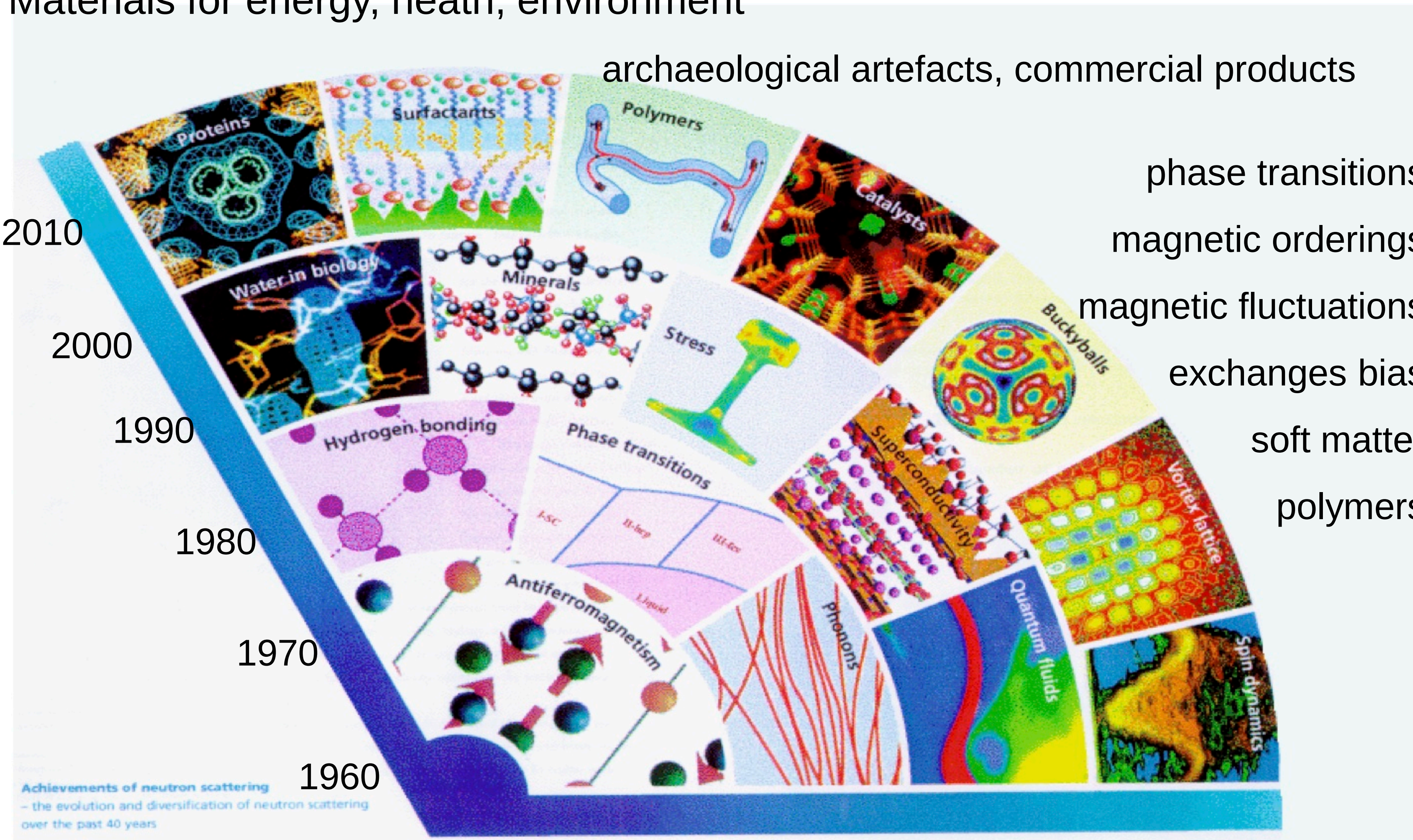
WHAT DO WE STUDY WITH NEUTRONS?

Materials for energy, heath, environment



WHY NEUTRONS?

Why do we use neutrons?

- Neutrons tell us about the positions and motions of atoms/magnetic moments in condensed matter

- Neutrons interact with nuclei and magnetic moments

  the two interactions have similar ‘strengths’

-Interaction with matter is gentle and simple:

  scattering data are easy to interpret

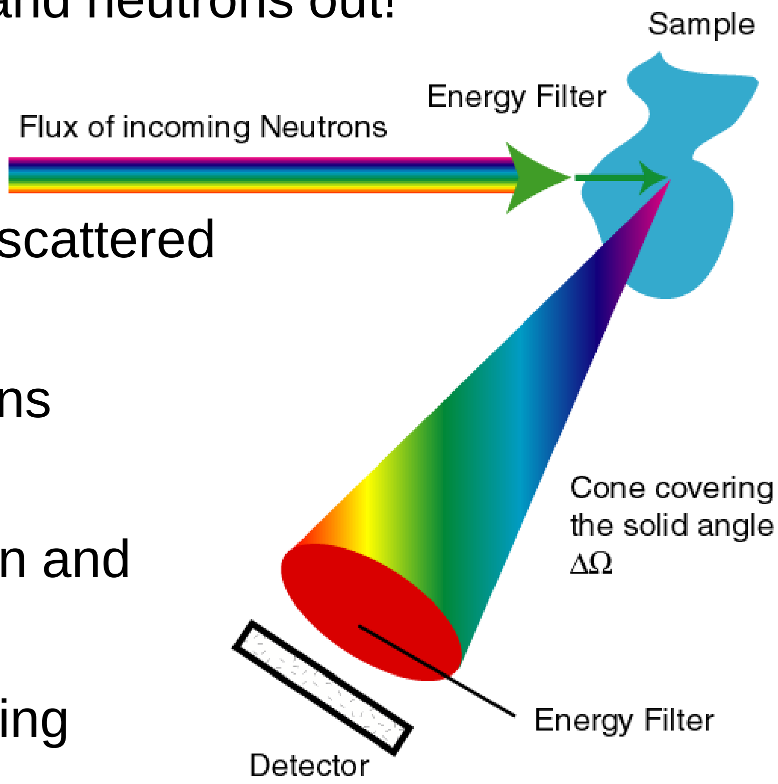
- Neutrons are penetrating: bulk materials can be studied

  any sample can be contained in special environment

- Experimental science: instrument design, data taking and data analysis ……

WHAT DO WE MEASURE?

Scattering experiments: neutrons in and neutrons out!



We measure the number of neutrons scattered by a sample

against the number of incident neutrons (neutron flux)

as a function of the change in direction and energy of the scattered neutrons

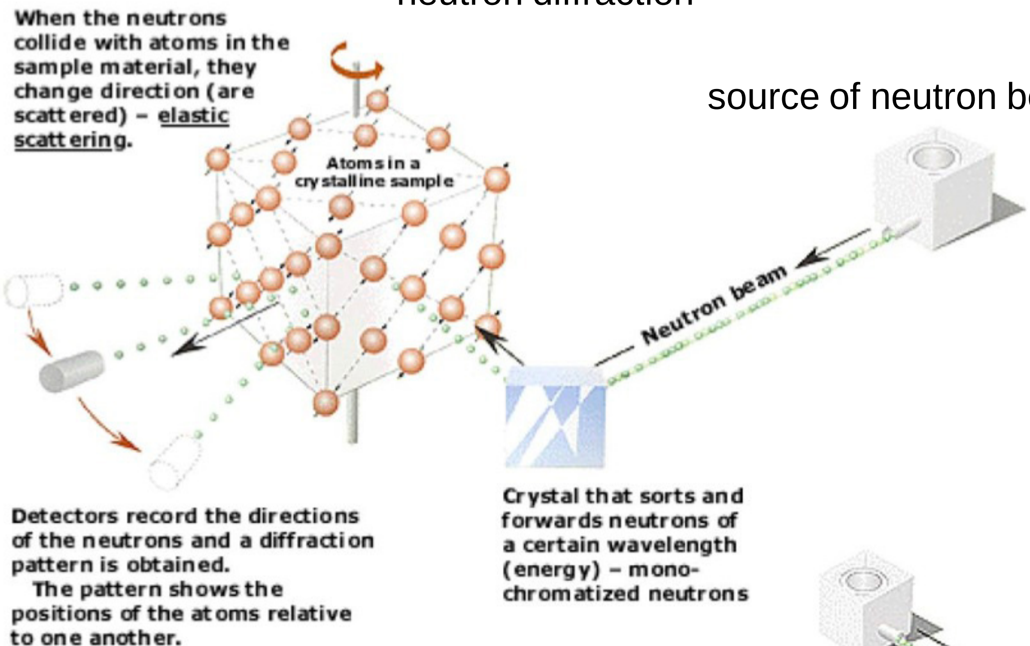
as a function of polarisation or polarising magnetic fields

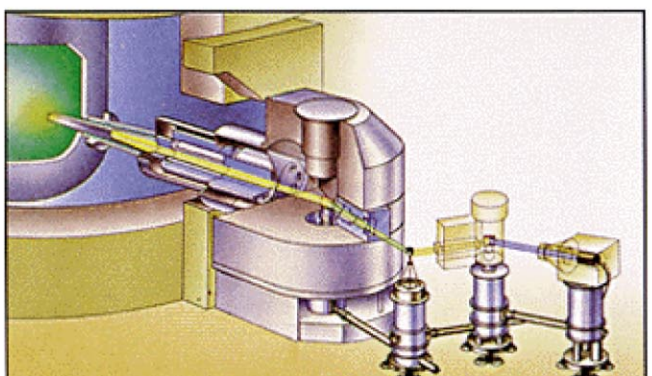
- Scattered intensities involve positions and motions of scattering centres: atoms/magnetic moments
- Scattered intensities are proportional to Fourier transforms (in space and time) pair correlation functions

HOW DO WE MEASURE?

neutron diffraction

source of neutron beams

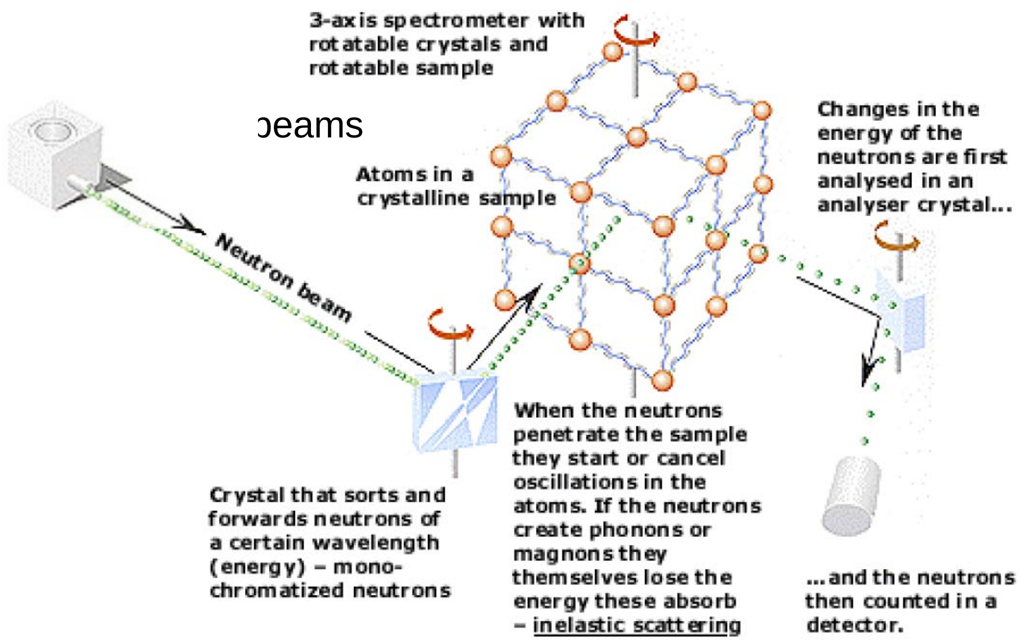




3-axis spectrometer

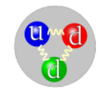
neutron spectroscopy

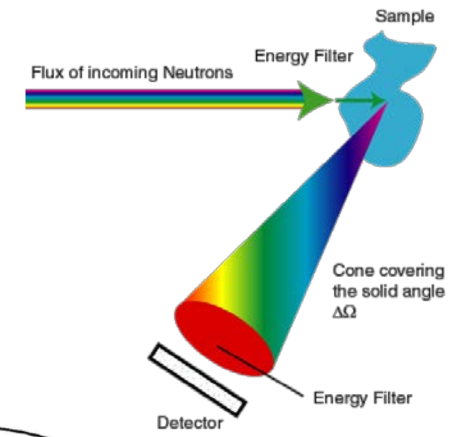
Inelastic scattering

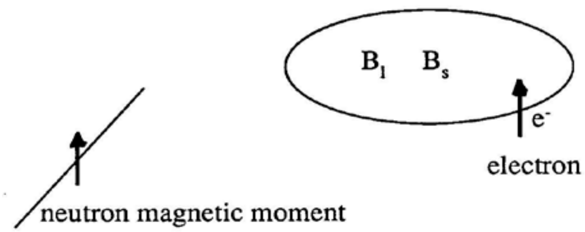


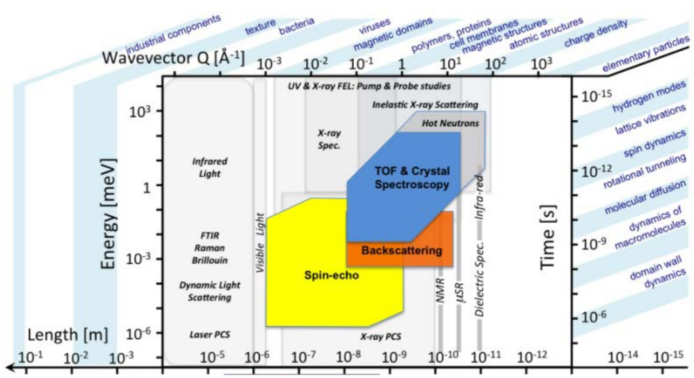
OVERVIEW

- a bit history
- neutron properties
- interactions between neutrons and matter
- measured quantities
- scattering by atoms/nuclei
- scattering by magnetic moments
- key messages









A bit of history:

W. Bothe & H. Decker -1930

discovered very penetrating radiation emitted when α particles hit light elements

I. Curie & F. Juliot -1932

observed creation of p^+ in paraffin sheets & thought new radiation was γ-rays

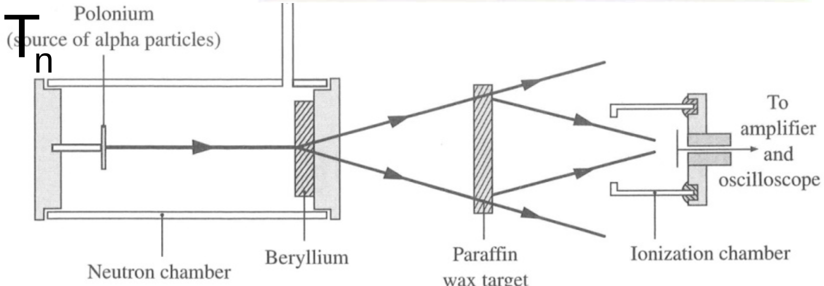
J. Chadwick -1932 a few months later

discovers the 'neutron', a neutral but massive particle

Nobel Prize in Physics

$${}^{4}_{2}He + {}^{10}_{5}B \rightarrow {}^{14}_{7}N + {}^{1}_{0}n \qquad {}^{4}_{2}He + {}^{9}_{4}Be \rightarrow {}^{16}_{6}C + {}^{1}_{0}n$$

$$(m_{He} + m_{B})c^2 + T_{He} = (m_N + m_n)c^2 + T_N + T_n$$



$$m_n = 1.0067 \pm 0.0012 \text{ a.m.u}$$

NEUTRONS: INTRODUCTION

A bit of history:

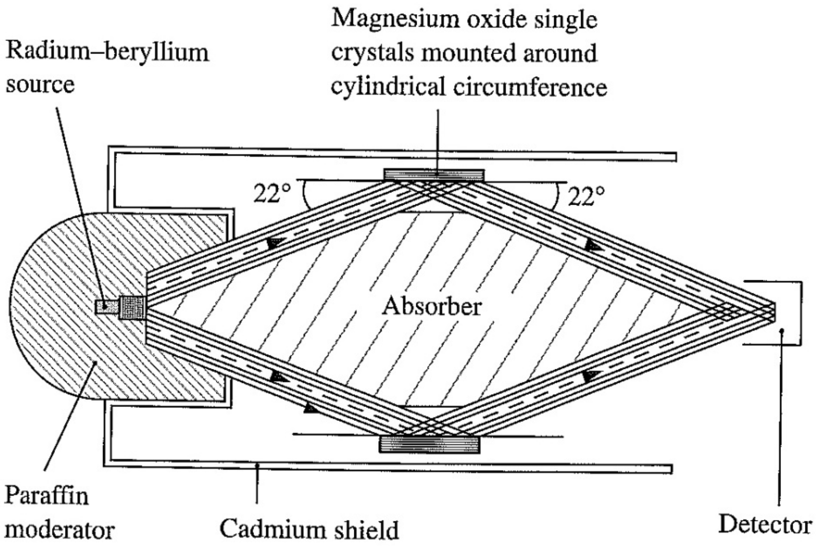
E. Fermi showed that neutrons moderated by paraffin could be captured by various elements, producing artificial radioactive nuclei

importance of neutron energy range

D.P. Mitchell & N. Powers / H. v. Halban & P. Preiswerk -1936

showed that thermal neutrons can be diffracted by crystalline matter

MgO crystals oriented (200) planes
22˚corresponds to Bragg angle for peak of wavelength distribution of thermal neutrons
~0.16nm



A bit of history:

• O. Hahn, F. Strassmann & L. Meitner -1938

discovered the fission of ^{235}U nuclei through thermal neutron capture

• H. v. Halban, F. Joliot & L. Kowarski -1939

showed that ^{235}U nuclei fission produced 2.4 n^0 on average – chain reaction

• E. Fermi & al. -1942

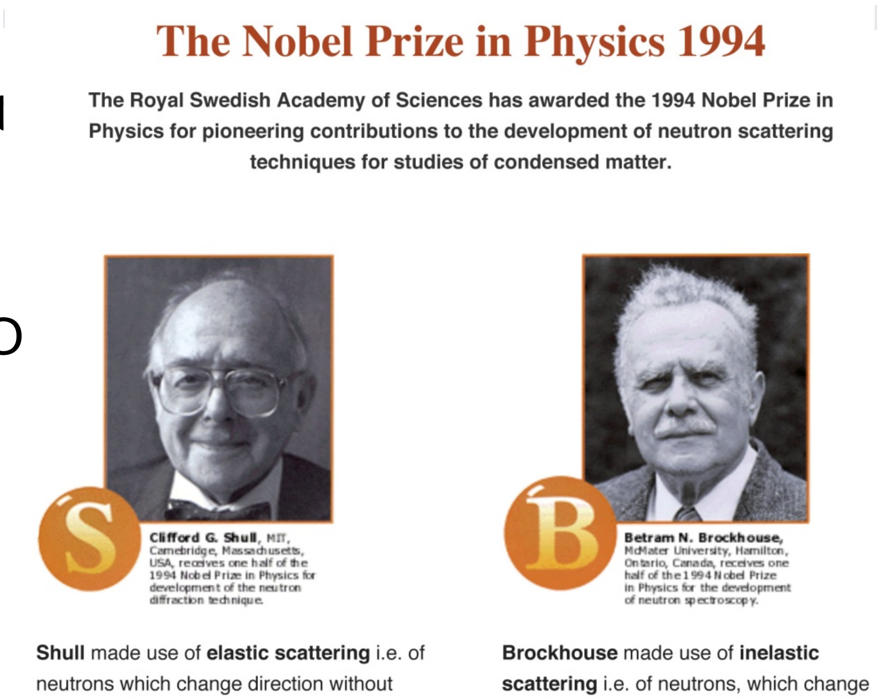
first self-sustained chain reaction react

• C.G. Shull -1942

Proof of antiferromagnetic order in MnO

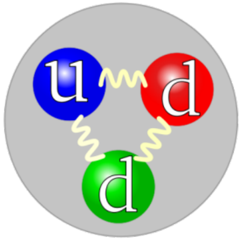
• C.G. Shull & B.N. Brockhouse -1994

Nobel Prize in Physics



NEUTRONS: NEUTRON PROPERTIES

free neutrons are unstable: β-decay proton, electron, anti-neutrino



life time: 886 ± 1 sec

wave-particle duality: neutrons have particle-like and wave-like properties

- mass: m_n = 1.675 x 10^{-27} kg = 1.00866 u. (unified atomic mass unit)
- charge = 0
- spin =1/2 magnetic dipole moment: $\mu_n = -1.913\ \mu_N$
- velocity (v) kinetic energy (E) temperature (T) wavevector (k) wavelength (λ)

$$E = m_n v^2/2 = k_B T = (hk/2\pi)^2/2m_n \qquad k = 2\pi/\lambda = m_n v/(h/2\pi)$$

$$\lambda\,(\mathrm{nm}) = 395.6/v(\mathrm{m/s}) = 0.286/(E)^{1/2}\ (E \text{ in eV}) \qquad 1\,(\overset{0}{\mathrm{A}}) \approx 82\,\mathrm{meV} \approx 124\,\mathrm{THz} \approx 950\ \mathrm{K}$$

$$E\ (\mathrm{meV}) = 0.02072\ k^2\ (k \text{ in nm}^{-1})$$

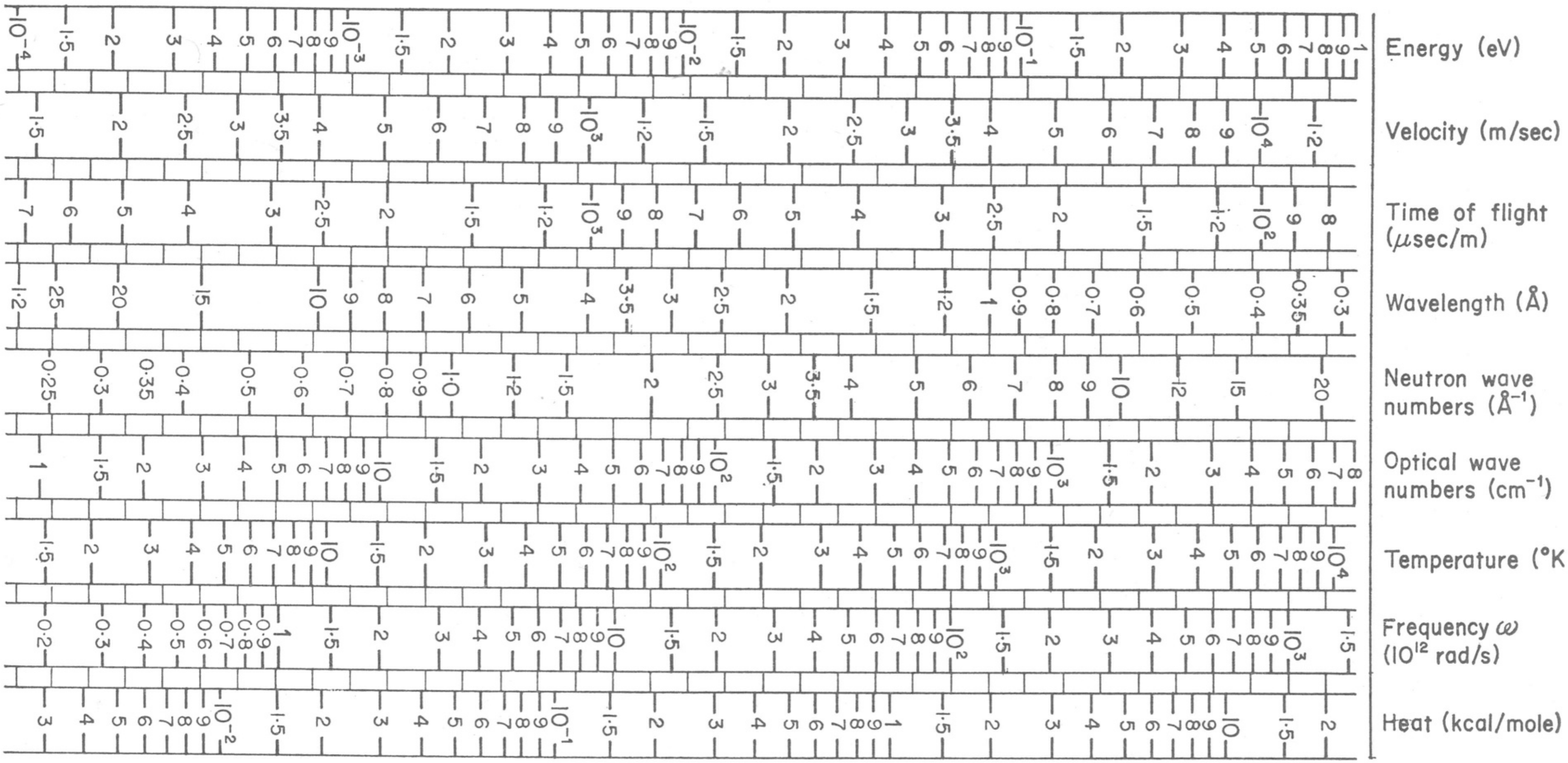
Example λ = 4 Å v=1000m/s
E= 5 meV

$$T = \frac{L}{v} = 252.77\ \mu\mathrm{sec} \cdot \lambda\left[\overset{o}{\mathrm{A}}\right] \cdot L[\mathrm{m}]$$

fortunately large value!

monochromatisation: diffraction or time of flight

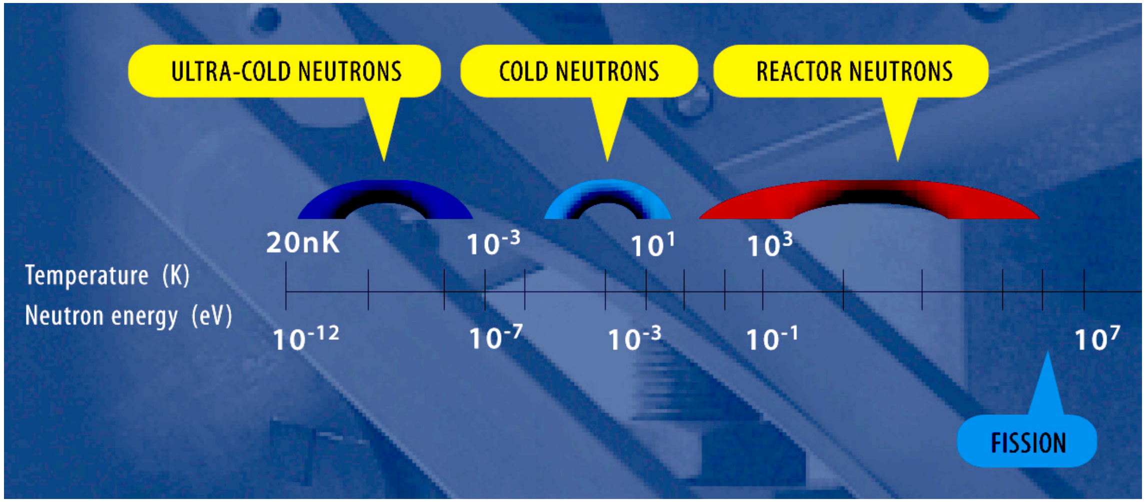
Conversion chart

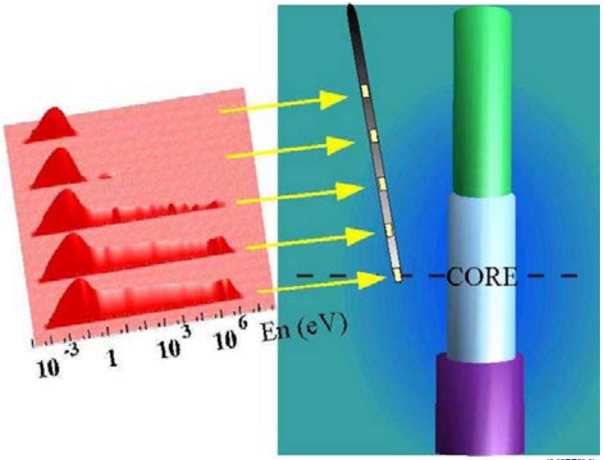


P. A. Egelstaff ed. - Thermal Neutron Scattering Academic Press 1965

Neutron energy ranges

	Energy	Temperature (K)	Wavelength (nm)	velocity (m/s)
Ultra cold neutrons	< 10 µeV	< 0.05	> 30	< 15
Cold neutrons	100 - 5000 µeV	1 - 60	0.4 - 3	150 - 1000
Thermal neutrons	5 - 50 meV	60 - 600	0.13 - 0.4	1000 - 4000
Hot neutrons	0.05 - 0.5 eV	600 - 6000	0.04 - 0.13	4000 - 10000
Epi-Cadmium neutrons	0.5 - 1 eV	> 6,000	< 0.005	> 13 km/s
"Slow' neutrons	1 - 10 eV			
Resonance neutrons	10 - 300 eV			
Intermediate neutrons	0.3 - 1 MeV			
Fast neutrons	1 - 20 MeV			
Relativistic neutrons	> 20 MeV			





ULTRA-COLD NEUTRONS

the very cold side

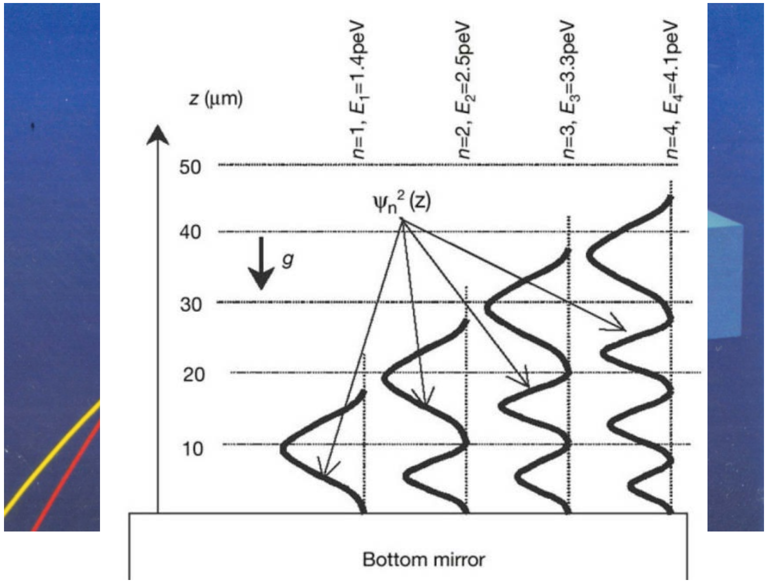
v ≈ 20m/s $\qquad$ $E_{kin} \approx 2\ \mu eV$ $\qquad$ T= 0.023 K

λ = 200 Å

effect of gravity - neutrons are massive!

mirror ~ potential well for ultra-cold neutrons

neutrons are ‘stacked’ at distinct height levels (in the micrometer range!)

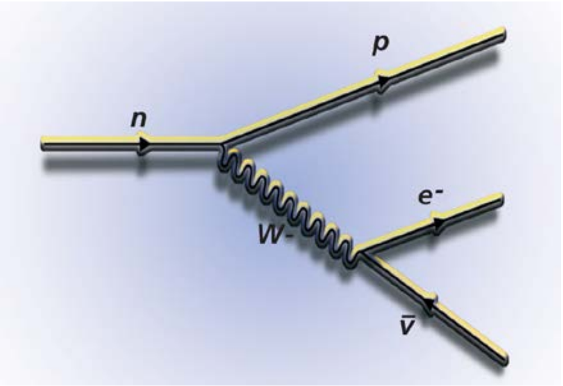


note: cold neutron beams are bent by gravity ~ 1.2 cm at 100 m for 20 Å neutrons

neutrons : objects to study fundamental interactions

neutron β-decay

free neutrons are not forever



KEY MESSSAGES

- neutrons are not elementary particles
- they are not for ever
- neutrons are not only powerful probe, they can be studied as objects

NEUTRON SCATTERING: INTERACTIONS

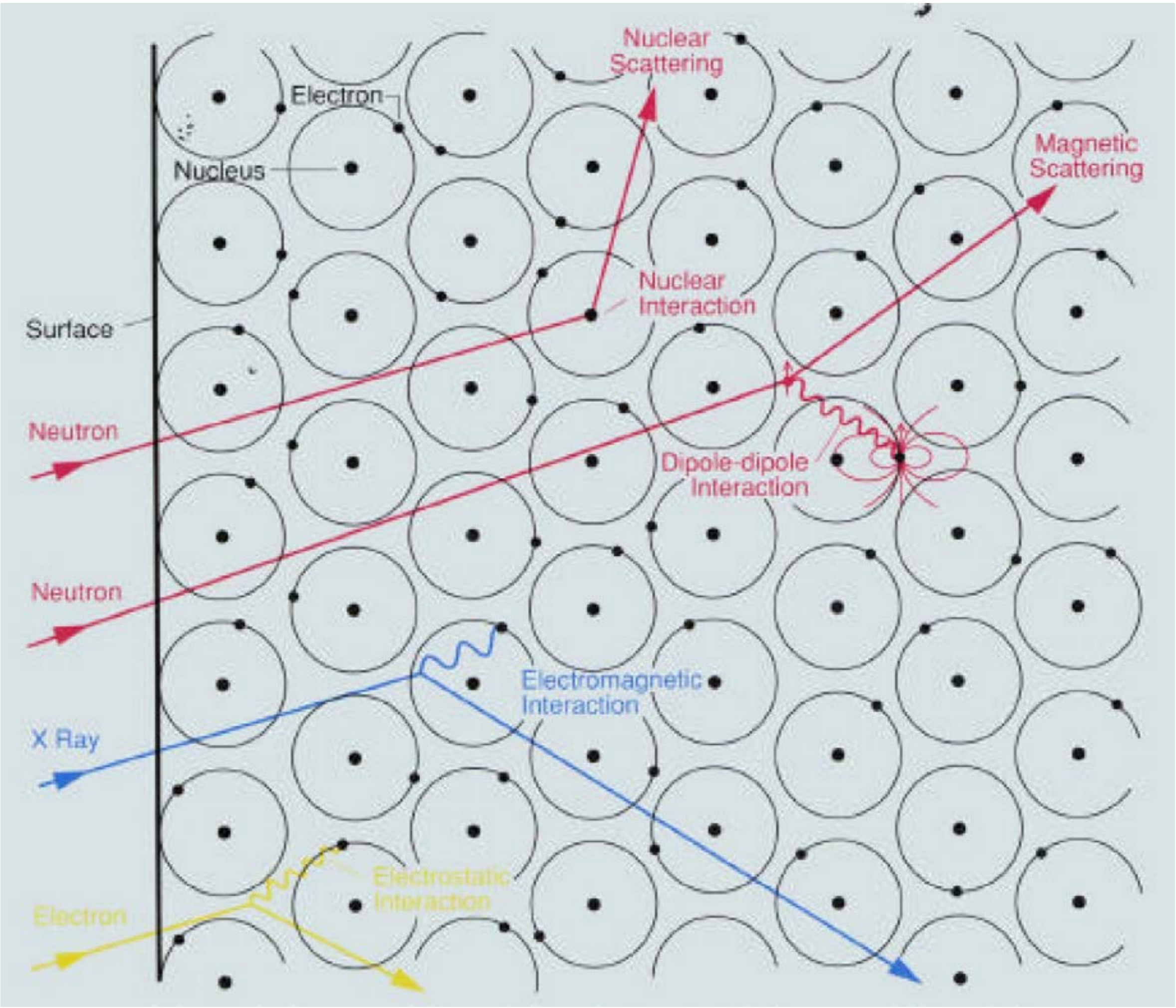
Neutron scattering exploits 'cool' neutrons: 0.05 meV< E_n < 500 meV

