Principles of Synchrotron Radiation

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X-Ray and Neutron Science Summer Program
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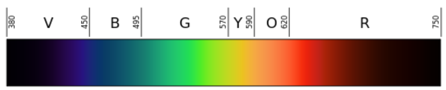
Properties of radiation

spectrum



flux

(photons/second)

polarization

“directionality” of radiation field

linear, circular

partial/full polarization

coherence

(single wavefront?
ability to make an interference pattern)

brightness

flux divided by source size

Synchrotron light sources give some control over all these properties, in may cases providing the only such source for particular parameters.

Other lectures tell you why x-rays are useful.

Here, I will talk about
where x-rays come from!

Outline

(1) X-rays from electrons

electricity and magnetism
some relativity

(2) Where do the electrons come from?

particle accelerators
electron storage ring
beam dynamics

radiation

+

synchrotron

=

synchrotron
radiation

PART 1

radiation

Radiation from charged particles- generalities

Maxwell's equations:

$$\nabla \cdot E = 4\pi\rho$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t}$$

$$\nabla \times B = \frac{4\pi}{c}J + \frac{1}{c}\frac{\partial E}{\partial t}$$

In vacuum, one derives the wave equation, where one gets plane waves representing electromagnetic radiation.

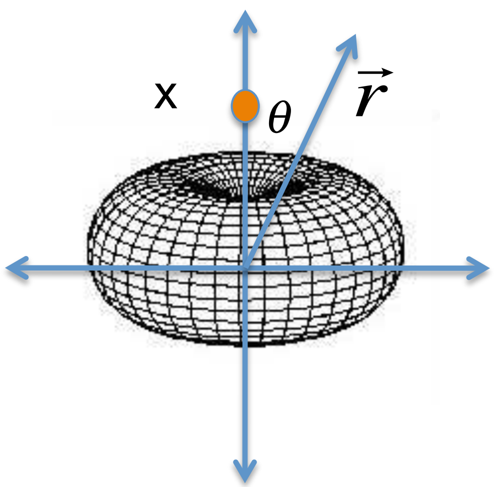
A source for such radiation requires a time dependent current.

Accelerating charged particles will thus provide a source of radiation.

Dipole radiation

Example of radiation source:

Consider a charge oscillating in sinusoidal motion.



$x(t) = A\cos(\omega t)$

It produces a radiation pattern:

$$\langle S \rangle \sim \frac{\sin^2(\theta)}{r^2}\hat{r}$$

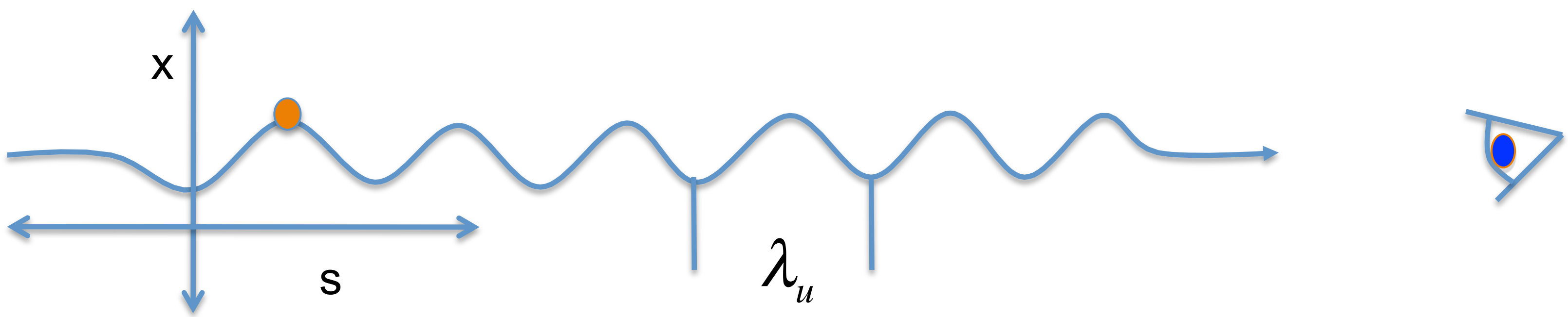
The radiation will have a frequency $f = \dfrac{\omega}{2\pi}$

and wavelength $\lambda = \dfrac{c}{f}$ remember $c = \lambda f$

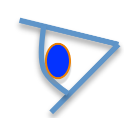
Polarization is linear in direction $\hat{\theta}$

c = 2.99792e8 m/sec = speed of light

Now, consider a moving, wiggling charge



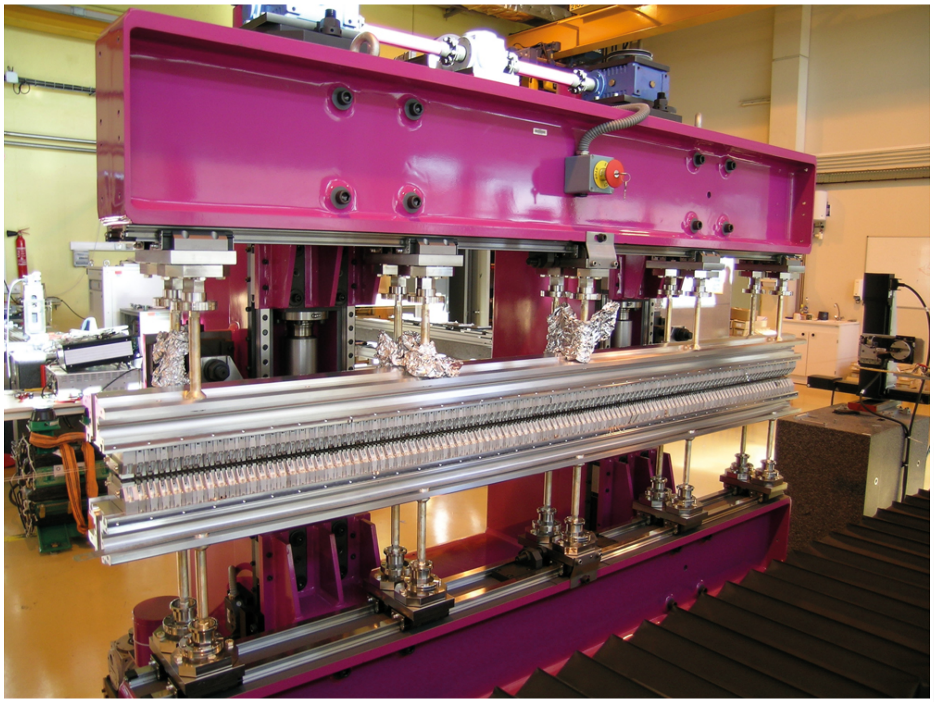
Observed from a distance in the plane
of oscillation, this looks (almost) like the oscillating dipole
again!



