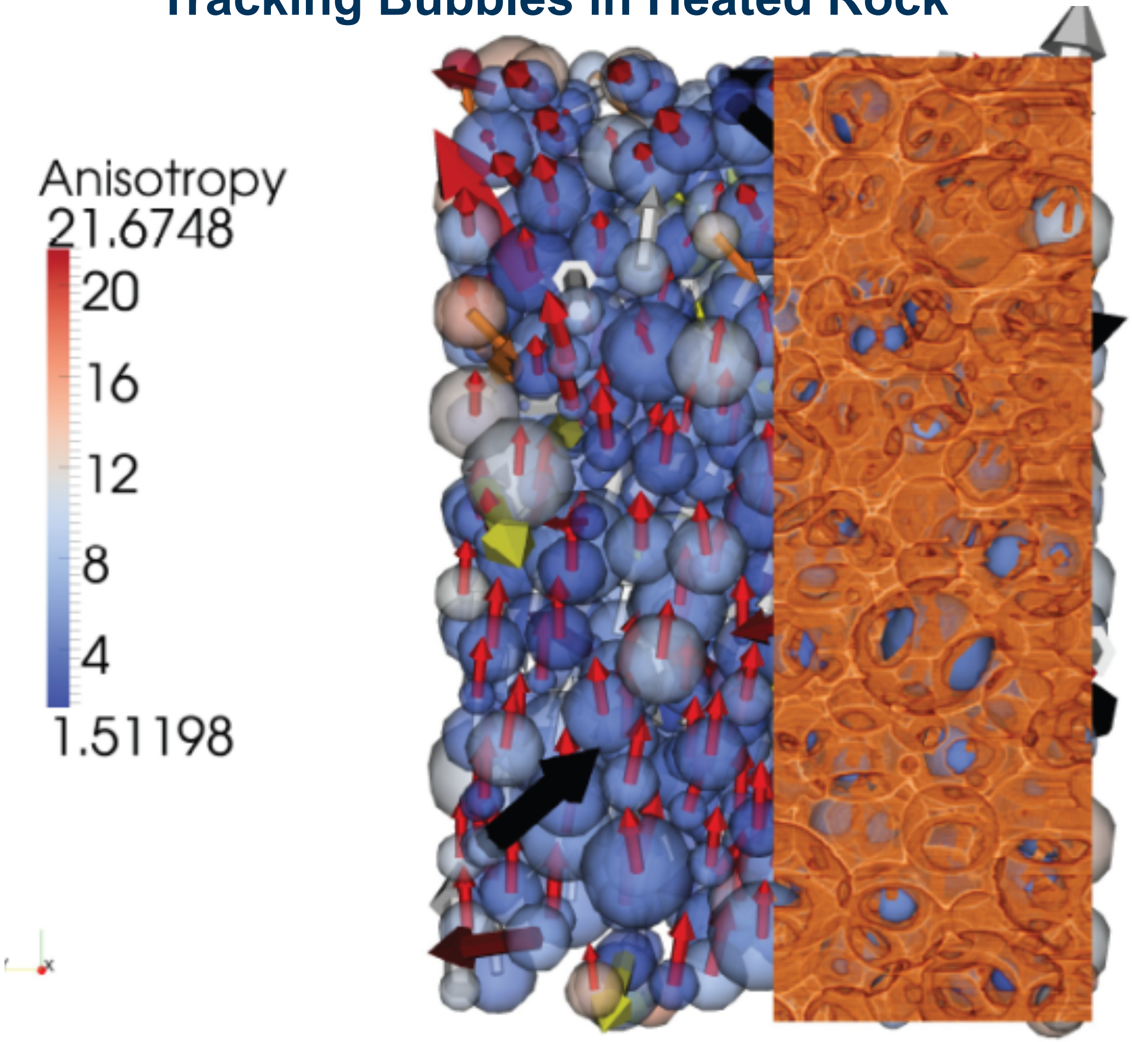
Mimicking volcano eruptions

2 cross firing 150W, 980nm cw, class 4 lasers

Gas Evolution in Heated Volcanic Rock



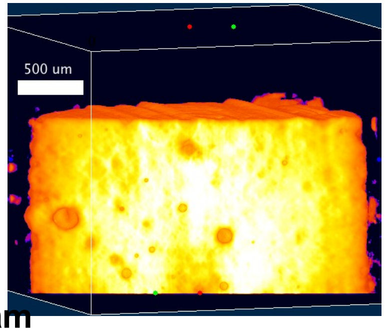
Tracking Bubbles in Heated Rock



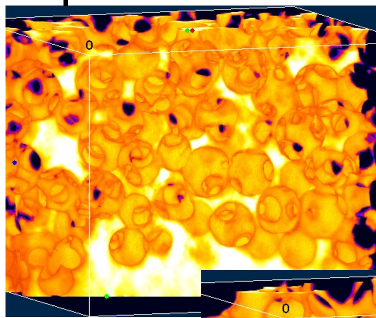
Thanks to M. Pistone (ETHZ) and J. Fife (PSI)