0.4 Å < λ < 40 Å

Neutrons

X-rays

Electrons



Neutrons interact with nuclei:

capture: absorption, emission of particles

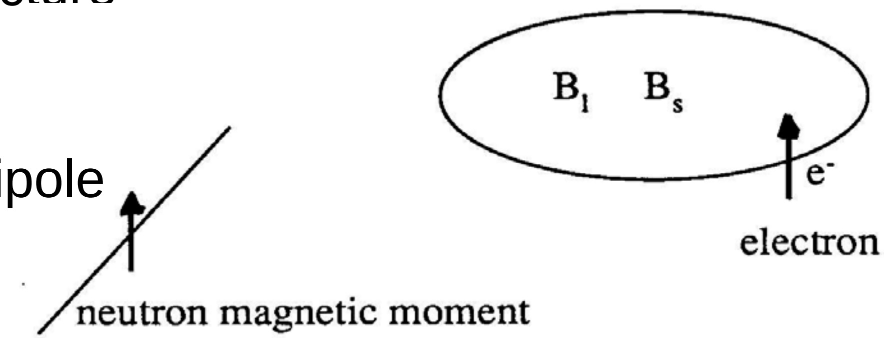
diffusion arising from very short range nuclear forces

neutron wavelength much longer than nucleus size

can't solve neutron structure

Neutrons interact with electrons:

magnetic interactions dipole-dipole



'interactions' lead to 'scattering'

Strength of interactions? Measured through 'scattering cross sections'

In the case of neutrons, nuclear and magnetic scattering are 'equivalent'

Neutrons have no charge, but they interact (extremely weakly) with charges/electrical fields spin-orbit/Schwinger scattering

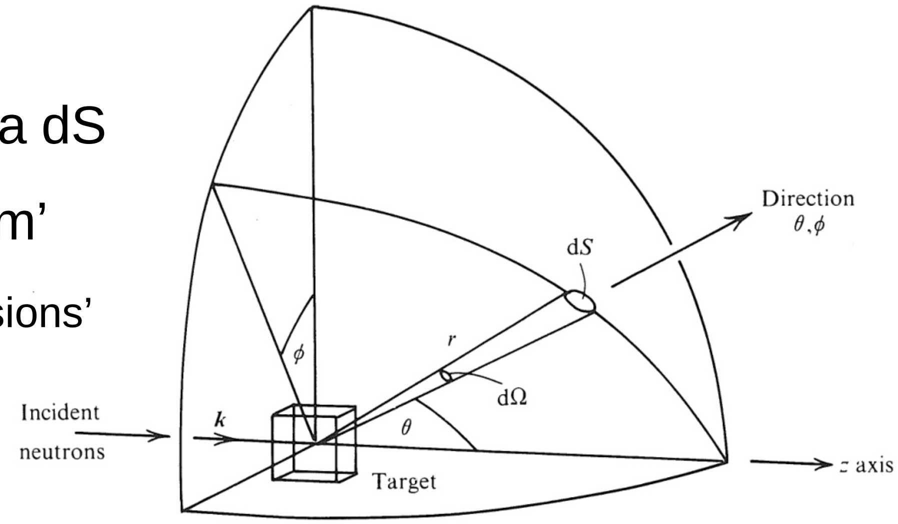
see non-resonant magnetic X-ray scattering

Scattering is measured in terms of cross-sections

neutron counter set up (θ, Φ) with area dS

at a distance *r* from 'scattering system'

large compared compared to sample 'dimensions'



Φ = number of incident neutrons per area per second

σ = number of neutrons scattered per second / Φ an area (barns)

$$\frac{d\sigma}{d\Omega} = \frac{\text{number of neutrons scattered per second into } d\Omega}{\Phi \; d\Omega}$$

$$\frac{d^2\sigma}{d\Omega dE} = \frac{\text{number of neutrons scattered per second into } d\Omega \;\&\; dE}{\Phi \; d\Omega \, dE}$$

applies to all types of scattering events

we have ignored the initial and final neutron spin states – to be seen later …..

HOW NEUTRONS INTERACT WITH MATTER – NUCLEAR SCATTERING

nuclear scattering from a single (fixed) nucleus

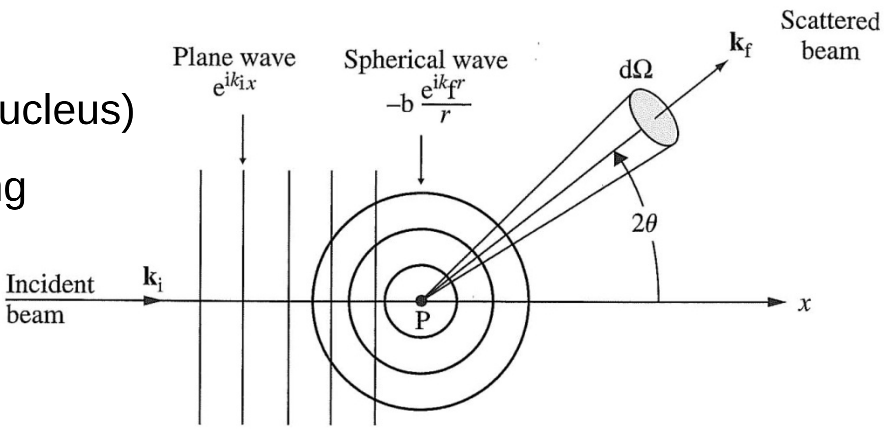
range of nuclear forces << neutron wavelength

point-like scattering s-wave scattering

elastic scattering (fixed nuclei and no change in nucleus)

same velocity before and after scattering

no absorption (far away from resonance)



incident plane wave

$e^{i\mathbf{k}_i . x}$ incident flux = neutron density x velocity = v = $\frac{h}{m_n}k$ neutron density = $\left|e^{i\mathbf{k}_i . x}\right|^2$

spherical scattered wave $-b\,\frac{e^{i\mathbf{k}_f . r}}{r}$ b : scattering length $V(\mathbf{r}) = \frac{2\pi\hbar^2}{m_r} b\,\delta(\mathbf{r})$

of the order of nucleus's 'size'

number of neutrons per second into *dΩ* $v\left|\Psi_{scatt}\right|^2 r^2\, d\Omega = v\,\frac{b^2}{r^2} r^2\, d\Omega = v\; b^2\, d\Omega$

cross sections $\frac{d\sigma}{d\Omega} = b^2$ $\sigma = 4\pi b^2$ b expressed in *fm* 10^{-15}m

scattering amplitude does not vary with scattering angle nuclear scattering from an assembly of nuclei:

atom at $\mathbf{R}_i$ incident wave: $e^{i\mathbf{k}_i.\mathbf{R}_i}$ scattered wave at $\mathbf{R}_i$: $e^{i\mathbf{k}_i\cdot\mathbf{R}_i}\left[-b_i \frac{e^{i\mathbf{k}_f.(\mathbf{r}-\mathbf{R}_i)}}{|\mathbf{r}-\mathbf{R}_i|}\right]$

$$\Psi_{scatt} = \sum_i e^{i\mathbf{k}_i\cdot\mathbf{R}_i}\left[-b_i \frac{e^{i\mathbf{k}_f.(\mathbf{r}-\mathbf{R}_i)}}{|\mathbf{r}-\mathbf{R}_i|}\right]$$

cross section

$$\frac{d\sigma}{d\Omega} = \frac{vdS|\Psi_{scatt}|^2}{vd\Omega} = \frac{dS}{d\Omega}\left|e^{i\mathbf{k}_f.\mathbf{r}}\sum_j b_j \left[\frac{e^{i(\mathbf{k}_i-\mathbf{k}_f).\mathbf{R}_j}}{|\mathbf{r}-\mathbf{R}_j|}\right]\right|^2 = \sum_{i,j} b_i^* b_j e^{-i(\mathbf{k}_i-\mathbf{k}_f).(\mathbf{R}_i-\mathbf{R}_j)}$$

wavevector transfer $\boldsymbol{K}$ is defined by $\boldsymbol{K} = \mathbf{k}_i - \mathbf{k}_f$

beware! X-ray boys use different sign convention!

$$\frac{d\sigma}{d\Omega} = \sum_{i,j} b_i^* b_j e^{-i\boldsymbol{K}.(\mathbf{R}_i-\mathbf{R}_j)}$$

Fourier transform

orders of magnitude:

nuclear scattering lengths, b's, depend on isotope, nuclear eigenstate, and nuclear spin orientation relative to neutron spin

Nuclide	Combined spin	b/fm	Nuclide	Combined spin	b/fm
^{1}H	1	10.85	^{23}Na	2	6.3
	0	−47.50		1	−0.9
^{2}H	$\frac{3}{2}$	9.53	^{59}Co	4	−2.78
	$\frac{1}{2}$	0.98		3	9.91

coherent and incoherent scattering

consider an assembly of similar atoms/ions – spins/isotopes are uncorrelated at different sites

$$\frac{d\sigma}{d\Omega} = \sum_{i,j \text{ averaged over all states}} b_i^* b_j e^{-iK.(\mathbf{R}_i - \mathbf{R}_j)}$$

for a single nucleus $b_i = \langle b \rangle + \delta b_i$ where $\langle \delta b_i \rangle = 0$ taken as real number

$b_i b_j = \langle b \rangle^2 + \langle b \rangle (\delta b_i + \delta b_j) + \delta b_i \delta b_j$ with $\langle \delta b_i \delta b_j \rangle = 0$ unless i=j

$$\langle \delta b_i^2 \rangle = \langle b_i - \langle b \rangle \rangle^2 = \langle b^2 \rangle - \langle b \rangle^2$$

$$\frac{d\sigma}{d\Omega} = \langle b \rangle^2 \sum_{i,j} e^{-iK \cdot (\mathbf{R}_i - \mathbf{R}_j)} + \left(\langle b^2 \rangle - \langle b \rangle^2 \right) N \qquad \sigma_{coh} = 4\pi \langle b \rangle^2 \quad \sigma_{incoh} = 4\pi \left(\langle b^2 \rangle - \langle b \rangle^2 \right)$$

coherent scattering: correlations between different sites

incoherent scattering: correlations on the same site (at different times)

particular to neutron scattering

sources of incoherent scattering:

isotopic distribution and nuclear spin

NUCLEAR SCATTERING – COHERENT/INCOHERENT

If single isotope and zero nuclear spin, no incoherent scattering

If single isotope and non-zero nuclear spin I

nucleus+neutron spin: I+1/2 and I-1/2 scattering length b^+ and b^-

If neutrons and nuclei are un-polarised:

probability 'plus' $f^+ = \frac{I+1}{2I+1}$ probability 'minus' $f^+ = \frac{I}{2I+1}$

$$\langle b \rangle = \frac{1}{2I+1}\left[(I+1)b^+ + Ib^-\right] \qquad \langle b^2 \rangle - \langle b \rangle^2 = \frac{I(I+1)}{(2I+1)^2}\left(b^+ - b^-\right)^2$$

To reduce incoherent scattering (background):

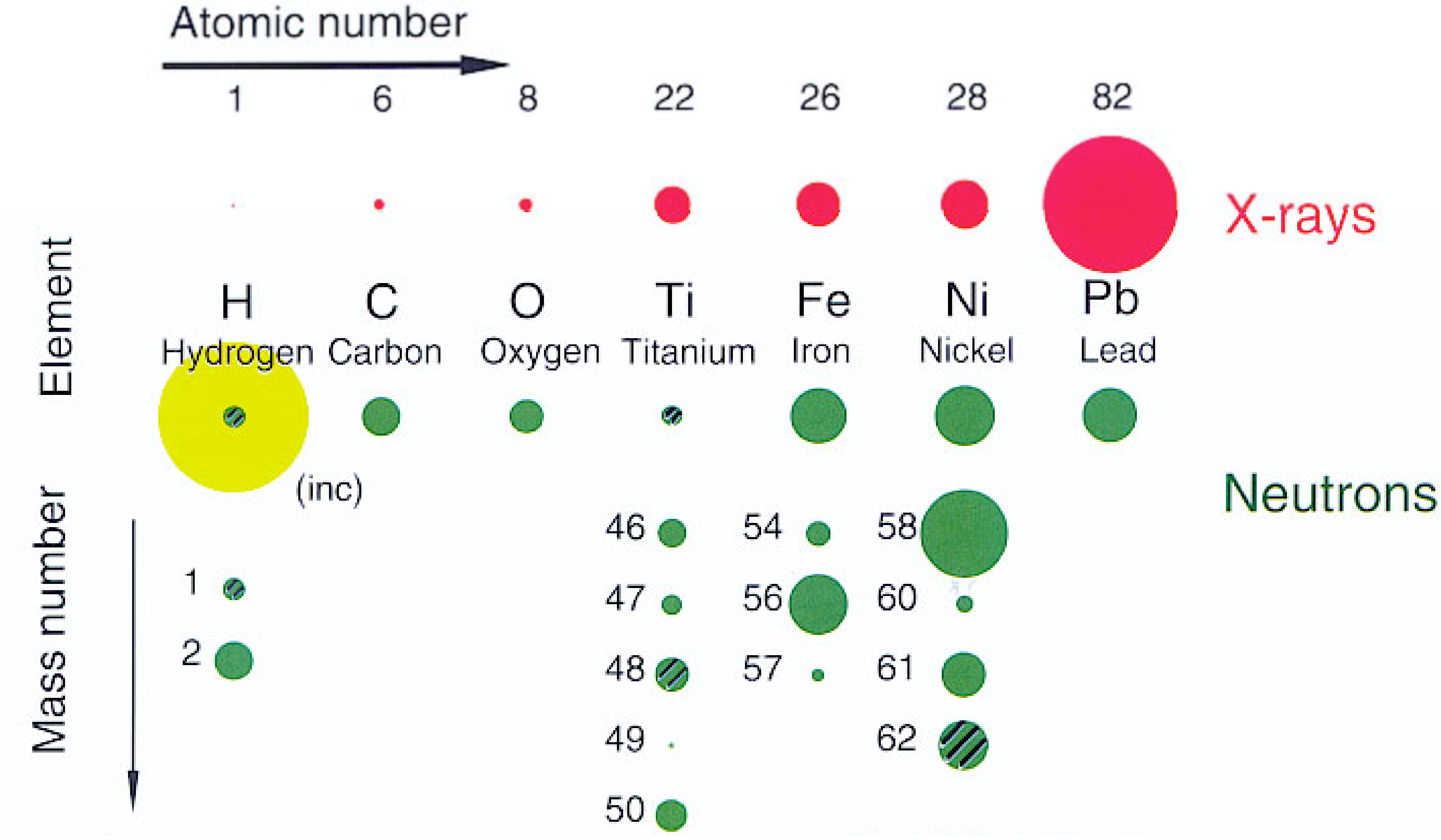
use isotope substitution

use zero nuclear spin isotopes

polarise nuclei and neutrons

NUCLEAR SCATTERING – COHERENT/INCOHERENT

examples of scattering lengths:



neighbouring elements can be easily identified

NUCLEAR SCATTERING – SCATTERING LENGTHS

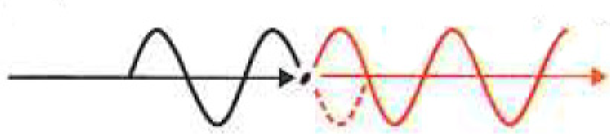
most neutron scattering lengths are positive

(same for X-rays)

phase changes by after scattering

no change in phase at scattering point

positive b



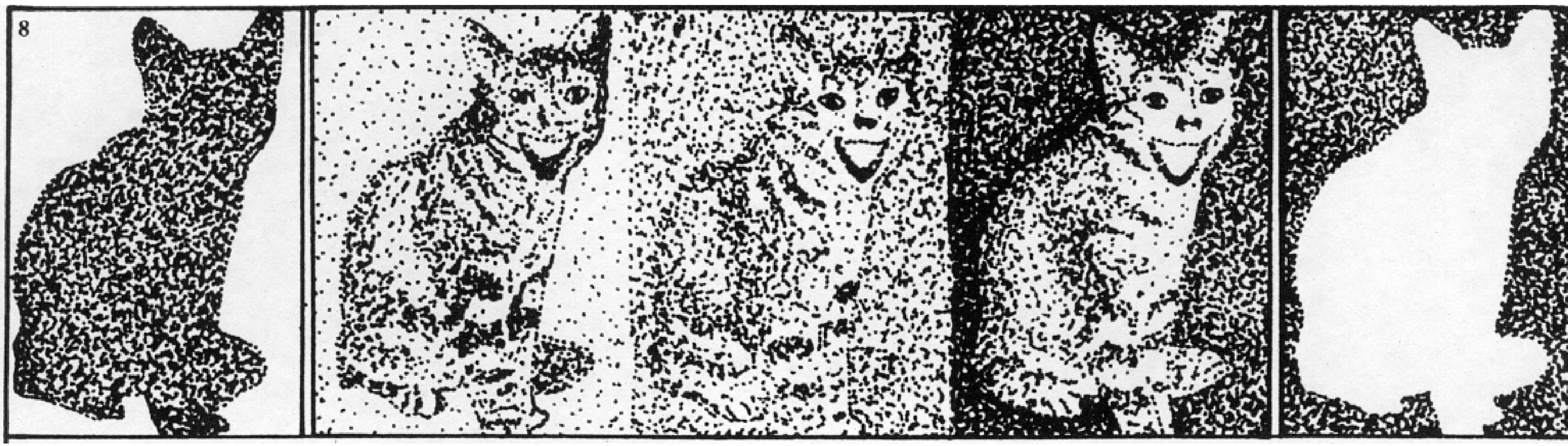
negative b e.g. hydrogen

ZSymbA	p or $T_{1/2}$	I	b_c	b_+	b_-	c	σ_{coh}	σ_{inc}	σ_{scatt}	σ_{abs}
0-N-1	**10.3 MIN**	**1/2**	**-37.0(6)**	**0**	**-37.0(6)**		**43.01(2)**		**43.01(2)**	**0**
1-H			**-3.7409(11)**				**1.7568(10)**	**80.26(6)**	**82.02(6)**	**0.3326(7)**
1-H-1	99.985	1/2	-3.7423(12)	10.817(5)	-47.420(14)	+/-	1.7583(10)	80.27(6)	82.03(6)	0.3326(7)
1-H-2	0.0149	1	6.674(6)	9.53(3)	0.975(60)		5.592(7)	2.05(3)	7.64(3)	0.000519(7)
1-H-3	12.26 Y	1/2	4.792(27)	4.18(15)	6.56(37)		2.89(3)	0.14(4)	3.03(5)	< 6.0E-6
2-He			**3.26(3)**				**1.34(2)**	**0**	**1.34(2)**	**0.00747(1)**
2-He-3	0.00013	1/2	5.74(7)	4.374(70)	9.835(77)	E	4.42(10)	1.532(20)	6.0(4)	5333.0(7.0)
2-He-4	0.99987	0	3.26(3)				1.34(2)	0	1.34(2)	0
3-Li			**-1.90(3)**				**0.454(10)**	**0.92(3)**	**1.37(3)**	**70.5(3)**
3-Li-6	7.5	1	2.0(1)	0.67(14)	4.67(17)	+/-	0.51(5)	0.46(5)	0.97(7)	940.0(4.0)
3-Li-7	92.5	3/2	-2.22(2)	-4.15(6)	1.00(8)	+/-	0.619(11)	0.78(3)	1.40(3)	0.0454(3)

NUCLEAR SCATTERING – CONTRAST VARIATION

Mixing ^1H and ^2H (H and D) allows contrast variation

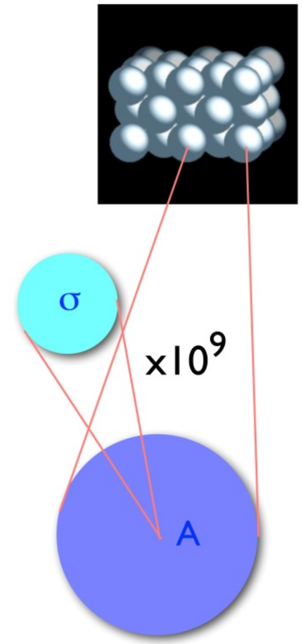
- full contrast : external shapes
- intermediate contrast : details



100 % H_2O ⟶ 100 % D_2O

useful for imaging and scattering (small angle neutron scattering, liquids)

SCATTERING LENGTHS



Orders of magnitude - numbers for neutron scattering

cross sections are in ~ barns 1 barn = 10^{-24} cm^2

area per atom ~ 10 $Å^2$ = 10 10^8 barns

1 atom gives 10^{-9} probability scattering when beam hits it!

to obtain 1% scattering (over 4π) requires 10^7 layers of atoms

~ 0.1 cm of sample!