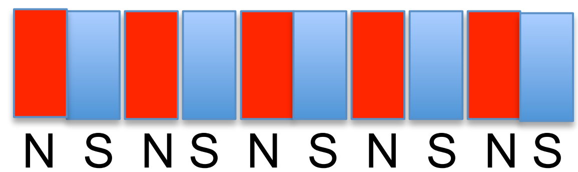
The motion creates two important differences:
1) wavelength shifted by Doppler effect, and in case
of relativistic speed, there is a time contraction effect.
Net effect: $\lambda = \frac{\lambda_u}{\gamma^2}$

2) Pattern of radiation gets distorted from motion. For high energy
gets bent into cone of angle $\frac{1}{\gamma}$

Undulator magnet causes electron wiggle



magnetic array or alternating field direction



Brief review of relativity

electron moving with velocity v

define $\beta = \dfrac{v}{c}$ ← speed of light

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

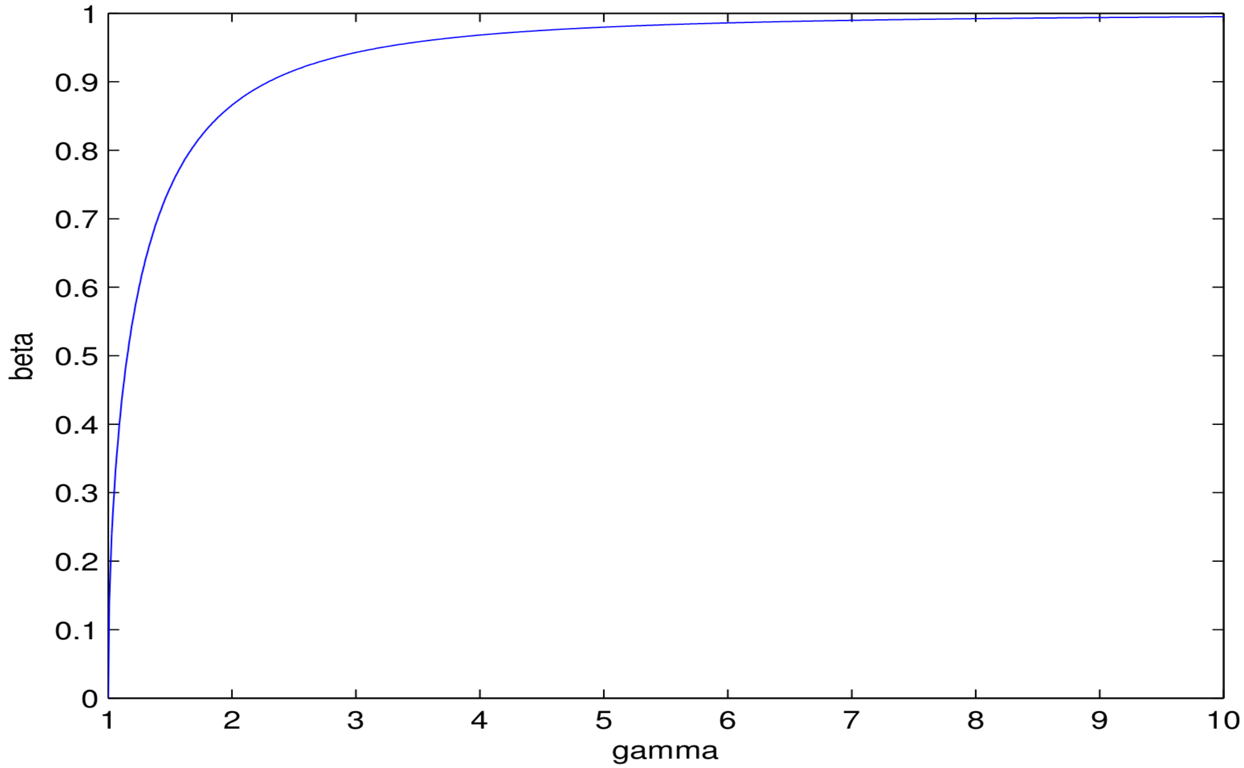
$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$E = \gamma mc^2 \implies \gamma = \frac{E[MeV]}{0.5109989} = 1.96 \times E[MeV]$$

Kinetic energy:

$$E_k = (\gamma - 1)mc^2 \quad \rightarrow E_k = \frac{1}{2}mv^2, \quad for \quad \beta << 1$$

Relativity (2)

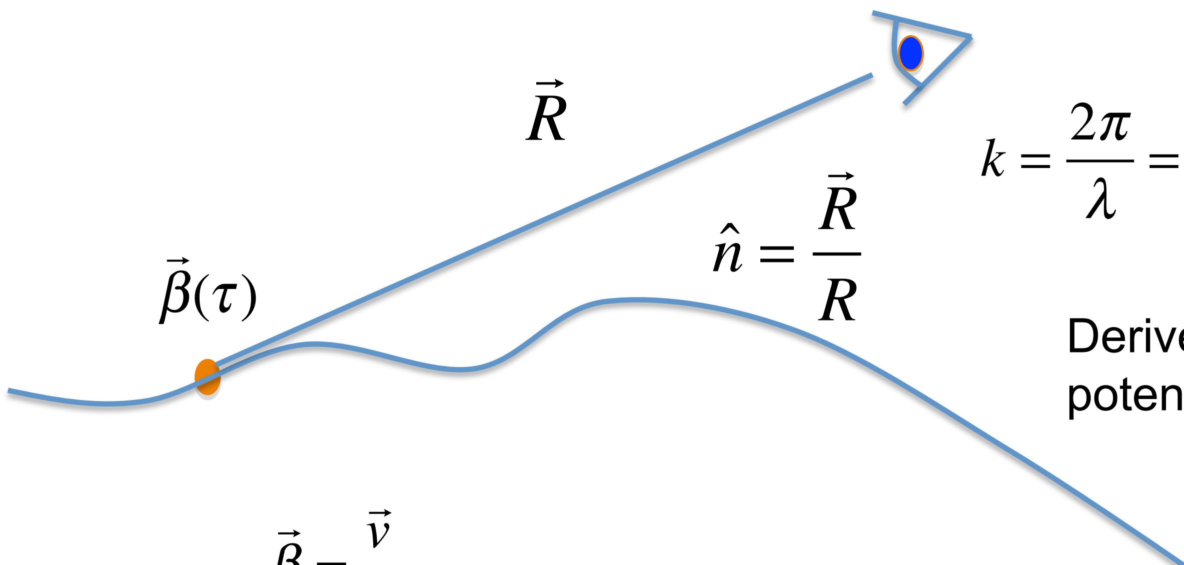


Note that for gamma>5, velocity increase becomes negligible

However, effects as gamma gets large:
length contraction
time dilation

General expression for Radiation from a trajectory

$$\vec{E}(\vec{R},\omega) = \int_{-\infty}^{\infty} \left[\vec{\beta}(\tau) - \frac{\hat{n}(\tau)}{R}\left(1 + \frac{ic}{\omega R}\right) \right] e^{i\omega\left(\tau + \frac{R}{c}\right)} d\tau$$



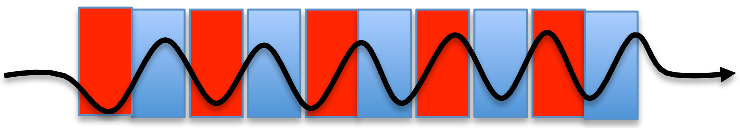
$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

Derived from Liénard-Wiechert potentials

$$\vec{\beta} = \frac{\vec{v}}{c}$$

So, given the electron orbit, we can compute the radiated electric field at a given frequency