3D follow-up of dynamic processes

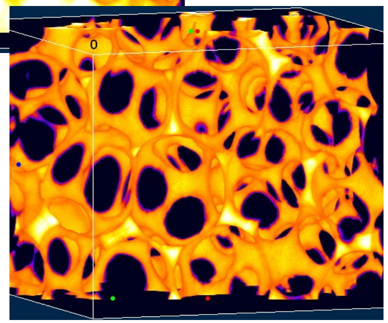
Nucleation

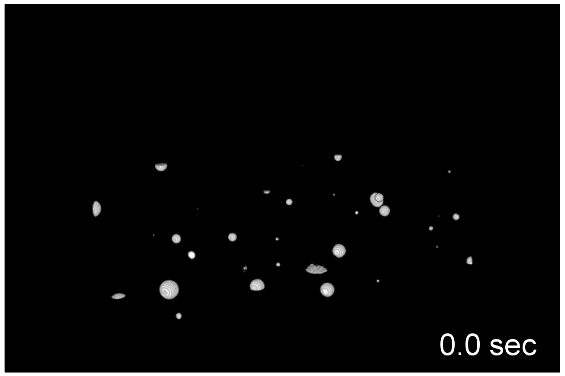


”Liquid” foam



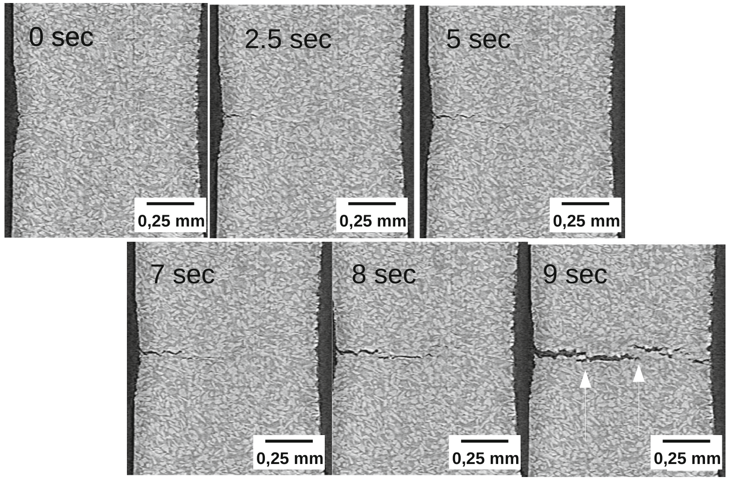
Solid foam

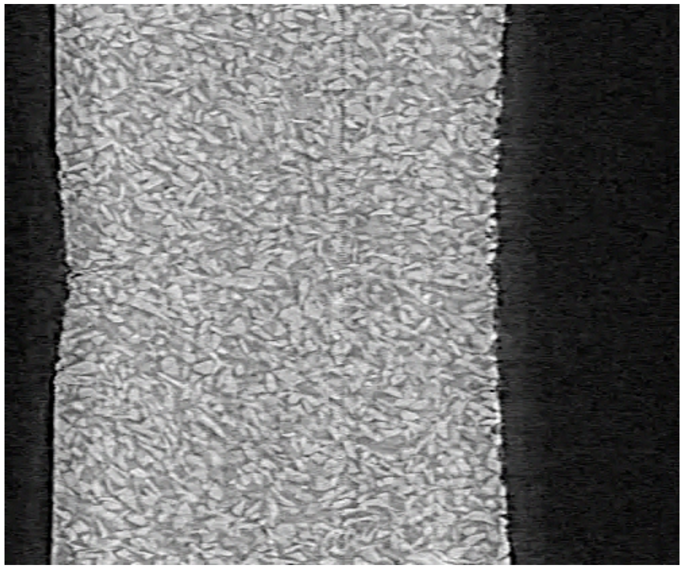




Data courtesy of E. Solorzano and S. Alonso, Univ. Valladolid