Take a single atom of C: b = 6.65 pm = 6.65 10^{-15}m = 6.65$10^{-13}$cm

σ = 5.56 barns

The probability to observe a single scattered neutron per second requires a huge flux: $I_0 = \Phi\sigma \quad \Phi = I_0/\sigma = 1/\left(5.56\,10^{-24}\right) \approx 1.8\,10^{23}$ particules/cm^2/sec

actual neutron flux ≈ 10^7 n/cm^2/sec, we must either wait for

$1.8\,10^{16}$ sec ≈ 570 million years   or use ~2 10^{16} atoms ~ 0.4 μg only

SCATTERING LENGTHS

Numbers for neutron scattering

typical neutron flux ~10^7 n/cm^2/sec

sample volumes in the fraction of cm^3 range

counting time for ‘incoherent scattering’ from Vanadium (σ ~ 5 barns)

sample volume 1x1x0.1 cm^3 i.e. ~ 8.7 10^{21} atoms

count rate ~ 4 10^5 n/sec over 4π

detector angular aperture ~ 1% leads to ~ 4 10^3 n/sec

Questions about statistics:

• experimental data are ‘counts in the detector’, independent events but with a fixed probability (scattering cross sections!): Poisson’s like

• usual goal is to achieve 1% error per information unit:

- requires ~10,000 counts per bin
- i.e. ~ 0.5 -10 minutes for typical elastic peak ($\frac{d\sigma}{d\Omega}$)
- i.e. at least 10 times longer for inelastic studies ($\frac{d^2\sigma}{d\Omega dE}$)

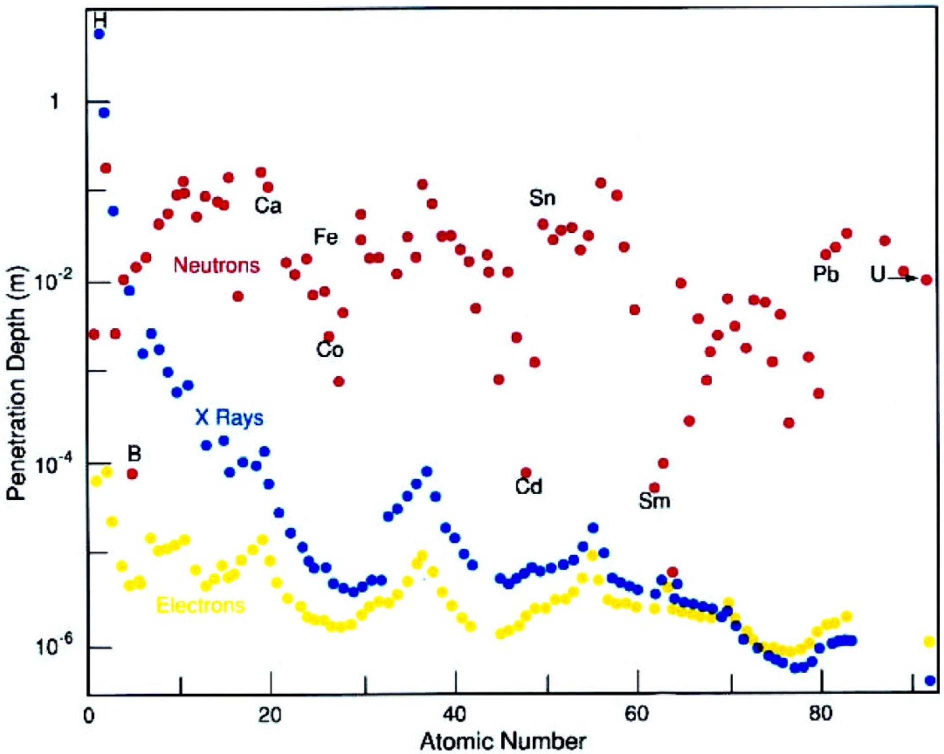
SCATTERING LENGTHS - CONSEQUENCES

Absorption

essentially neutron capture

random variation with

atomic number and isotopes



Applications for detection

neutrons are captured by nuclei

capture creates charged particles

recoiling particles ionise gaseous materials

$$^{3}_{2}He + ^{1}_{0}n \rightarrow ^{3}_{1}H + p + 0.764\ MeV$$

$$^{10}_{5}B + ^{1}_{0}n \rightarrow ^{7}_{3}Li + ^{4}_{2}He + 2.3\ MeV$$

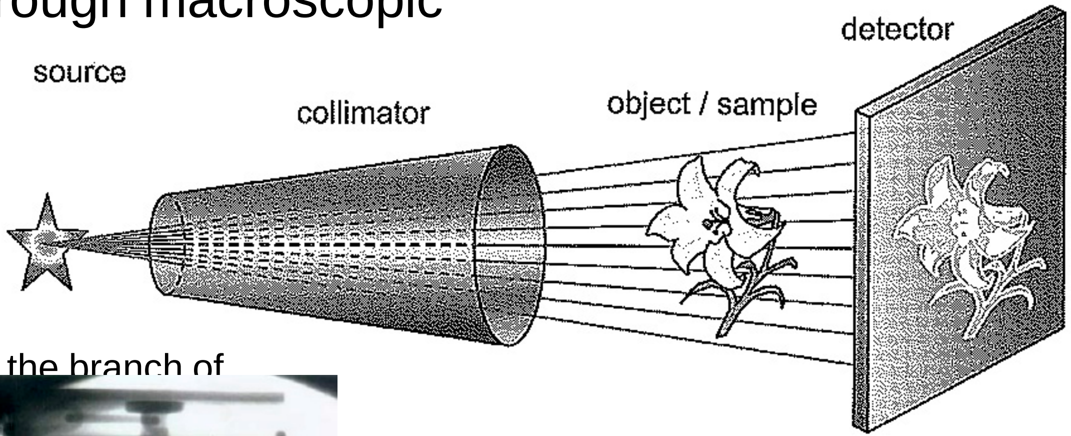
KEY MESSAGES

- neutrons interact with nuclei
    - random variation of b’s with atomic number
    - isotropic scattering amplitude
    - contrast and isotopic substitution
    - low absorption
    - coherent and incoherent scattering

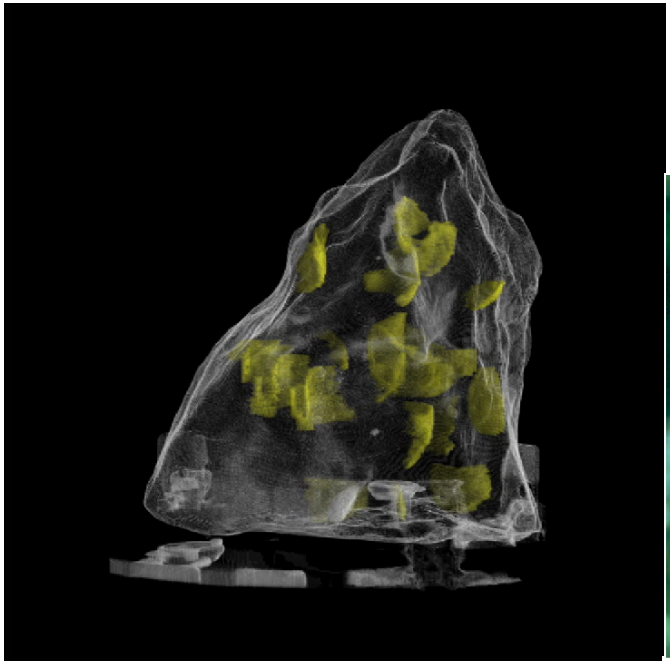
Imaging with neutrons

selectivity of neutrons – selective imaging through absorption

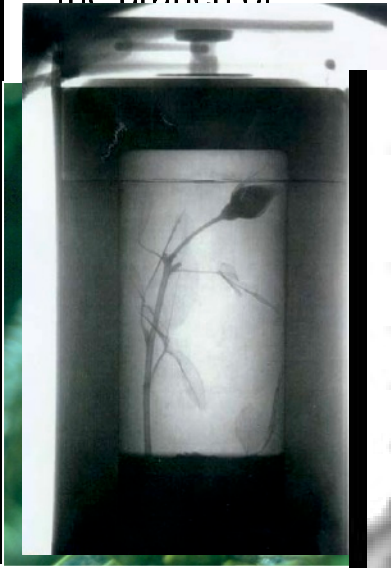
direct transmission through macroscopic



a piece of rock from the Antartic



the branch of



SCATTERING LENGTHS - CONSEQUENCES

Refractive index for neutrons

For a single nucleus, Fermi pseudo-potential $V(\mathbf{r}) = \frac{2\pi\hbar^2}{m_r} b\,\delta(\mathbf{r})$

Inside matter, $\overline{V} = \frac{2\pi\hbar^2}{m}\rho$ scattering length density $\rho = \frac{1}{\text{volume}}\sum_i b_i$

neutrons obey Schrödinger's equation $\left[\nabla^2 + \frac{2m}{\hbar^2}\left(E - \overline{V}\right)\right]\Psi(\mathbf{r}) = 0$

In vacuo, $\overline{V} = 0$ and $E = E_{cin}$ $k_i^2 = 2mE/\hbar^2$

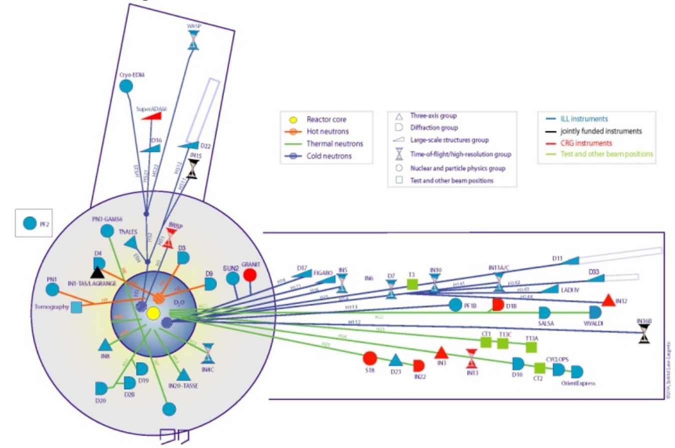
In the medium, $k_f^2 = 2m\left(E - \overline{V}\right)/\hbar^2 = k_i^2 - 4\pi\rho$ $n = k_f/k_i \approx 1 - \lambda^2\rho/2\pi$

With b>0, n<1 and neutrons are externally reflected by most materials

SCATTERING LENGTHS - CONSEQUENCES

Applications

neutron guides: critical angle $\gamma_c \approx \lambda\sqrt{\rho/\pi}$ Ni (Ni^{58}) $\gamma_c \approx 0.1° Å^{-1}$

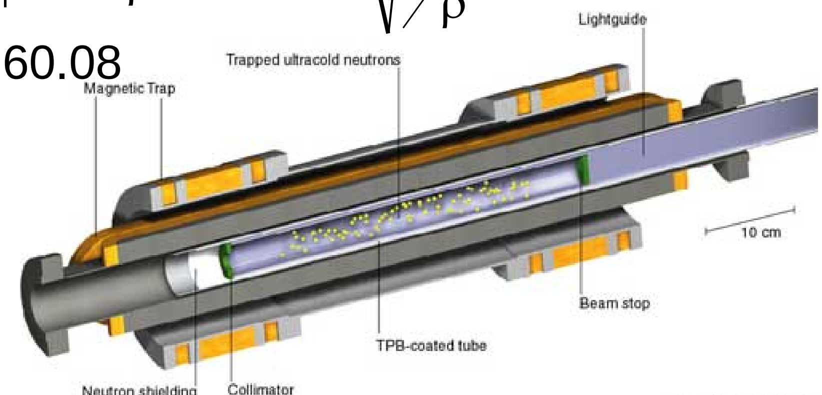


Neutron bottles: a bottle imposes n = o, $k_i^2 = 4\pi\rho$ or $\lambda = \sqrt{\pi/\rho}$

example SiO_2 density: 2.66 molecular weight: 60.08

$$N = 10^{-24}(2.66/60.08)N_{Avogadro} = 0.0267\ Å^{-3}$$

$$\rho = N(b_{Si} + 2b_O) = 4.21\,10^{-6}\ Å^{-2}$$



Nature 403, 62 (2000)

This works with very cold neutrons: $\lambda > 864$ Å v~ 4.6 m/s E~ 0.1μeV ~ 1mK!!!

HOW TO 'ACCUMULATE' INTENSITIES?

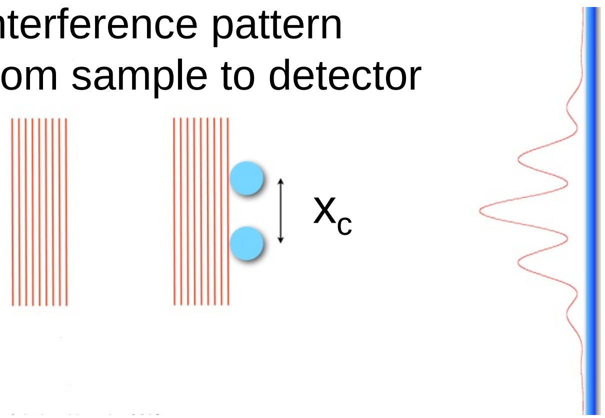
So far, we have added individual scattering intensities. How to combine them?

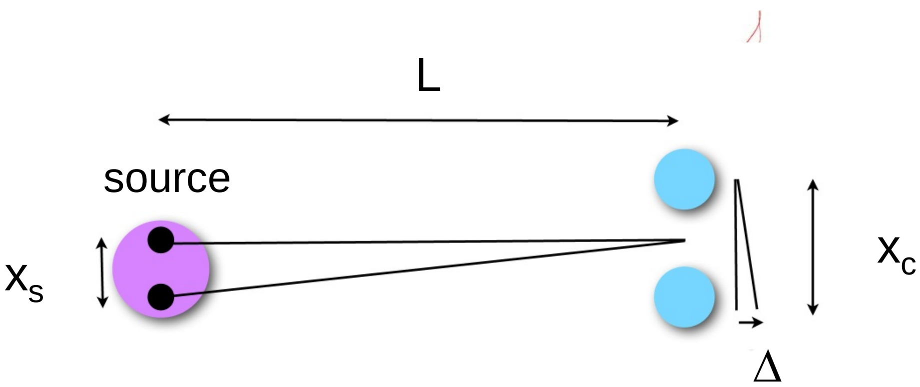
neutron sources are chaotic:

emission over 4π, at ill-defined times with wide distribution of energies, neutrons are moderated

Do we have to take the neutron source into account?

interference pattern
from sample to detector





$x_c / \Delta_{max} = L / x_s \qquad x_c = (\lambda / 2\pi) L / x_s = (1/k_i) L / x_s$

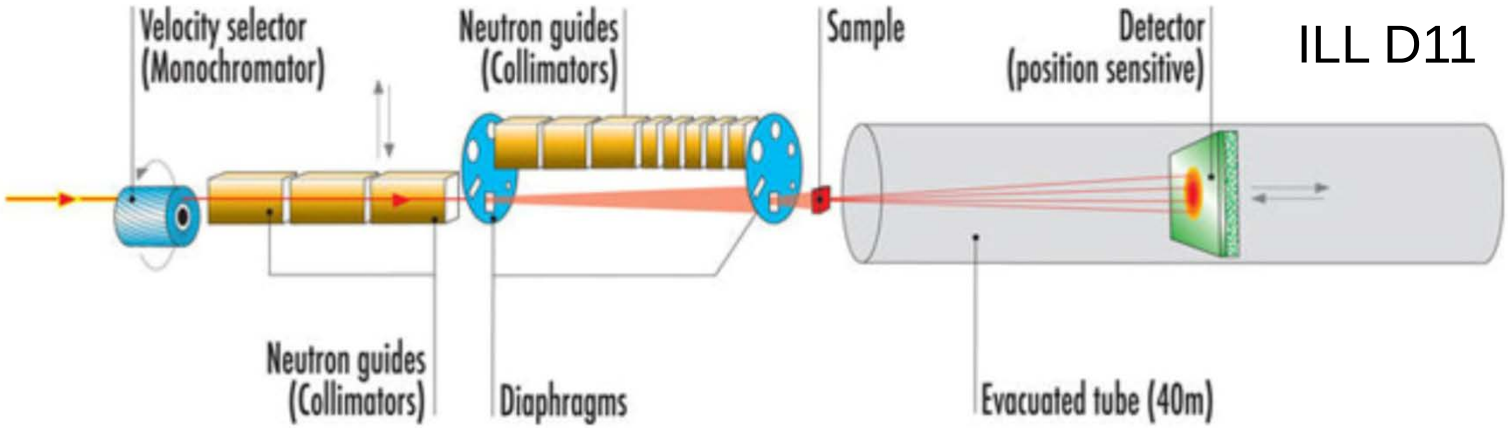
typical values: $x_c \sim y_c \sim 10$ nm (X-ray tomography with pin-hole source $x_c \sim 500 \mu m$)

$$z_c = \lambda^2 / \Delta\lambda \approx 50 \times \lambda \approx 10 \text{ nm}$$

particular case: small angle neutron scattering

what is the largest object that can be measured?