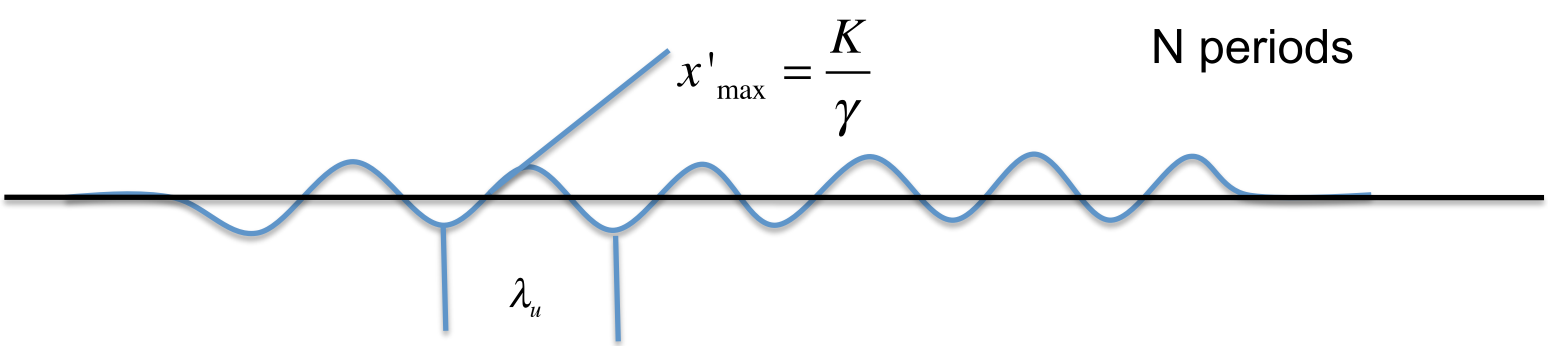
Undulator/wiggler orbit



$$B_y = B_0 \cos(\frac{2\pi s}{\lambda_u})$$

(simplest planar undulator)

$$K = \frac{eB_0\lambda_u}{2\pi mc^2} = 0.934\, B_0[T]\lambda_u[cm]$$



$$x' = \beta_x(s) = \frac{K}{\gamma}\sin\left(\frac{2\pi s}{\lambda_u}\right)$$

transverse velocity

$$\beta_s(s) = 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2} + \frac{K^2}{4\gamma^2}\cos(\frac{4\pi s}{\lambda_0})$$

long. velocity modulation

Undulator/Wiggler spectrum

$$\lambda_1(\theta) \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + (\gamma\theta)^2\right) \quad \textbf{Fundamental undulator wavelength} \qquad (2.28)$$

$$S_N(\omega) = \sum_{n=0}^{N-1} e^{in\omega T} = \frac{\sin\left(N\omega T/2\right)}{\sin\left(\omega T/2\right)} e^{i(N-1)\omega T/2}$$

for K<1, all radiation contained in same cone

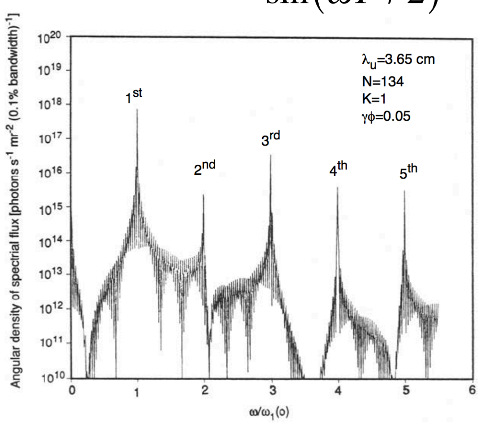


Figure 2.15: Angular density of spectral flux for an undulator with the parameters indicated in the inset, and for an electron current of I=0.4 A at $\mathcal{E}$=1.5 GeV. (From Ref.[2])

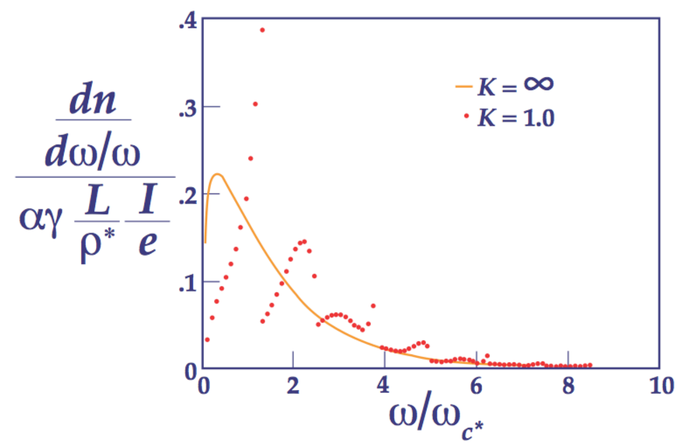
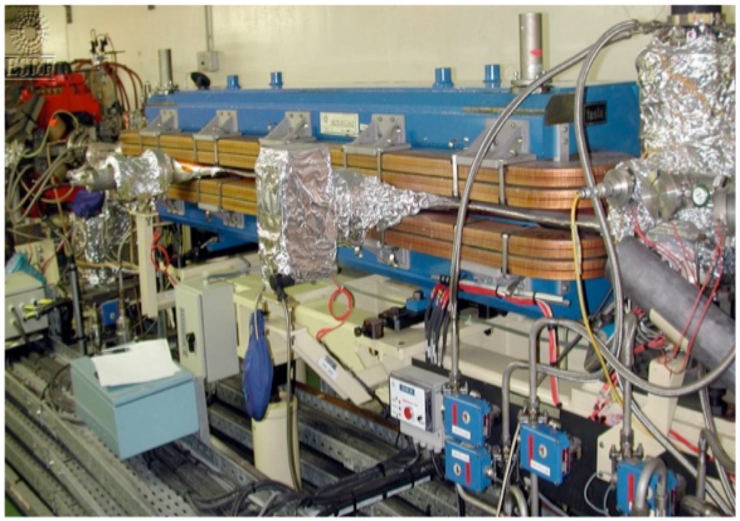
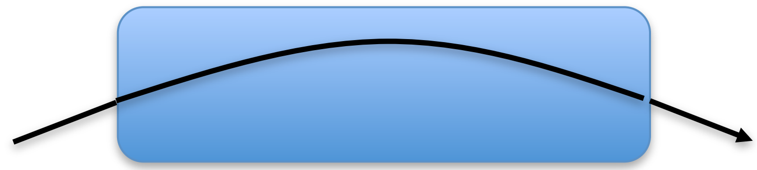


Figure 2b — Angle-integrated spectrum of synchrotron radiation from a sinusoidal undulator, $K = 1.0$ and $K = \infty$

Transition between undulator and Wiggler spectrum

We also get radiation out of a dipole magnet

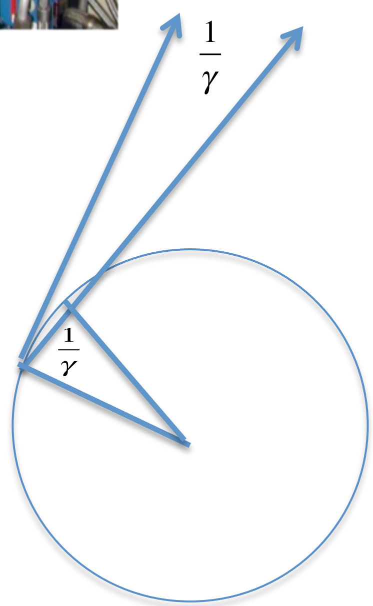




$$B_y = B_0$$

combining with time compression, we get a characteristic time

$$dT' = \frac{\rho}{2c\gamma^3}(1+(\alpha\gamma)^2)$$

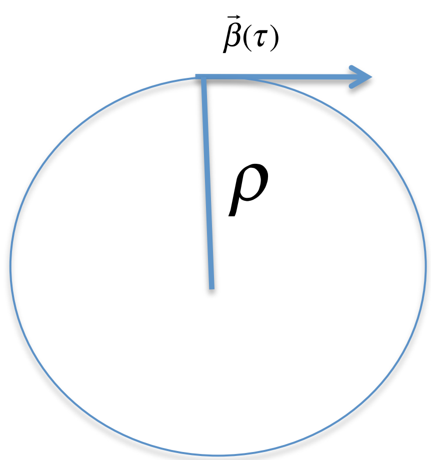


orbit length $dL = \frac{\rho}{\gamma}$

characteristic time $dT' = \frac{\rho}{c\gamma}$

Dipole magnet orbit

Consider electron in constant magnetic field



$$\vec{r}(\tau) = \left(\rho \sin\frac{\beta c \tau}{\rho}, \rho\left(1 - \cos\frac{\beta c \tau}{\rho}\right), 0 \right)$$

$$B\rho[\mathrm{Tm}] = 3.3357\, p[\mathrm{GeV/c}]$$

B=.85 T

p/c=6.04 GeV

$$\rho = 23\, m$$

$$\omega_0 = \frac{Be}{\gamma m}$$

Dipole magnet spectrum

critical frequency defined as $$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$ (18.8 KeV for current ESRF)

$$(E = hf = \frac{hc}{\lambda})$$

Computing spectrum, one finds

h=Planck's constant

$$\wp(\omega) = \frac{P_\gamma}{\omega_c} S(\frac{\omega}{\omega_c})$$

$$S(\xi) = \frac{9\sqrt{3}}{8\pi}\xi\int_\xi^\infty K_{5/3}(\bar{\xi})d\bar{\xi}$$