In-situ 20 Hz tomographic imaging





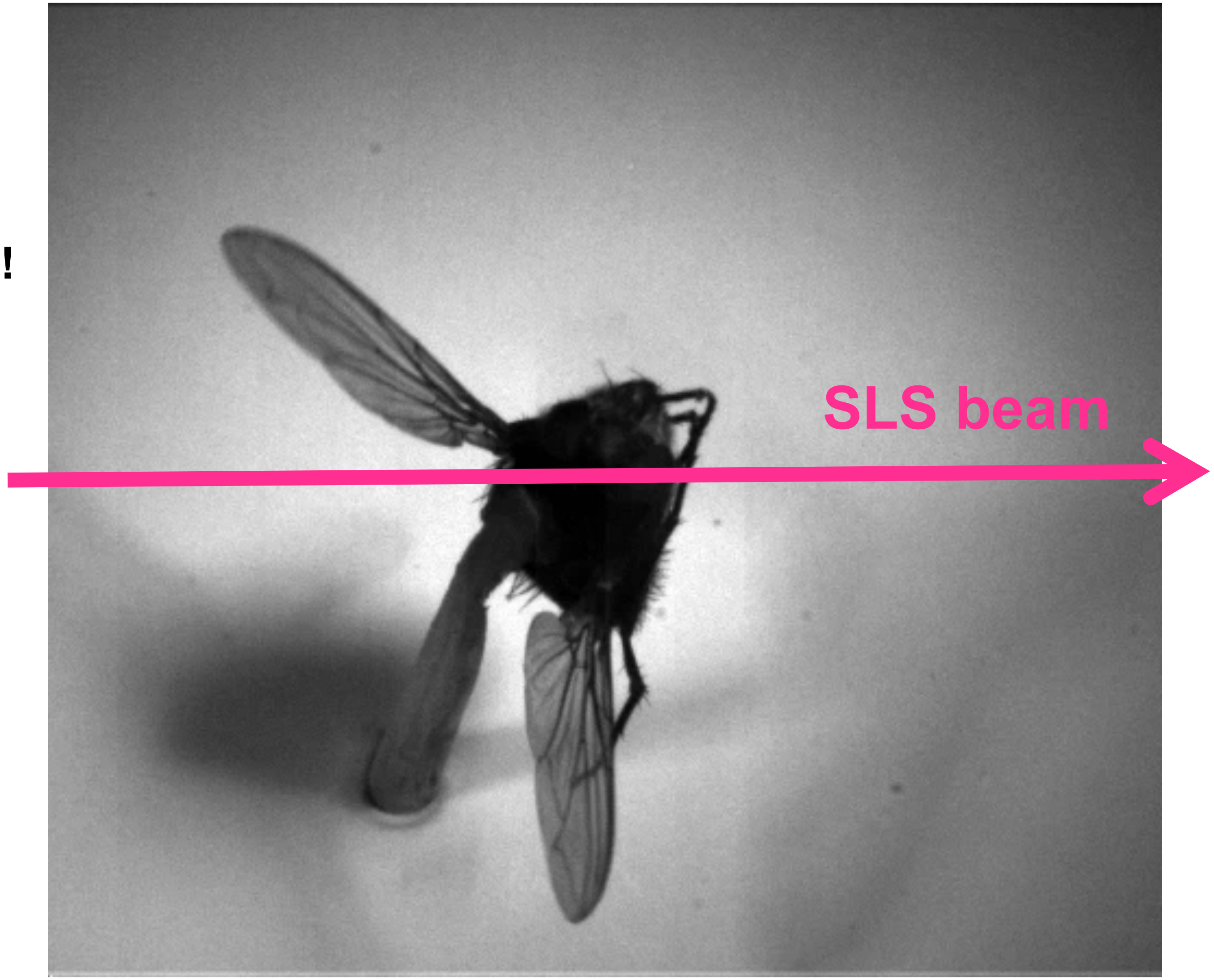
- **Crack propagation dynamics under tensile load**
- **20 (!) 3D volumes per second**

Movie playing in real time (9 seconds, 180 frames)

E. Maire, et. al., Int J Fract 1 (2016)

How does a fly really fly?

Wings beat at 150 Hz !!



2500 X-ray images per second…