ILL D11



size given by the lateral coherence length $x_c = (\lambda/2\pi) L / x_s$

L~40 m, λ = 12Å, x_s~10^{-2} m (adjustable) $x_c \sim 1\mu m$

Typically, objects smaller than 1μm are studied by scattering methods

objects larger than 1μm are 'imaged'

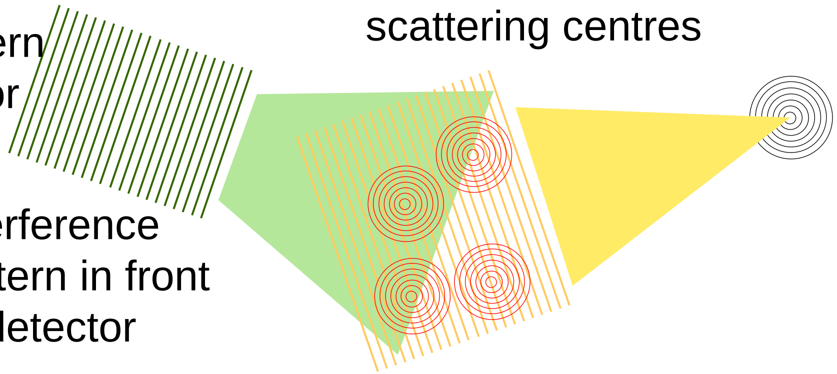
In general, ignore coherence of neutron sources

and focus on constructive interference of scattered waves

HOW TO ‘ACCUMULATE’ INTENSITIES?

Assume plane waves for incident neutrons

‘far away’ from source



How to combine scattered waves (amplitudes and phases)?

HOW TO 'ACCUMULATE' INTENSITIES?

Different ways to combine scattered waves

Born approximation – kinematic theory

- neutron wavefunction un-perturbed inside sample
- in general OK, away from Bragg reflections and total reflection

Dynamical theory of scattering

- takes into account of the change in the neutron wave in the system
- see refractive index – but generalised for all scattering vectors
- important near total reflection
- may be needed near Bragg reflections from perfect crystals with highly collimated beams

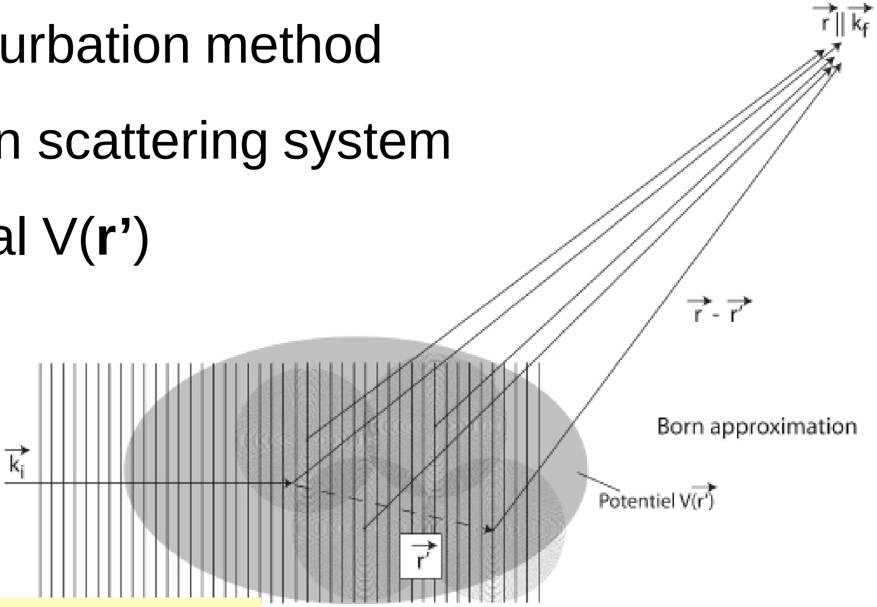
In most cases, kinematic theory applies for neutrons

HOW TO 'ACCUMULATE' INTENSITIES?

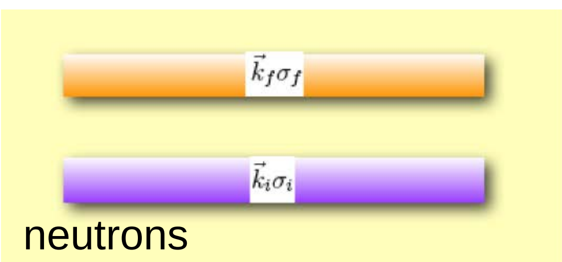
Born approximation, i.e. first order perturbation method

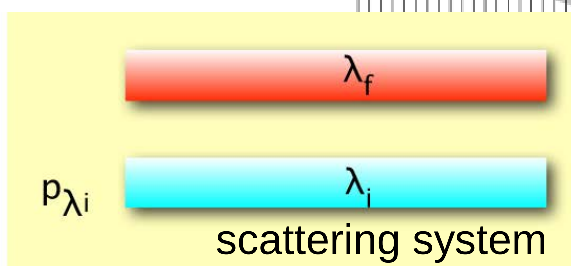
The incident wave travel unperturbed in scattering system

scattering centres with potential V(**r'**)



From the definition, $$\frac{d\sigma}{d\Omega} = \frac{\sum_{k_f \, in \, d\Omega} W_{k_i,\lambda_i \to k_f,\lambda_f}}{\Phi \; d\Omega}$$





Use Fermi's Golden rule to calculate transition probabilities $\sum_{k_f \, in \, d\Omega} W_{k_i,\lambda_i \to k_f,\lambda_f}$

$$\sum_{k_f \, in \, d\Omega} W_{k_i,\lambda_i \to k_f,\lambda_f} = \frac{2\pi}{h} \rho_{\mathbf{k}_f} \left| \langle \mathbf{k}_f \lambda_f | V | \mathbf{k}_i \lambda_i \rangle \right|^2$$

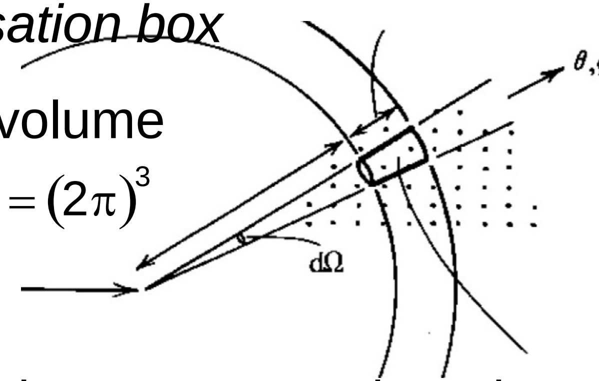
where $\rho_{\mathbf{k}_f}$ is the density of **k**-states in $d\Omega$ per unit energy range

HOW TO 'ACCUMULATE' INTENSITIES?

some algebra and manipulations

$e^{i\mathbf{k}_i\cdot\mathbf{r}}$ as incident wavelength - plane wave in *normalisation box*

neutron states are periodic in **k**-space, unit cell volume



neutron flux $\Phi = \frac{h}{m_n} k$

$v_k = (2\pi)^3$

$\rho_{\mathbf{k}_f} dE_f = \frac{1}{v_k} k_f^2 \, dk_f \, d\Omega$ is the number of n-states in dΩ between E_f and $E_f + dE_f$

kinetic energy: $dE_f = \frac{h^2}{m_n} k_f \, dk_f$ therefore: $\rho_{\mathbf{k}_f} = \frac{1}{(2\pi)^3} k_f \frac{m_n}{h^2} d\Omega$

scattering cross section with $\mathbf{k}_i$, λ_i and λ_f fixed $\left(\frac{d\sigma}{d\Omega}\right)_{\lambda_i \to \lambda_f} = \frac{k_f}{k_i}\left(\frac{m_n}{2\pi h^2}\right)^2 \left|\langle \mathbf{k}_f \lambda_f | V | \mathbf{k}_i \lambda_i \rangle\right|^2$

energy of neutrons+ scattering system must be conserved $E_i + E_{\lambda_i} = E_f + E_{\lambda_{fi}}$

$$\left(\frac{d^2\sigma}{dE_f \, d\Omega}\right)_{\lambda_i \to \lambda_f} = \frac{k_f}{k_i}\left(\frac{m_n}{2\pi h^2}\right)^2 \left|\langle \mathbf{k}_f \lambda_f | V | \mathbf{k}_i \lambda_i \rangle\right|^2 \delta\left(E_i - E_f + E_{\lambda_i} - E_{\lambda_f}\right)$$

partial differential cross section for all scattering potentials more algebra and manipulations

insert a potential V – Fermi pseudo-potential - short range, scalar and central

V has the form $V = \sum_j V_j(\mathbf{r} - \mathbf{R}_j) = \sum_j V_j(\mathbf{x}_j)$ with $\mathbf{x}_j = \mathbf{r} - \mathbf{R}_j$

$$\langle \mathbf{k}_f \lambda_f | V | \mathbf{k}_i \lambda_i \rangle = \sum_j \int \chi^*_{\lambda_f} \exp(-i\mathbf{k}_f \cdot \mathbf{r}) V(\mathbf{x}_j) \chi_{\lambda_i} \exp(i\mathbf{k}_i \cdot \mathbf{r}) \, d\mathbf{R}_1 d\mathbf{R}_2 d\mathbf{R}_3 \ldots d\mathbf{R}_j \ldots d\mathbf{r}$$

$$\langle \mathbf{k}_f \lambda_f | V | \mathbf{k}_i \lambda_i \rangle = \sum_j V_j(\boldsymbol{K}) \langle \lambda_f | \exp(i\boldsymbol{K} \cdot \mathbf{R}_j) | \lambda_i \rangle \quad \text{where} \quad \boldsymbol{K} = \mathbf{k}_i - \mathbf{k}_f$$

$$V_j(\boldsymbol{K}) = \int V_j(\mathbf{x}_j) \exp(i\boldsymbol{K} \cdot \mathbf{x}_j) \, d\mathbf{x}_j$$

$$\langle \lambda_f | \exp(i\boldsymbol{K} \cdot \mathbf{R}_j) | \lambda_i \rangle = \int \chi^*_f \exp(i\boldsymbol{K} \cdot \mathbf{R}_j) \chi^*_f \, d\mathbf{R}_1 d\mathbf{R}_2 d\mathbf{R}_3 \ldots d\mathbf{R}_j \ldots$$

Fermi pseudo-potential $V_j(\mathbf{x}_j) = \frac{2\pi h^2}{m} b_j \delta(\mathbf{x}_j)$ $\quad V_j(\boldsymbol{K}) = \frac{2\pi h^2}{m} b_j$

$$\left(\frac{d^2\sigma}{dE_f \, d\Omega}\right)_{\lambda_i \to \lambda_f} = \frac{k_f}{k_i} \left| \sum_j b_j \langle \lambda_f | \exp(i\boldsymbol{K} \cdot \mathbf{R}_j) | \lambda_i \rangle \right|^2 \delta(E_i - E_f + E_{\lambda_i} - E_{\lambda_f})$$

even more algebra and manipulations

introduce time and energy to reach thermodynamics

$$\delta\left(E_i - E_f + E_{\lambda_i} - E_{\lambda_f}\right) = \frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty} \exp\left\{ i\left(E_{\lambda_f} - E_{\lambda_i}\right) t/\hbar \right\} \exp(-i\omega t)\,dt \qquad \hbar\omega = E_i - E_f$$

$$\left(\frac{d^2\sigma}{dE_f\,d\Omega}\right)_{\lambda_i \to \lambda_f}$$

$$= \frac{k_f}{k_i}\sum_{jj'} b_j b_{j'} \left\langle \lambda_i \middle| \exp\left(-i\boldsymbol{K}\cdot\mathbf{R}_{j'}\right) \middle| \lambda_f \right\rangle \left\langle \lambda_f \middle| \exp\left(i\boldsymbol{K}\cdot\mathbf{R}_{j}\right) \middle| \lambda_i \right\rangle \times \frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty} \exp\left\{ i\left(E_{\lambda_f} - E_{\lambda_i}\right) t/\hbar \right\} \exp(-i\omega t)\,dt$$

$$\left(\frac{d^2\sigma}{dE_f\,d\Omega}\right)_{\lambda_i \to \lambda_f} = \frac{k_f}{k_i}\,\frac{1}{2\pi\hbar}\sum_{jj'} b_j b_{j'}$$

$$\times \int_{-\infty}^{+\infty} \left\langle \lambda_i \middle| \exp\left(-i\boldsymbol{K}\cdot\mathbf{R}_{j'}\right) \middle| \lambda_f \right\rangle \left\langle \lambda_f \middle| \exp(iHt/\hbar)\exp\left(i\boldsymbol{K}\cdot\mathbf{R}_{j}\right)\exp(-iHt/\hbar) \middle| \lambda_i \right\rangle \exp(-i\omega t)\,dt$$

where *H* is the Hamiltonian of the scattering system

Introduce time-dependent operators $\mathbf{R}_j(t) = \exp(iHt/\hbar)\exp\left(i\boldsymbol{K}\cdot\mathbf{R}_j\right)\exp(-iHt/\hbar)$

To get measured cross section, sum over all final λ_f (with fixed $\lambda_{i)}$ and average over all λ_i.

$$\delta\left(E_i - E_f + E_{\lambda_i} - E_{\lambda_f}\right) = \frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty} \exp\left\{i\left(E_{\lambda_f} - E_{\lambda_i}\right)t/\hbar\right\}\exp(-i\omega t)\,dt \qquad \hbar\omega = E_i - E_f$$

Introduce partition function $Z = \sum_\lambda \exp\left(-E_\lambda / k_B T\right)$, probability $p_\lambda = \frac{1}{Z}\exp\left(-E_\lambda / k_B T\right)$

$$\left(\frac{d^2\sigma}{dE_f\, d\Omega}\right) = \sum_{\lambda_i \lambda_f} p_{\lambda_i}\left(\frac{d^2\sigma}{dE_f\, d\Omega}\right)_{\lambda_i \to \lambda_f}$$

$$= \frac{k_f}{k_i}\frac{1}{2\pi\hbar}\sum_{jj'} b_j b_{j'} \int_{-\infty}^{+\infty}\left\langle \exp\left\{-i\boldsymbol{K}\cdot\mathbf{R}_{j'}(0)\right\}\exp\left\{i\boldsymbol{K}\cdot\mathbf{R}_j(t)\right\}\right\rangle \exp(-i\omega t)\,dt$$

A compact form, but not easy to calculate:

measure of ‘pair correlation functions’

information on scattering system contained in time-dependent ops.

and wavefunctions

Consider a simple system with a single element but different b's

$$\frac{d^2\sigma}{d\Omega dE_f} = \frac{k_f}{k_i}\frac{1}{2\pi h}\sum_{jj'}\langle b_{j'} b_j\rangle \int_{-\infty}^{+\infty}\langle j', j\rangle \exp(-i\omega t)\, dt \qquad \langle b\rangle = \sum_i f_i b_i \qquad \langle b^2\rangle = \sum_i f_i b_i^2$$

no correlation between b's on different sites $\langle b_{j'} b_j\rangle = \langle b\rangle^2,\ j' \neq j \qquad \langle b_{j'} b_j\rangle = \langle b^2\rangle,\ j' = j$

$$\left.\frac{d^2\sigma}{d\Omega dE_f}\right)_{coh} = \frac{\sigma_{coh}}{4\pi}\frac{k_f}{k_i}\frac{1}{2\pi h}\sum_{jj'}\int_{-\infty}^{+\infty}\left\langle \exp\left\{-i\mathbf{K}\cdot\mathbf{R}_{j'}(0)\right\}\exp\left\{i\mathbf{K}\cdot\mathbf{R}_j(t)\right\}\right\rangle \exp(-i\omega t)\, dt$$

$$\left.\frac{d^2\sigma}{d\Omega dE_f}\right)_{incoh} = \frac{\sigma_{incoh}}{4\pi}\frac{k_f}{k_i}\frac{1}{2\pi h}\sum_{j}\int_{-\infty}^{+\infty}\left\langle \exp\left\{-i\mathbf{K}\cdot\mathbf{R}_{j}(0)\right\}\exp\left\{i\mathbf{K}\cdot\mathbf{R}_j(t)\right\}\right\rangle \exp(-i\omega t)\, dt$$