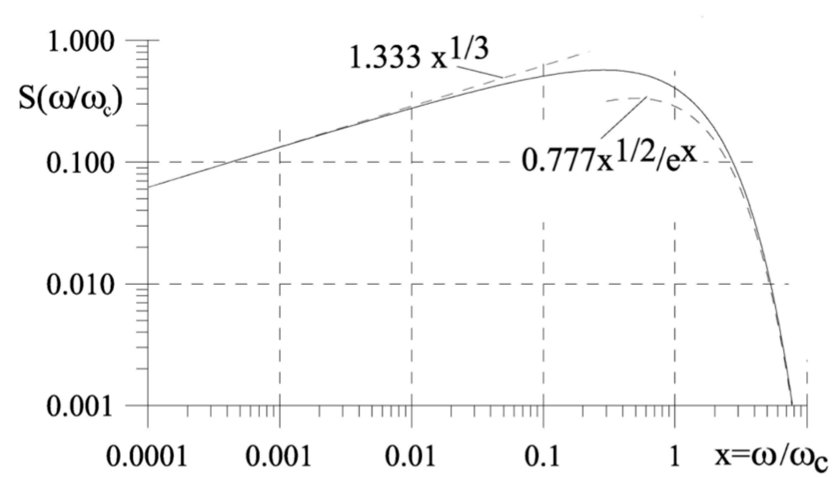


Fig. 22.11. Universal function: $S(\xi) = \frac{9\sqrt{3}}{8\pi}\xi\int_\xi^\infty K_{5/3}(x)\,dx$, with $\xi = \omega/\omega_c$

Note that spectrum is much broader than for the undulator.

PART 2

synchrotron

Particle accelerators and storage rings

Two things to understand:

1) Single electrons: How to store an electron and what kind of orbit will it have?

2) What kind of distribution of electrons will we get in the synchrotron?

How to store a high energy electron?

First accelerate: 6 GeV for ESRF

To move in a circle, we use dipole magnets

For transverse focussing/stability, use quadrupoles

To fix chromatic aberration, we need sextupoles

To give energy back lost to synchrotron radiation, and
to provide longitudinal stability, use RF cavities

ESRF Acceleration Complex

$$1-\beta = 3.6\times10^{-9}$$

$E_k = 0.025eV$, $\gamma = 1$

$E_k = 100keV$, $\gamma = 1.2$

$E_k = 11MeV$, $\gamma = 22.5$

$E_k = 200MeV$, $\gamma = 391$

$E_k = 6.03GeV$, $\gamma = 11{,}800$

electron gun

prebuncher

buncher

linac

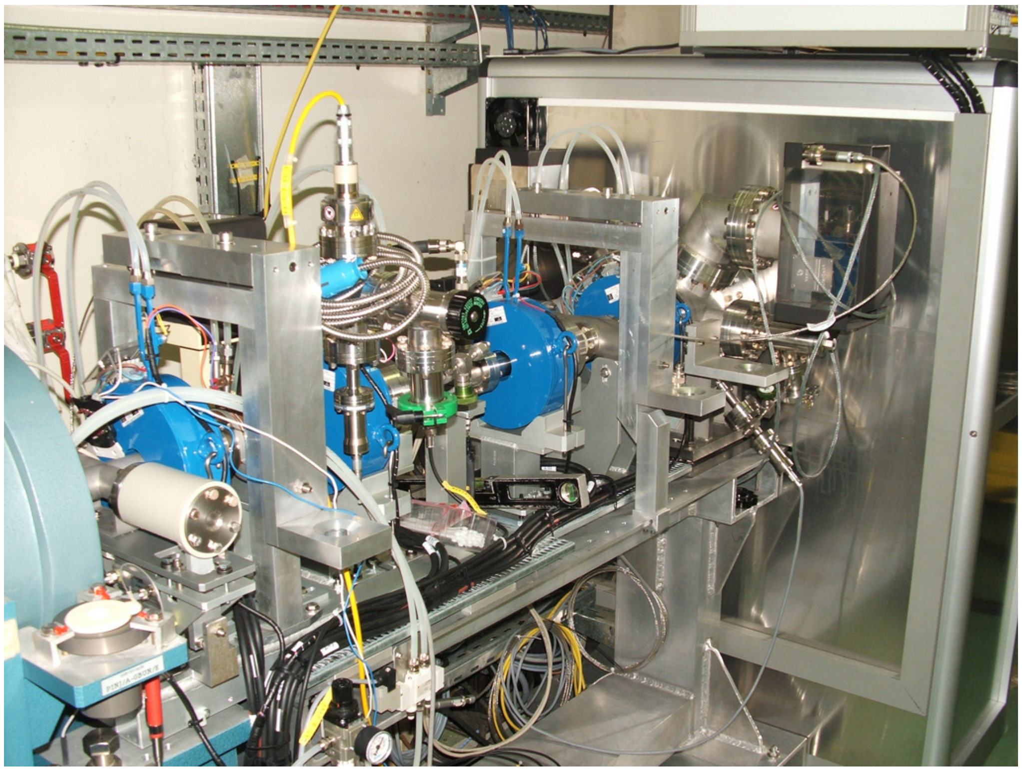
TL1

booster

TL2

storage ring

Electron Gun and pre-buncher



impulse to
gun determines
bunch shape and
length

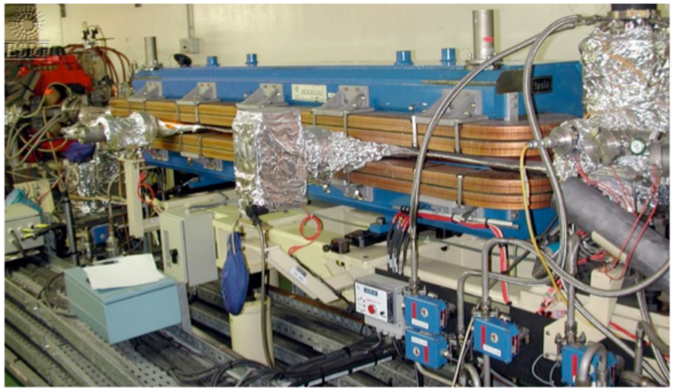
pre-buncher does not
accelerate

100 keV triode gun

$\gamma = 1.2$

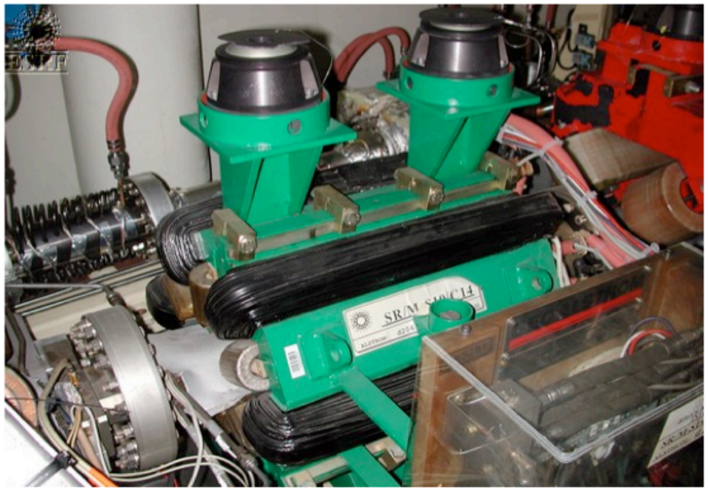
gun is triggered either at 10 Hz or at 1 Hz

Storage ring components



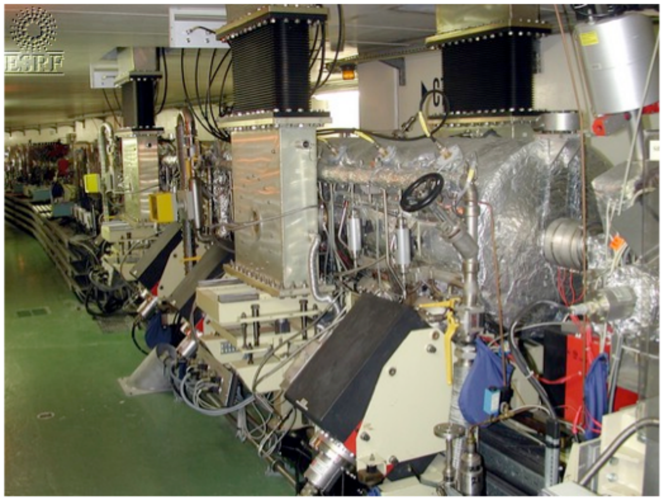
dipole

quadrupole

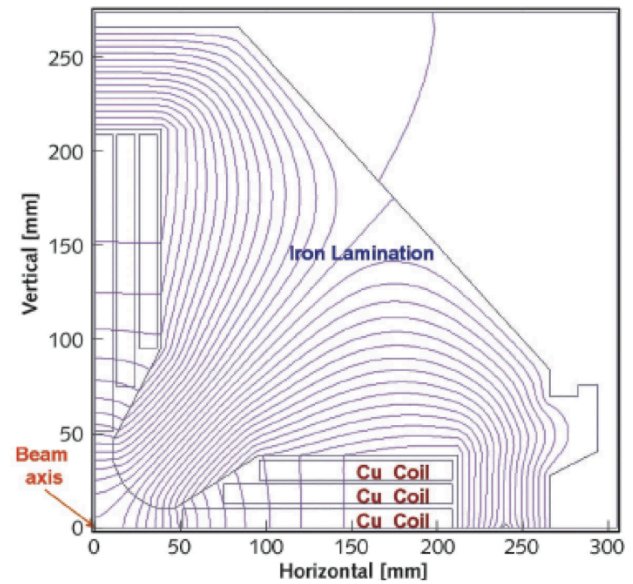


sextupole

RF cavity



Quadrupoles for strong focusing of electrons



¼ of an ESRF quadrupole

Field in body given by

$$\vec{B} = B_1(y\,\hat{x} + x\,\hat{y})$$

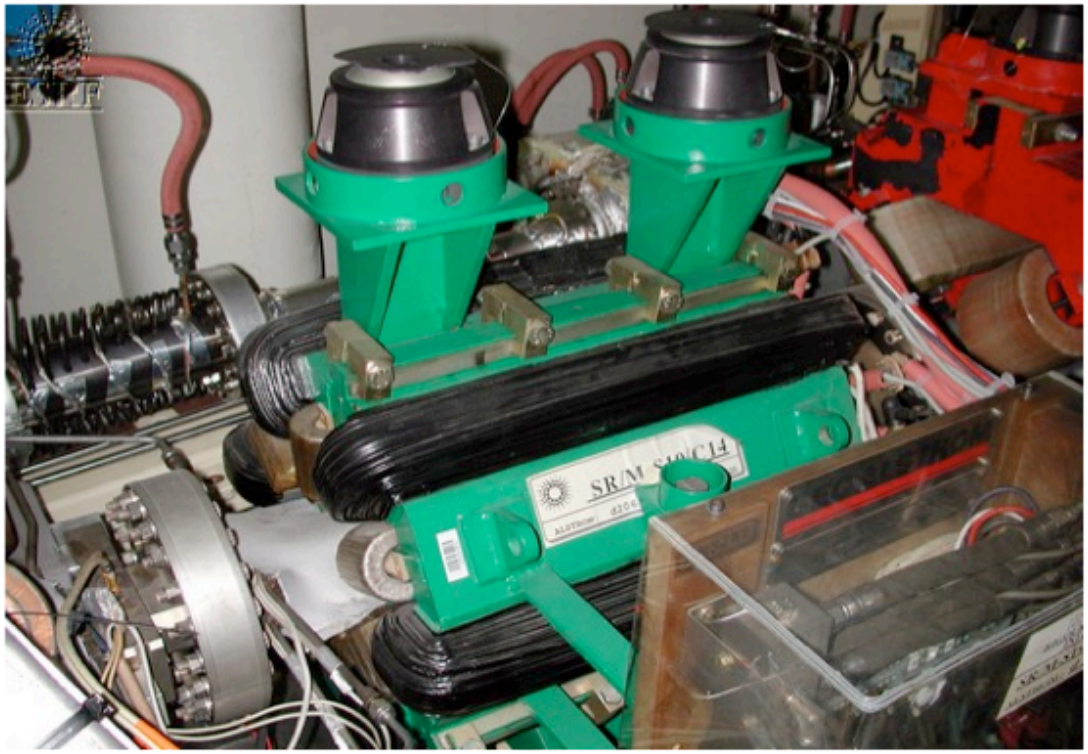
Apply Lorentz force law and we get focal strengths

$$k_x = -\frac{B_1}{B\rho}$$

$$k_y = \frac{B_1}{B\rho}$$

Opposite signs! Requires clever quad placement and polarity to get overall focussing!