Coherent scattering: correlation between the position of the same nucleus at different times and correlation between the positions of different nuclei at different times

interference effects

Incoherent scattering: only correlation between the position of the same nucleus at different times

no interference effects

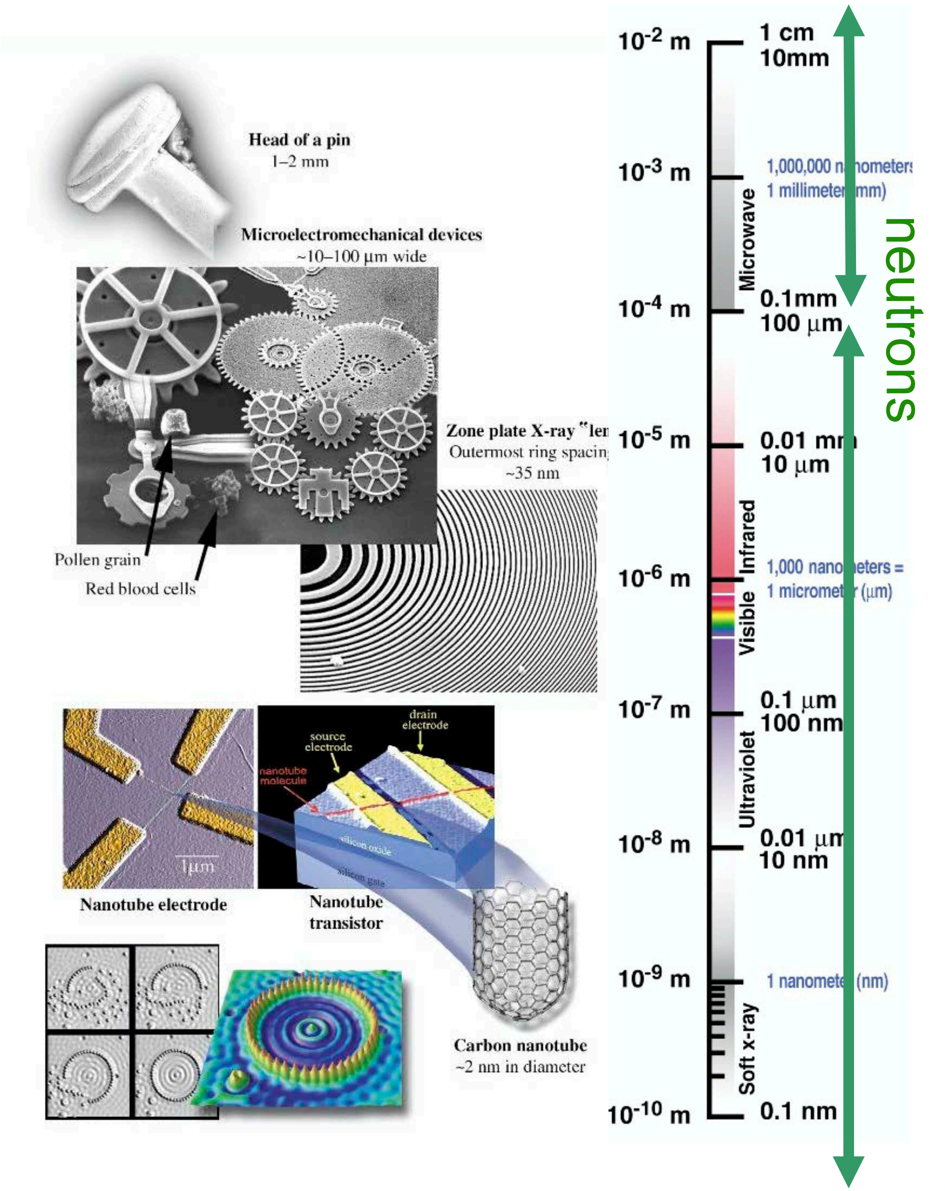
KEY MESSAGE

• neutron scattered intensities are proportional to space and time Fourier transforms of site correlation functions

KEY MESSAGE

neutrons cover a wide range of length scales

imaging/scattering



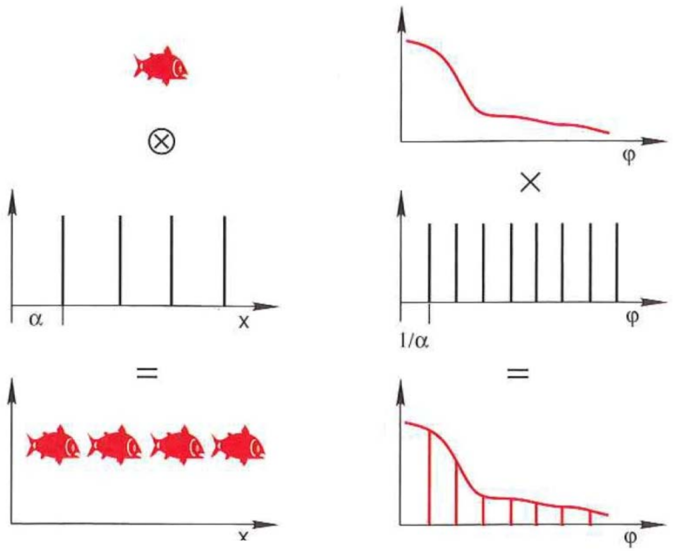
Examples of objects on a lattice - crystalline/ordered materials

Real space

1-d system:

convolution of objects and lattice of Dirac functions

N objects, N large

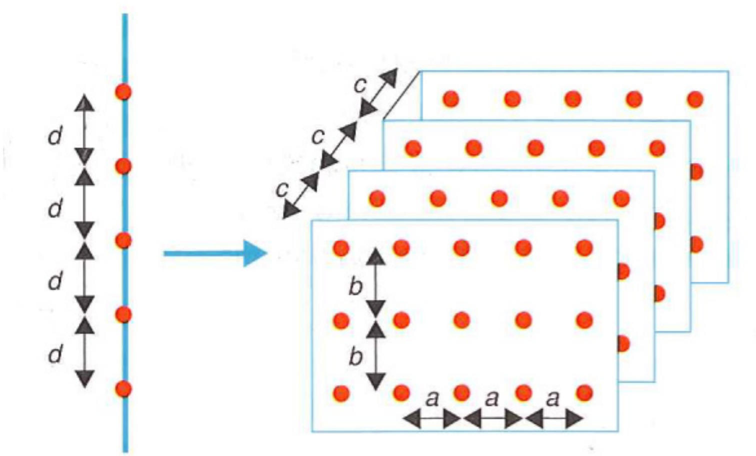


Reciprocal space

1-d system:

Fourier transforms

For N large



Similarly we define associated reciprocal spaces that reflect the symmetry and periodicities of real space lattices

Crystalline materials:

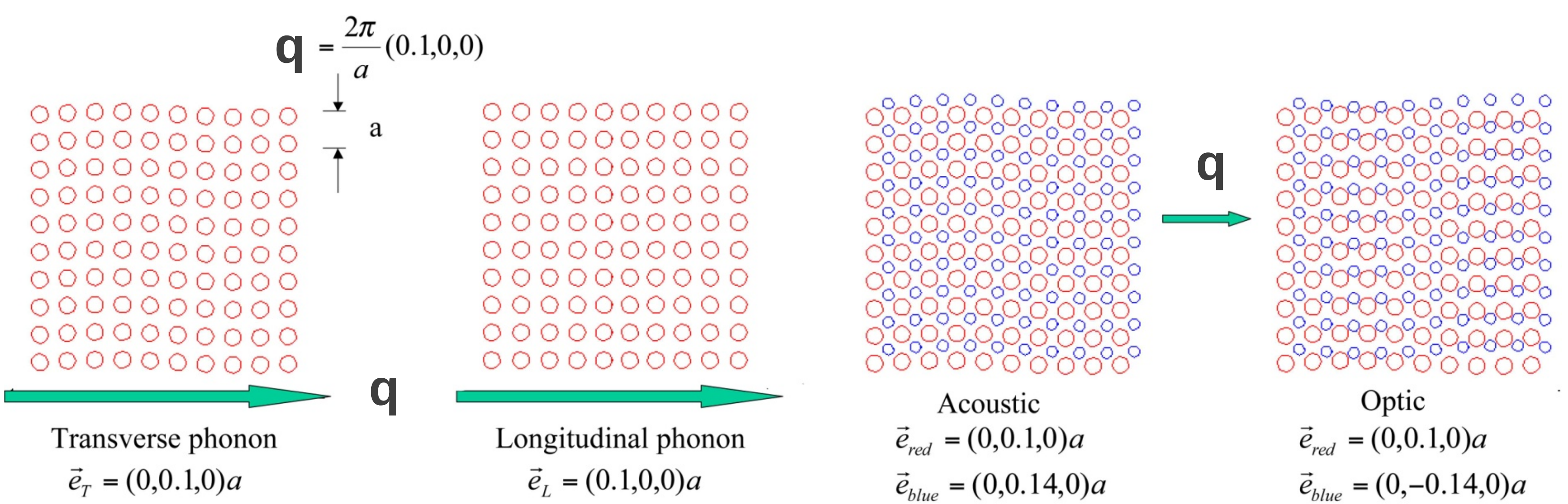
all atoms (nuclei) have an equilibrium position and they move about it

atom in cell *j*: $\mathbf{R}_j(t) = \mathbf{j} + \mathbf{u}_j(t)$ can be generalised to non-Bravais lattices

$$\sum_{jj'}\left\langle \exp\{-\mathrm{i}\boldsymbol{K}\cdot\mathbf{R}_{j'}(0)\}\exp\{\mathrm{i}\boldsymbol{K}\cdot\mathbf{R}_j(t)\}\right\rangle = \mathrm{N}\sum_j \exp(\mathrm{i}\boldsymbol{K}\cdot\mathbf{j})\left\langle \exp\{-\mathrm{i}\boldsymbol{K}\cdot\mathbf{u}_0(0)\}\exp\{\mathrm{i}\boldsymbol{K}\cdot\mathbf{u}_j(t)\}\right\rangle$$

$$\sum_{j}\left\langle \exp\{-\mathrm{i}\boldsymbol{K}\cdot\mathbf{R}_{j}(0)\}\exp\{\mathrm{i}\boldsymbol{K}\cdot\mathbf{R}_j(t)\}\right\rangle = \mathrm{N}\left\langle \exp\{-\mathrm{i}\boldsymbol{K}\cdot\mathbf{u}_0(0)\}\exp\{\mathrm{i}\boldsymbol{K}\cdot\mathbf{u}_0(t)\}\right\rangle$$

displacements **u**(*t*) can be expressed in terms of normal modes or phonons



NUCLEAR SCATTERING FROM CRYSTALLINE MATERIALS

Coherent part

exp U   exp V

$$\sum_{jj'}\left\langle \exp\left\{-\mathrm{i}\boldsymbol{K}\cdot\mathbf{R}_{j'}(0)\right\}\exp\left\{\mathrm{i}\boldsymbol{K}\cdot\mathbf{R}_{j}(t)\right\}\right\rangle = \mathrm{N}\sum_{j}\exp(\mathrm{i}\boldsymbol{K}\cdot\mathbf{j})\left\langle \exp\left\{-\mathrm{i}\boldsymbol{K}\cdot\mathbf{u}_{0}(0)\right\}\exp\left\{\mathrm{i}\boldsymbol{K}\cdot\mathbf{u}_{j}(t)\right\}\right\rangle$$

it can be shown (Squires) $\langle \exp\mathrm{U}\ \exp\mathrm{V}\rangle = \exp\langle \mathrm{U}^2\rangle \exp\langle \mathrm{UV}\rangle$

$$\left(\frac{d^2\sigma}{d\Omega dE_f}\right)_{coh} = \frac{\sigma_{coh}}{4\pi}\frac{k_f}{k_i}\frac{\mathrm{N}}{2\pi\hbar}\exp\langle U^2\rangle\sum_j \exp(\mathrm{i}\boldsymbol{K}\cdot\mathbf{j})\int_{-\infty}^{+\infty}\exp\langle \mathrm{UV}\rangle\exp(-\mathrm{i}\omega t)\,dt$$

Debye-Waller factor $2\mathrm{W} = -\langle \mathrm{U}^2\rangle = \left\langle \{\mathrm{K}\cdot\mathrm{u}\}^2\right\rangle$

$$\left(\frac{d^2\sigma}{d\Omega dE_f}\right)_{coh} = \frac{\sigma_{coh}}{4\pi}\frac{k_f}{k_i}\frac{\mathrm{N}}{2\pi\hbar}\exp(-2\mathrm{W})\sum_j \exp(\mathrm{i}\boldsymbol{K}\cdot\mathbf{j})\int_{-\infty}^{+\infty}\left(1+\langle \mathrm{UV}\rangle+\frac{1}{2!}\langle \mathrm{UV}\rangle^2+\ldots..\right)\exp(-\mathrm{i}\omega t)\,dt$$

zero-th order: coherent elastic scattering - Bragg scattering

1st order: coherent one-phonon scattering

……

Coherent elastic scattering

diffraction

$$\left.\frac{d^2\sigma}{d\Omega dE_f}\right)_{coh} = \frac{\sigma_{coh}}{4\pi}\frac{k_f}{k_i}\frac{N}{2\pi h}\exp(-2W)\sum_j \exp(i\boldsymbol{K}\cdot\mathbf{j})\int_{-\infty}^{+\infty}\exp(-i\omega t)\,dt$$

$$\int_{-\infty}^{+\infty}\exp(-i\omega t)\,dt = 2\pi h\,\delta(h\omega)$$

purely elastic scattering $|\mathbf{k}_i| = |\mathbf{k}_f|$

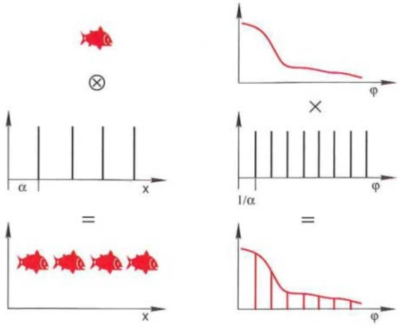
$$\left.\frac{d^2\sigma}{d\Omega dE_f}\right)_{coh\ el} = \frac{\sigma_{coh}}{4\pi}N\ \exp(-2W)\sum_j \exp(i\boldsymbol{K}\cdot\mathbf{j})\,\delta(h\omega)$$

$$\left.\frac{d\sigma}{d\Omega}\right)_{coh\ el} = \int_0^{\infty}\left.\frac{d^2\sigma}{d\Omega dE_f}\right)_{coh\ el} dE_f = \frac{\sigma_{coh}}{4\pi}N\ \exp(-2W)\sum_j \exp(i\boldsymbol{K}\cdot\mathbf{j})$$

$$\left.\frac{d\sigma}{d\Omega}\right)_{coh\ el} = \frac{\sigma_{coh}}{4\pi}N\frac{(2\pi)^3}{v_0}\exp(-2W)\sum_\tau \delta(\boldsymbol{K}-\tau)$$

N: number of unit cells

v_0: volume of the unit cell



non-Bravais lattice with different atoms *d* in the unit cell

$$\left.\frac{d\sigma}{d\Omega}\right)_{coh\ el} = N\frac{(2\pi)^3}{v_0}\sum_\tau \delta(\boldsymbol{K}-\tau)\left|F_N(\boldsymbol{K})\right|^2 \qquad \left|F_N(\boldsymbol{K})\right| = \sum_d \langle b_d\rangle \exp(i\boldsymbol{K}\cdot\mathbf{d})\exp(-W_d)$$

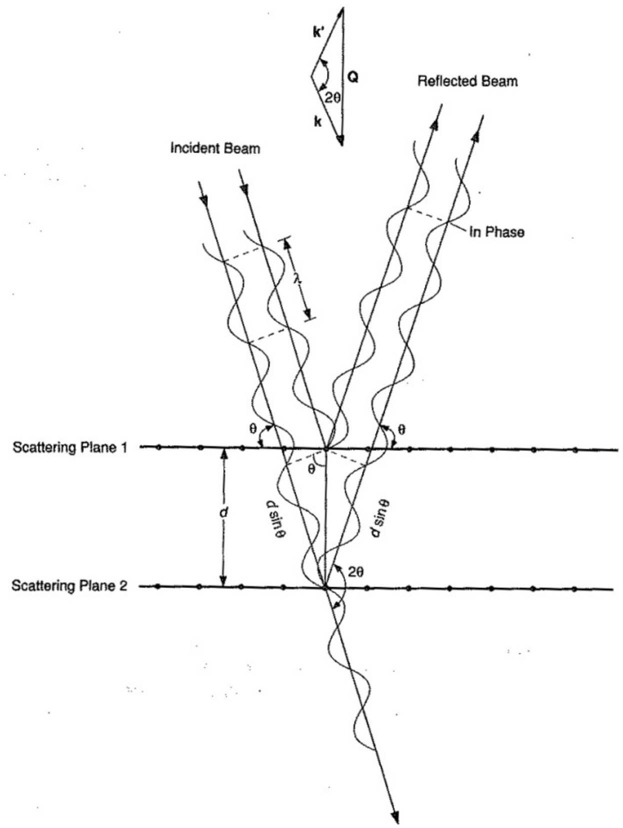
structure factor

Bragg's law

$$\lambda = 2d\sin\theta$$

Practical application

monochromators!