Sextupoles



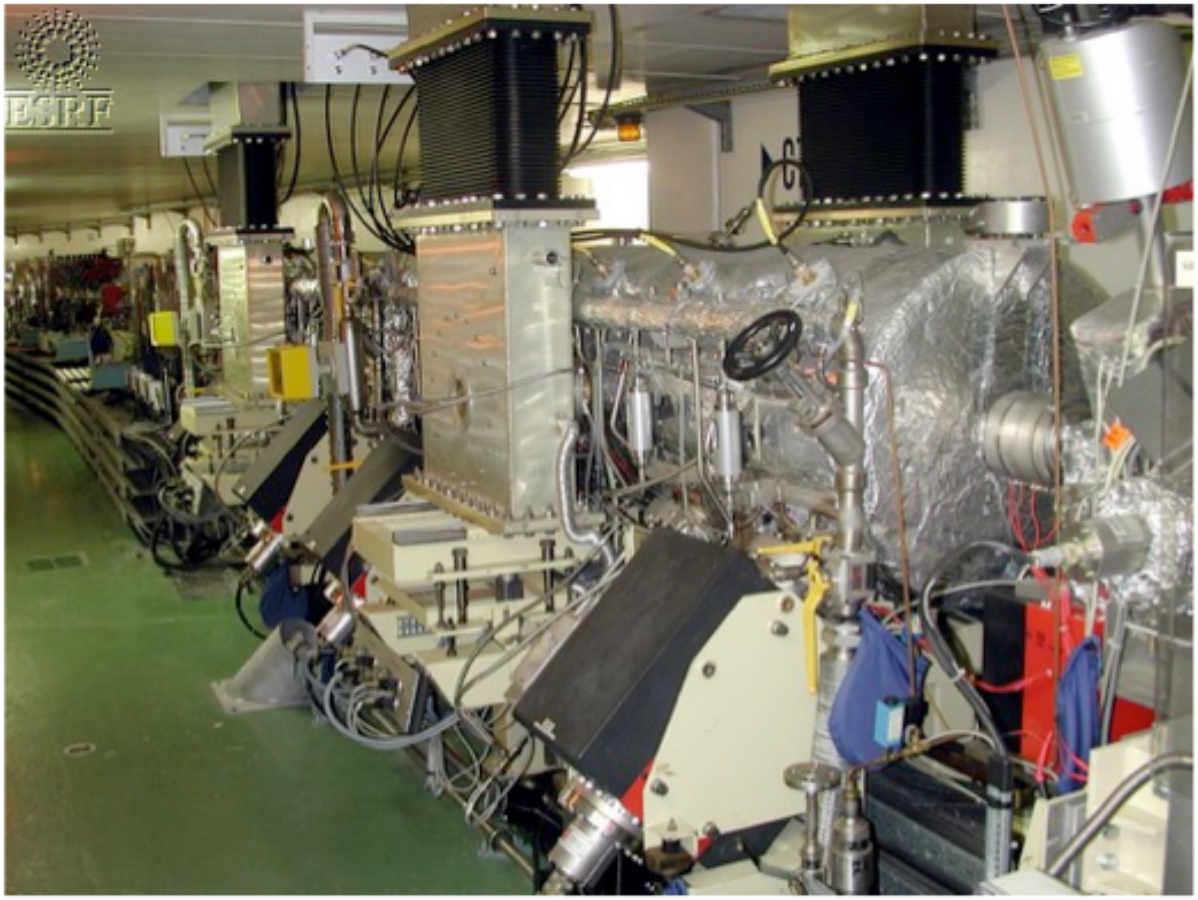
Sextupoles may be used to correct energy effect from quadrupoles (chromaticity). Then causes additional stability problems which need to be corrected!

Beam lifetime and dynamic aperture for injection

$$\vec{B} = B_2(xy\,\hat{x} + (x^2 - y^2)\,\hat{y})$$

hard problem in non-linear dynamics!

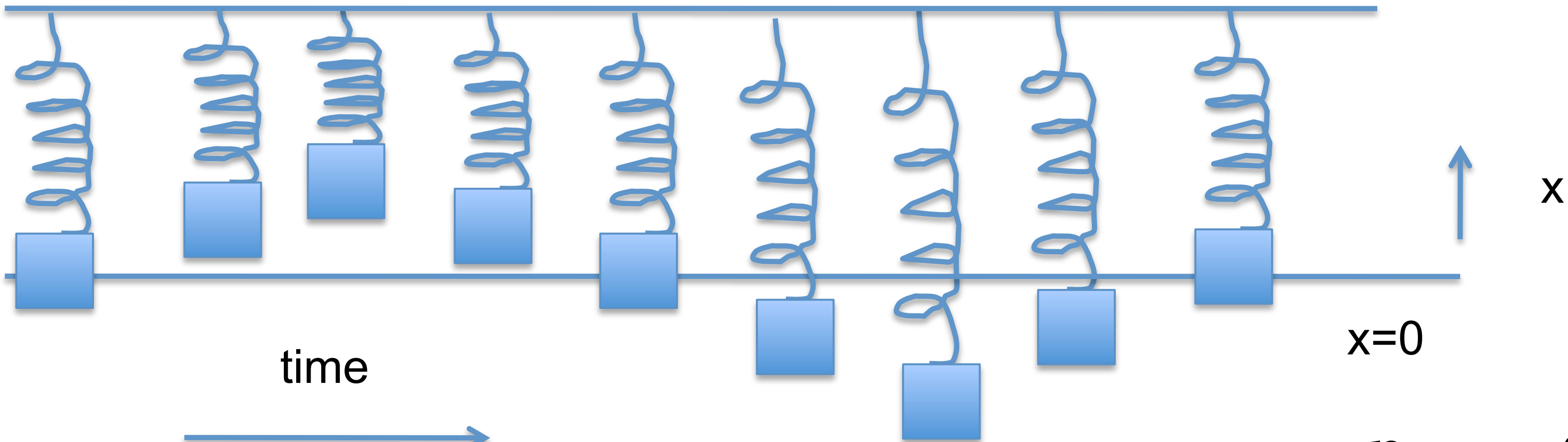
RF cavity



Gives energy back that was lost from radiation and provides longitudinal focussing.

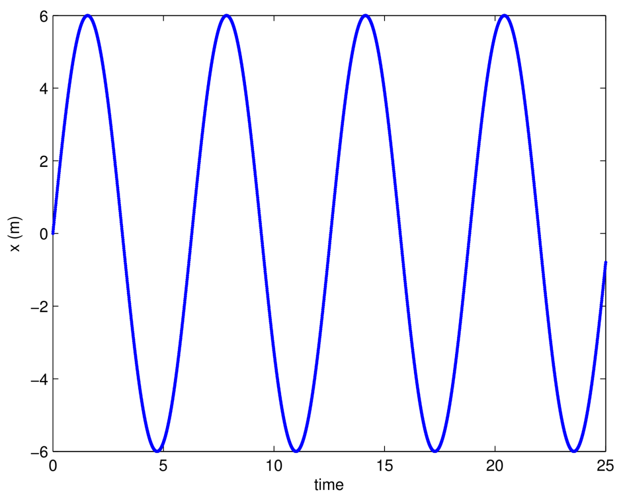
Most of the ESRF energy use (around 1.5 MW of power) is in these cavities;

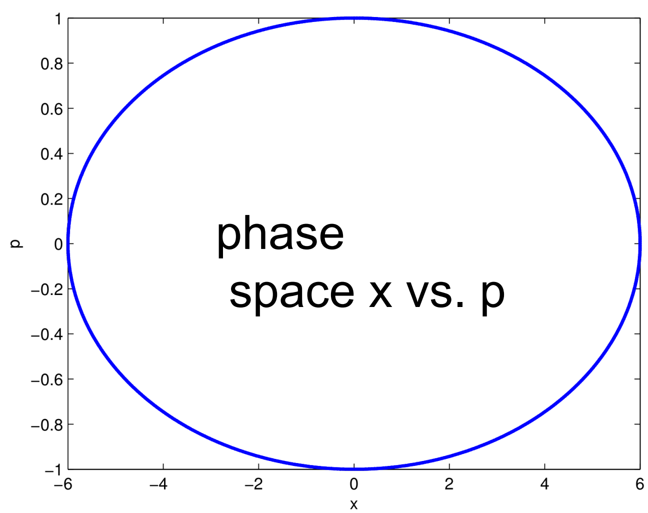
What happens to stored electrons? Phase space



$$p_{x,y} = \gamma m v_{x,y}$$

configuration space x vs. time





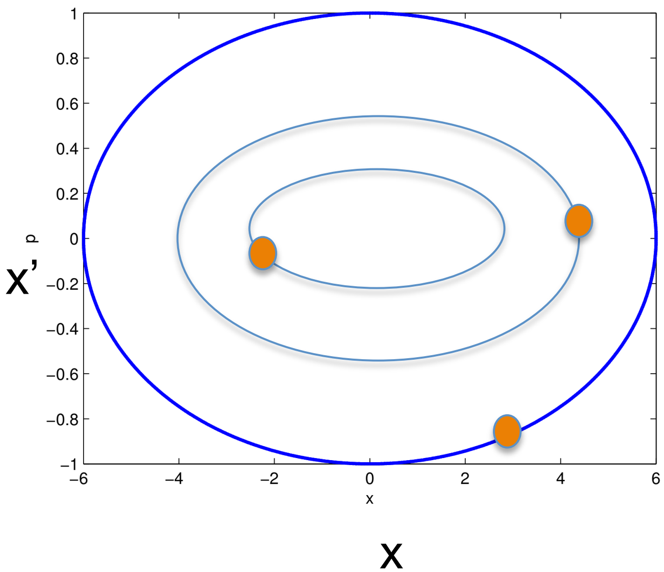
for electron, we normalize with

$$P_0 = \gamma m v_s$$

and use

$$x' = \frac{p_x}{P_0} = \frac{dx}{ds}$$

What kind of distribution of electrons will we have in a storage ring?



several stored electrons with different amplitudes.

Another effect: radiation

radiated power in a dipole:

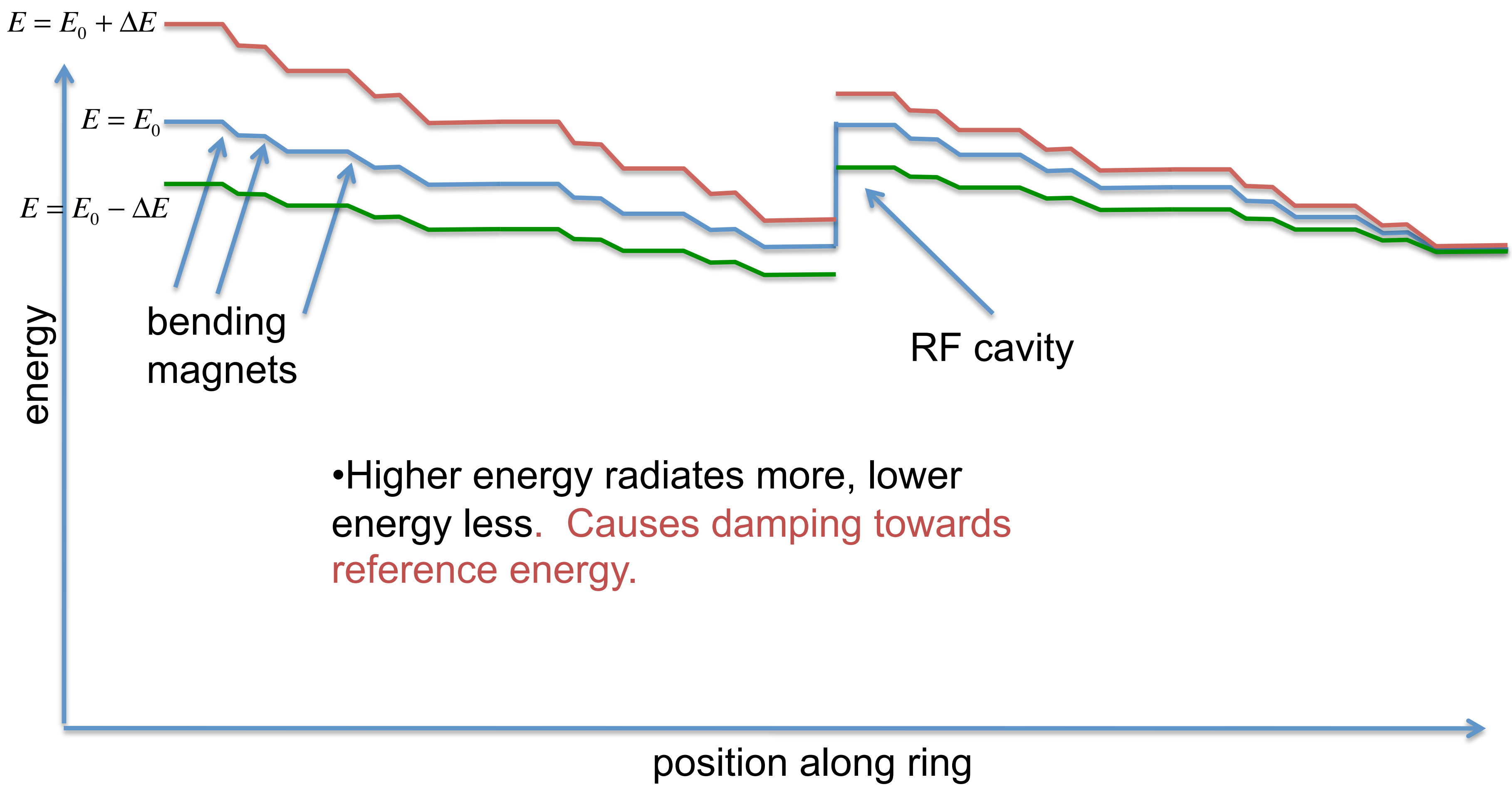
$$P_\gamma = \frac{cC_\gamma}{2\pi}\frac{E^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3}\frac{r_e}{(mc^2)^3} = 8.85 * 10^{-5}\ \frac{m}{GeV^3}$$

radiation constant

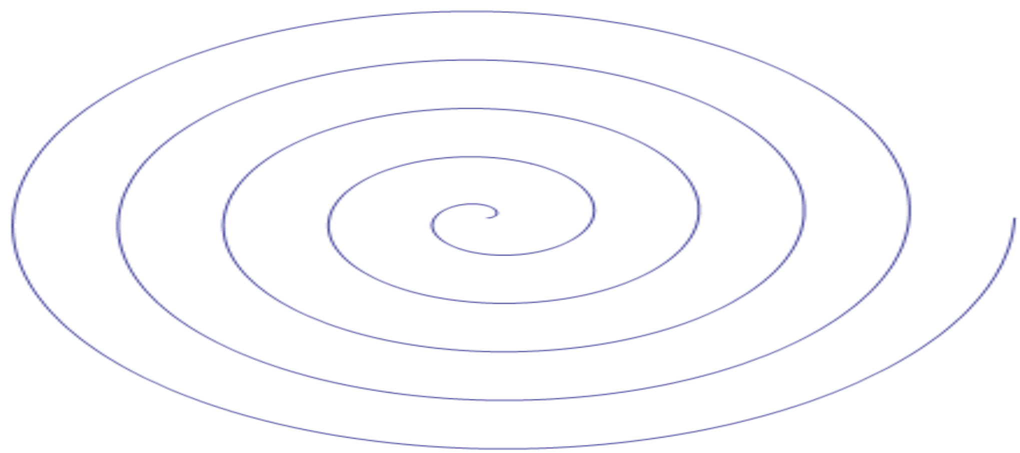
Higher energy radiates more
Lower energy radiates less:
Radiation Damping!

Radiation effect on Longutidinal dynamics



•Higher energy radiates more, lower energy less. Causes damping towards reference energy.

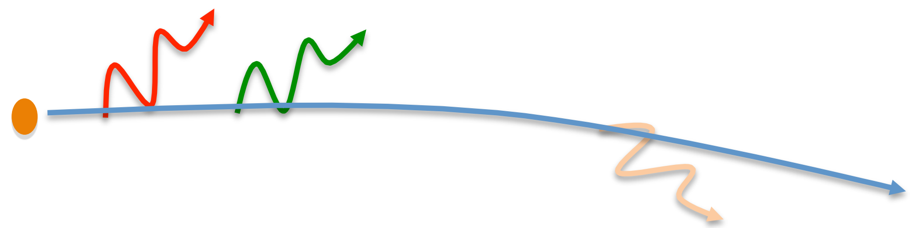
Radiation damping



All electrons damp towards the same orbit!

What sets the size of the electron beam?