Collecting intensities at Bragg peaks gives access to 'squared' values of Fourier components of the structure

Collect as many as possible to overcome the phase problem

Coherent elastic scattering (diffraction) provides:

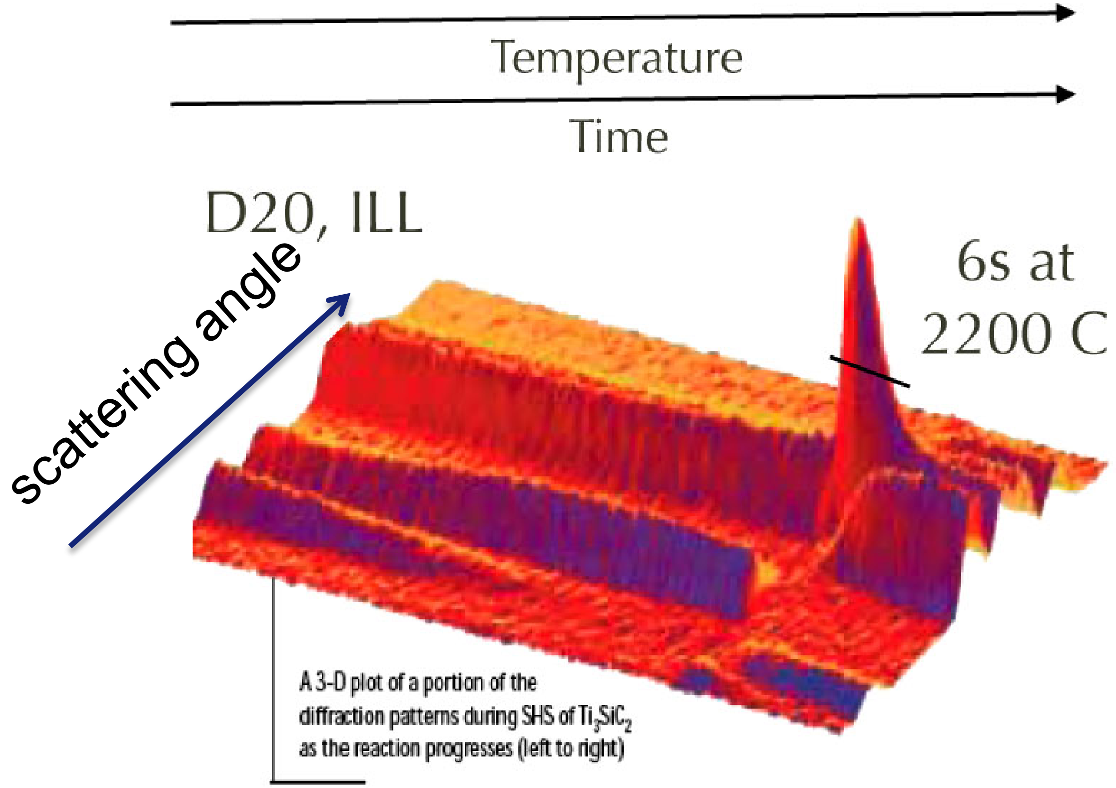
- periodicity in space, lattice symmetry and lattice constants
- positions of atoms in cells from $|F_N(\boldsymbol{K})|^2$
- powder and single crystals methods

But requires inversion of intensities into phases/amplitudes

Self propagating synthesis

High temperatures

Phase transitions visible



A 3-D plot of a portion of the diffraction patterns during SHS of Ti_3SiC_2 as the reaction progresses (left to right)

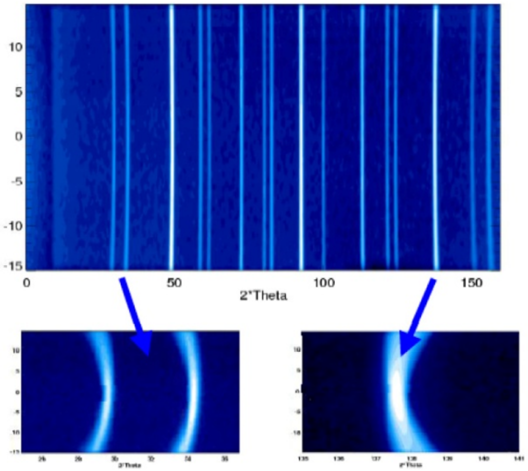
How does it work?

monochromatisation

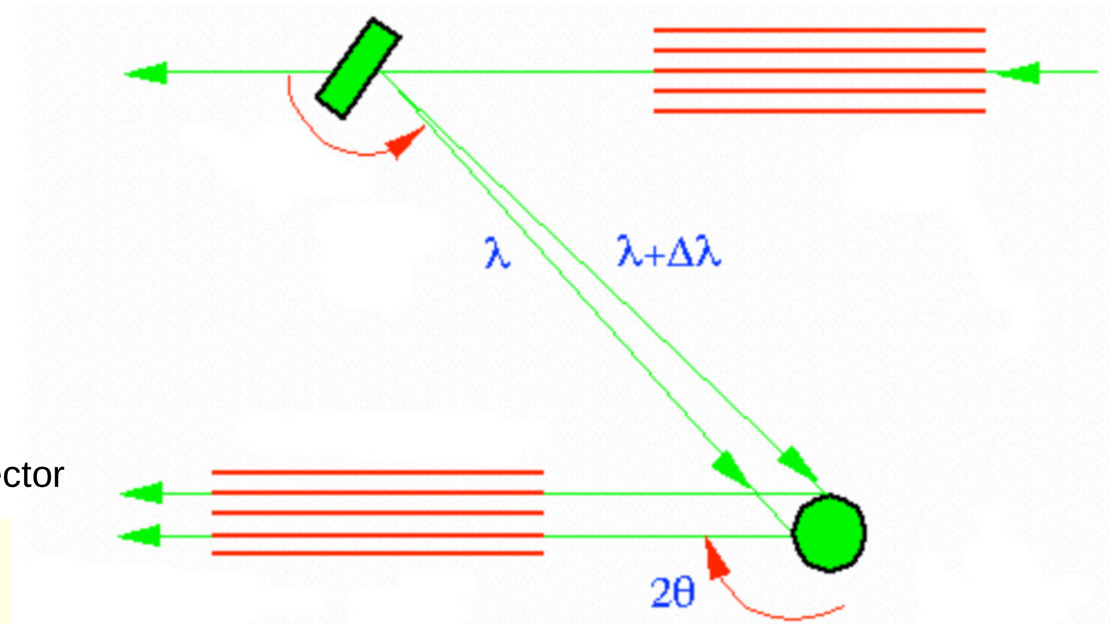
crystal – diffraction

or time of flight

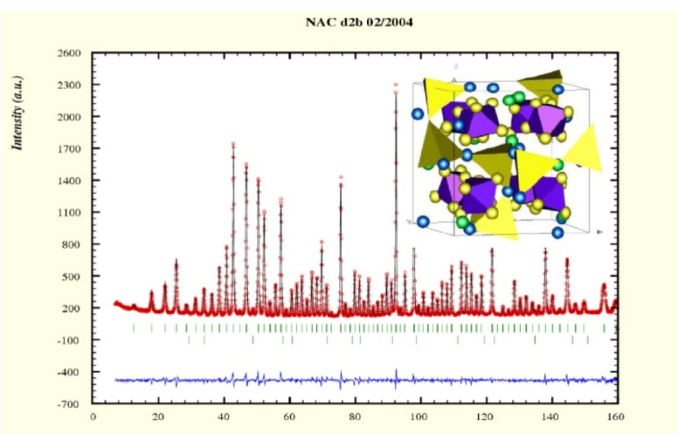
neutron source



Detector 2D



detector



Detector 1D

CRYSTALLINE MATERIALS : BEAM FILTERS

Maximum wavelength for which no Bragg scattering can occur

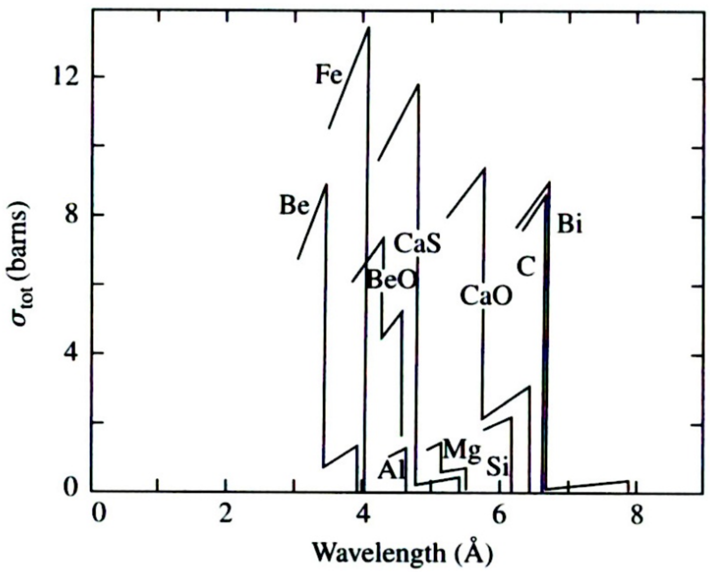
$$\lambda_{max} = 2d_{max}(\sin\theta)_{max} = 2d_{max}$$

Bragg cut-off

d_{max} is the maximum plane spacing

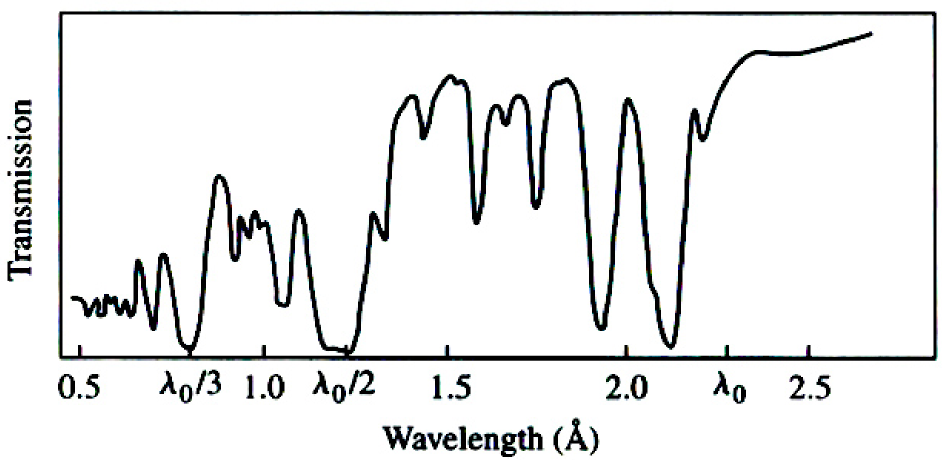
Beyond λ_{max} σ_{tot} drops

$$\sigma_{tot} = \sigma_{scatt} + \sigma_{absor}$$



Cut-off wavelengths for polycrystalline materials

Be: 3.9 Å   graphite: 6.7 Å



Transmission of pyrolytic graphite

KEY MESSAGES

- neutron diffraction is an essential tool for structures determination
- it complements other diffraction methods
- neutrons probe bulk samples

Inelastic coherent scattering (Bravais lattice)

$$\left.\frac{d^2\sigma}{d\Omega dE_f}\right)_{coh} = \frac{\sigma_{coh}}{4\pi}\frac{k_f}{k_i}\frac{N}{2\pi\hbar}\exp(-2W)\sum_j \exp(i\boldsymbol{K}\cdot\mathbf{j})\int_{-\infty}^{+\infty}\left(1+\langle UV\rangle+\frac{1}{2!}\langle UV\rangle^2+.....\right)\exp(-i\omega t)\,dt$$

one-phonon coherent scattering (Bravais lattice)

<UV> involves creation and annihilation of phonon modes

$$\langle UV\rangle = \frac{\hbar}{2MN}\sum_s \frac{(\boldsymbol{K}\cdot\mathbf{e}_s)}{\omega_s}\left[\exp\{-i(\mathbf{q}\cdot\mathbf{j}-\omega_s t)\}\langle n_s+1\rangle + \exp\{i(\mathbf{q}\cdot\mathbf{j}-\omega_s t)\}\langle n_s\rangle\right]$$

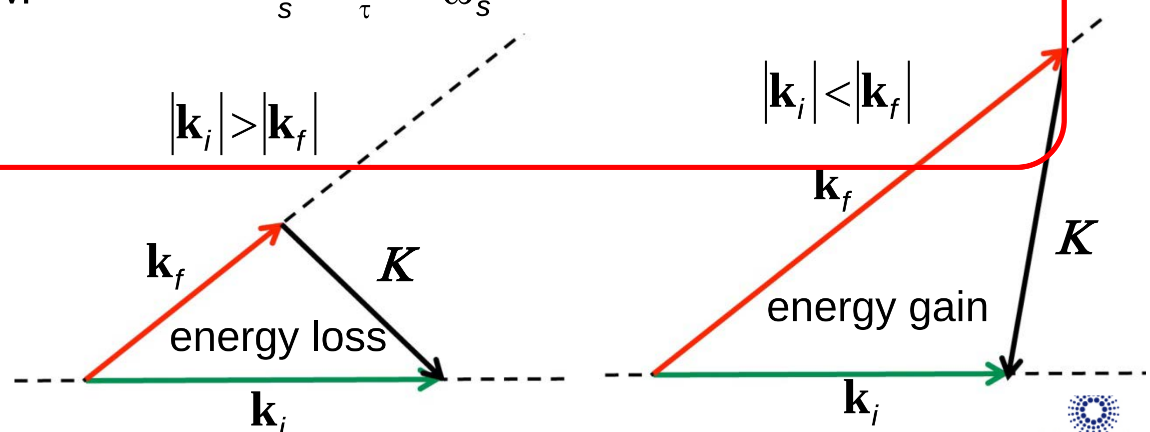
$$\left.\frac{d^2\sigma}{d\Omega dE_f}\right)_{coh\pm1} = \frac{\sigma_{coh}}{4\pi}\frac{k_f}{k_i}\frac{(2\pi)^3}{v_0}\frac{1}{2M}\exp(-2W)\sum_s\sum_\tau \frac{(\boldsymbol{K}\cdot\mathbf{e}_s)}{\omega_s}\langle n_s+1/2\pm1/2\rangle$$

$$\times\,\delta(\omega\mp\omega_s)\,\delta(\boldsymbol{K}\mp\mathbf{q}-\tau)$$

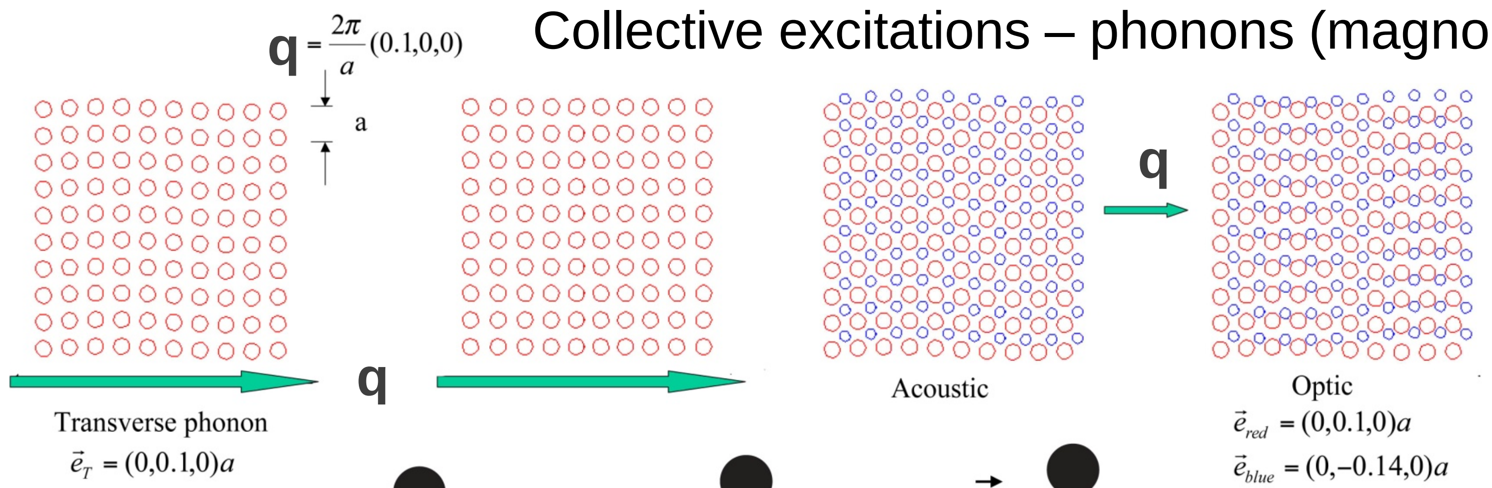
conservation laws!

$$\boldsymbol{K} = \tau \pm \mathbf{q}$$

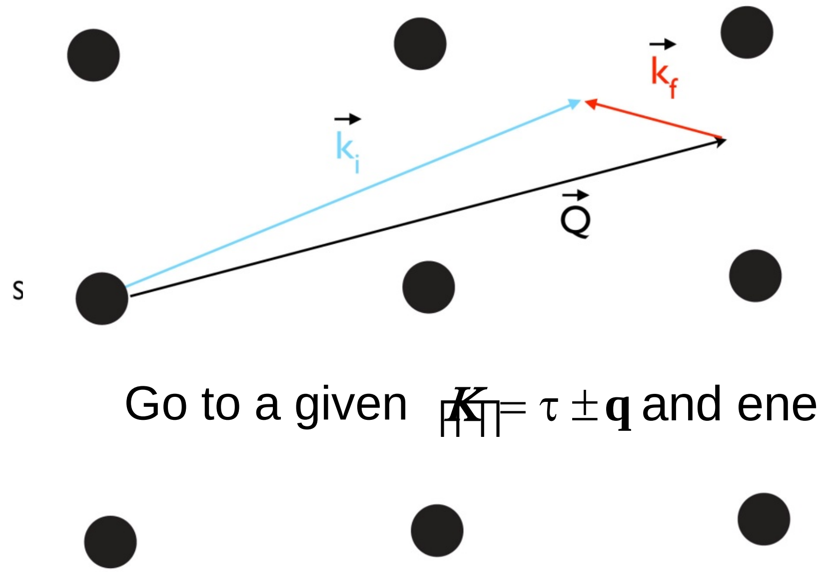
$$E_i - E_f = \hbar\omega = \pm\hbar\omega_s$$



Collective excitations – phonons (magnons)



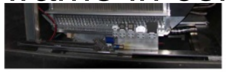
How to measure?