Graininess of photon emission



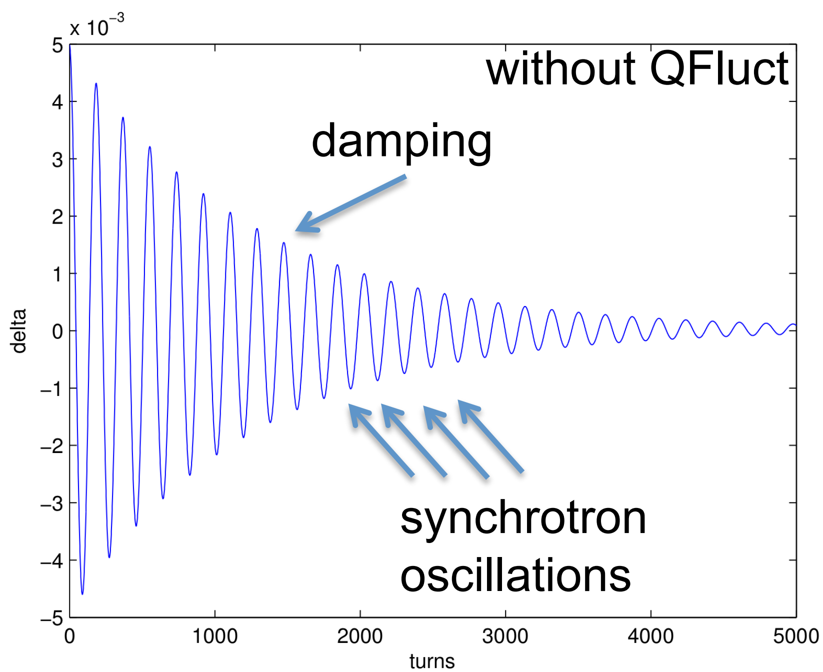
Two sources of randomness:
emission time of photons are random: Poisson process

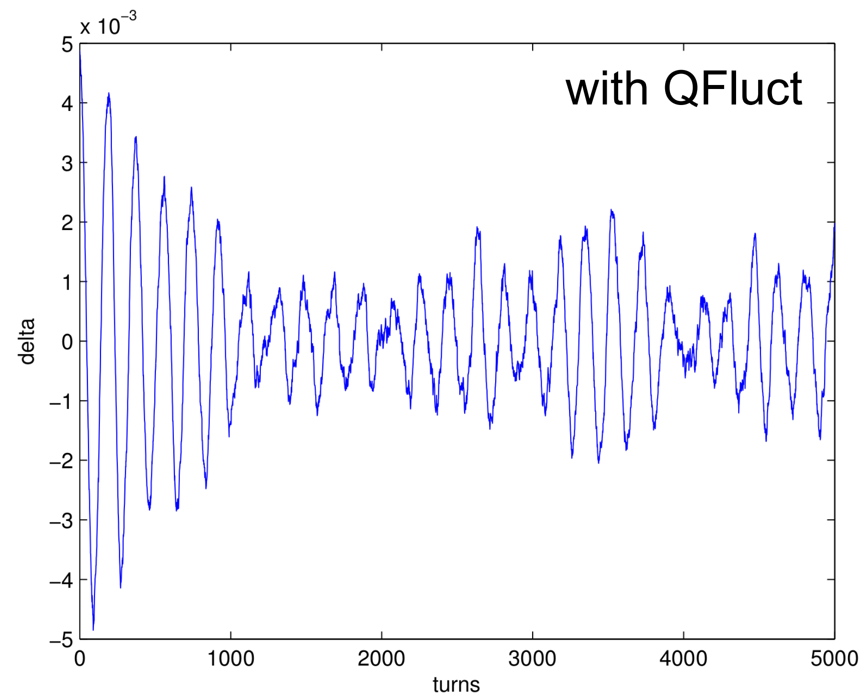
Energy emitted is also a random process, with the power spectrum as the probability distribution for each photon.

For ESRF, only about 800 photons per turn!
Or, about 1 photon emitted per meter!
(approx. 12 photons per dipole)

This quantum mechanical diffusion process accounts for the size of the electron beam, which (usually) determines the size of the x-ray beam!

Quantum fluctuation effect on electron dynamics





Electron motion and without quantum fluctuations.

Result of damping/diffusion

The electron beam reaches a unique Gaussian distribution– independent of how one injects into the ring.

This is a major difference between electron synchrotrons and proton synchrotrons (e.g. LHC)

By careful choice of where the dipoles and quadrupoles are, one can reduce the size of this equilibrium beam size (emittance = beam size in phase space). So called "Low emittance ring design"

In fact, due to developments in lattice design, ESRF is completely replacing the storage ring in 2018 to reduce the electron beam emittance. 4nm -> 150 pm

Coherent versus incoherent SR

In storage ring, average interparticle spacing is larger than radiation wavelength, thus each electron emits independently. Power scales with N, number of electrons.

An FEL (Free Electron Laser) is a different kind of accelerator where the electron beam is designed to have a microbunching structure on the scale of the radiation. This gives coherent radiation with a power proportional to N^2.

There are several XFEL projects, e.g. LCLS (US), European XFEL (Germany) , SACLA (Japan), and more

Because these are single pass (vs. storage ring) the energy requirement and repetition rate is typically much lower than storage rings.
It looks like both storage ring synchrotron sources and FEL sources will fulfill different requirements and can complement each other.

Further References

Synchrotron Radiation

Elleaume, Onuki, “Undulators, Wigglers and their Applications”, Taylor and Francis (2003)

Electron Storage rings

Matt Sands, “The Physics of Electron Storage Rings” Slac report 121 (1970)

General accelerator physics references

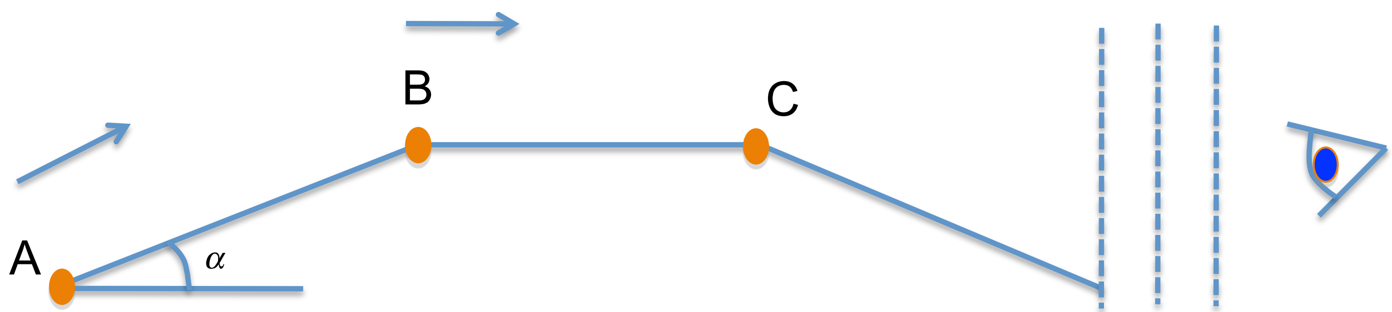
Helmut Wiedemann, ‘Particle Accelerator Physics’, 1&2, Springer-Verlag, 1993, (1999)

S.Y. Lee, ‘Accelerator Physics’, World Scientific, (1999)

Klaus Wille, ‘The Physics of Particle Accelerators’, Oxford University Press, (1996) Thank you for your attention!!

Extra Slides

Time compression factor



Electron moves at speed β, emitting wavefront at A, B, C

$$\Delta t = \frac{(c-v)}{c}\Delta t' \qquad \frac{v}{c} = \beta_e \cos\alpha$$

Δt time between wavefronts

$$\Delta t = (1-\beta_e \cos\alpha)\Delta t'$$

$\Delta t'$ time for electron to emit

$$\Delta t \approx \frac{1+(\alpha\gamma)^2}{2\gamma^2}\Delta t'$$

$$\frac{dt}{dt'} = (1-\beta\cos\alpha)$$ continuous generalization