Go to a given $K = \tau \pm \mathbf{q}$ and energy transfer $\hbar\omega$

Start elastically i.e. $k_i = k_f$

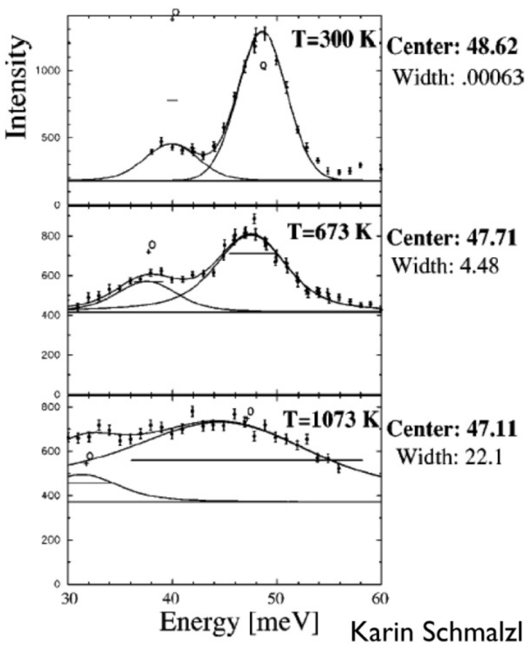
Orient sample: sample frame in coincidence with instrument frame

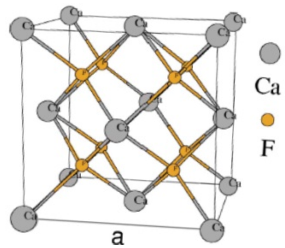


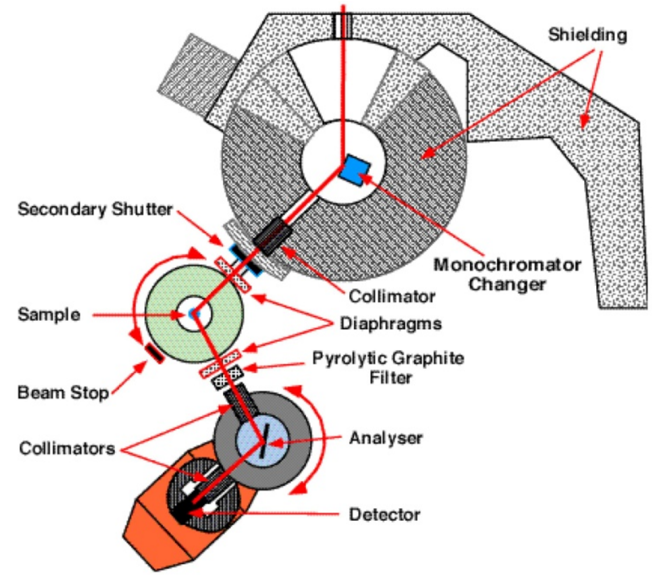
Detector

PHONON MODES

classical instruments: triple-axis machines







constant ‘*K*’

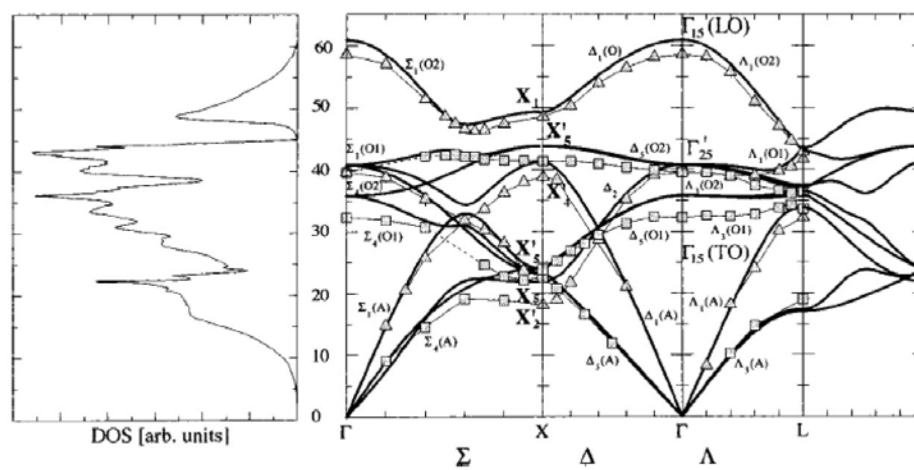
scan point by point

either constant ‘*K*’

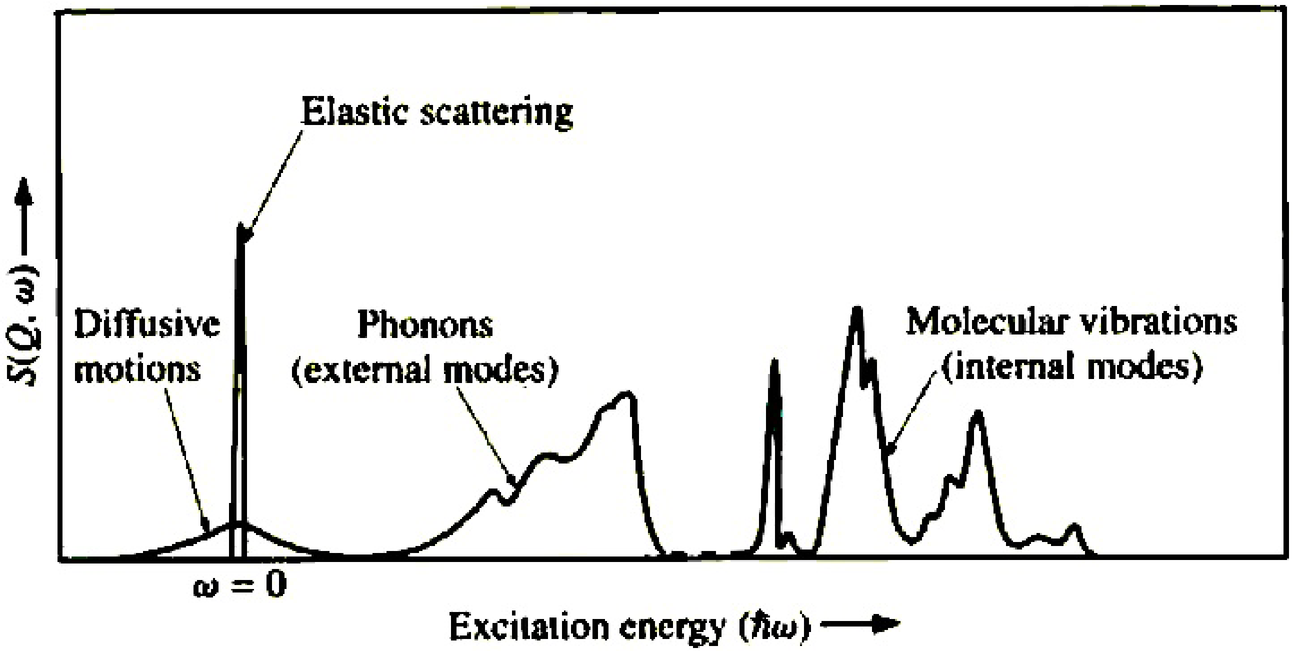
or constant ‘energy’

applications:

- lattice interactions, ‘soft mode’
- phase transitions
- superconductivity, …



Spectroscopy – internal modes – little dispersion of modes

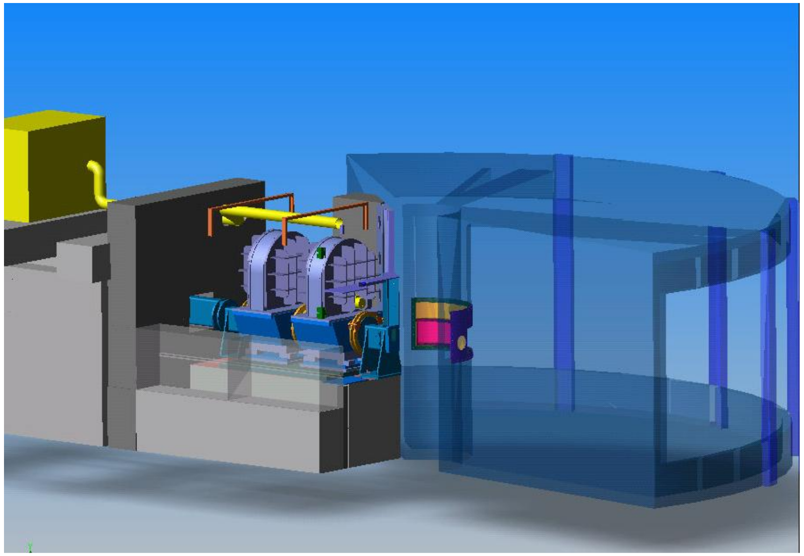


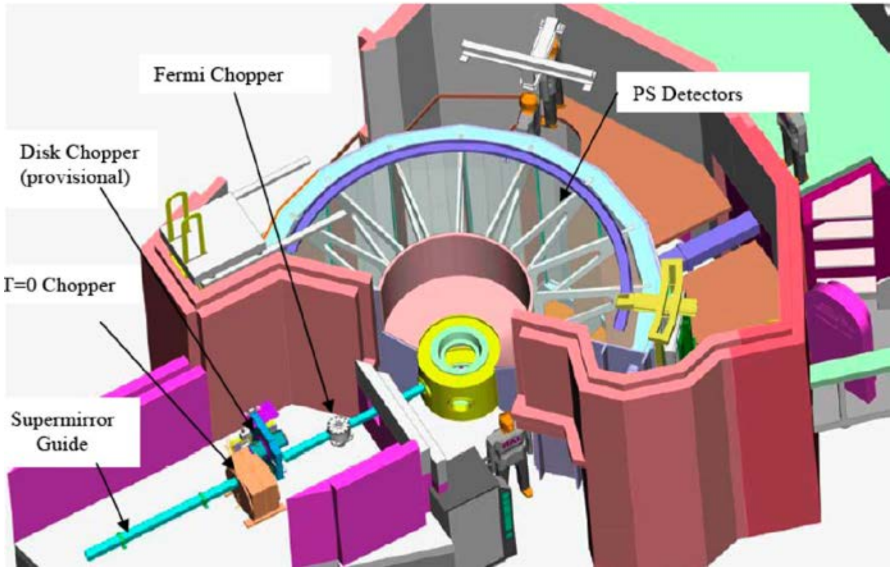
Excitations in hypothetical molecular crystal J.Eckert SpectroChim. Acta 48A, 271 (1992)

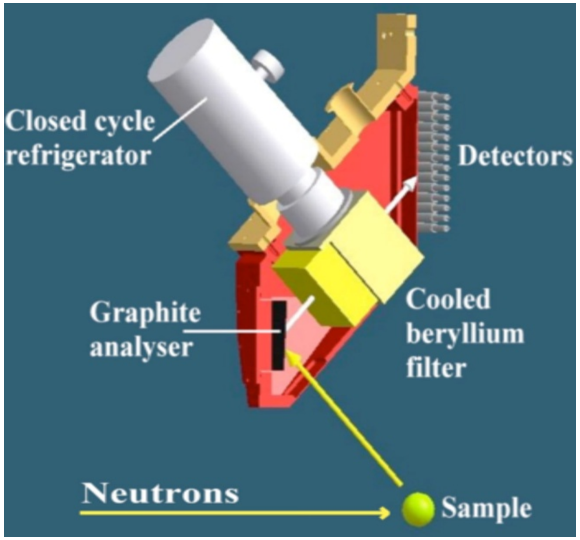
Use incoherent scattering

$$\left.\frac{d^2\sigma}{d\Omega dE_f}\right)_{incoh\pm1} = \frac{k_f}{k_i}\sum_s \delta(\omega \mp \omega_s)\frac{\langle n_s + 1/2 \pm 1/2\rangle}{2\omega_s}\sum_r \frac{(\sigma_{incoh})_r}{4\pi}\frac{1}{M_r}\left|\mathbf{K}\cdot\mathbf{e}_r\right|^2 \exp(-2W_r)$$

more global picture: time of flight and large detector coverage





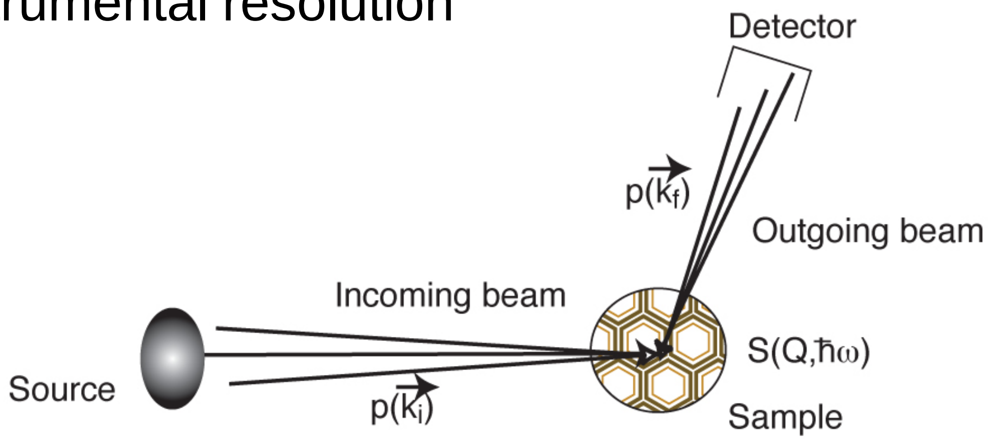


Time domain is reached through inelastic scattering

the range of energy transfer that can be covered determines the 'accessible' time domain (Fourier transform!)

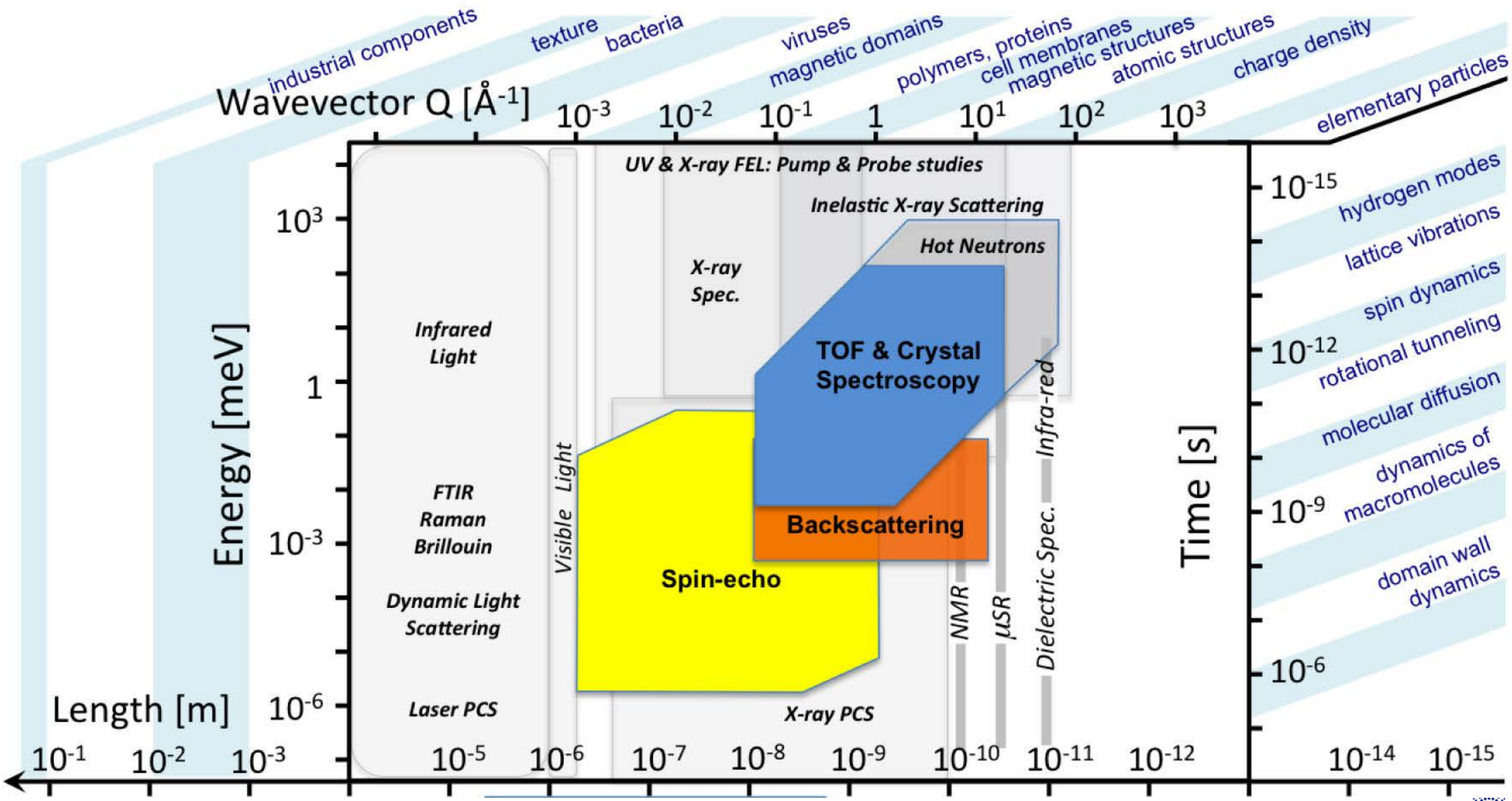
Depends on 'incident' neutron energy

and instrumental resolution



Typical resolution in wavelength (and energy) are ~ a few percents

except special techniques (backscattering, NSE) complementarity with other methods



KEY MESSAGES

- accessible time and space domains cover a wide range of applications
- no single probe can cover the whole (K,ω) space that would allow an ideal Fourier transformation
- use complementary probes

DISORDERED MATERIALS

liquids, glasses, ..no equilibrium positions for atoms

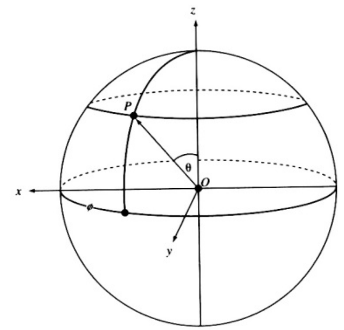
local order but no long range order

consider mono-atomic system – $\frac{d\sigma}{d\Omega} = \sum_{i,j} b_i^* b_j e^{-i\boldsymbol{K}.(\mathbf{R}_i - \mathbf{R}_j)}$

for most liquids, glasses, ... scattering depends on magnitude of

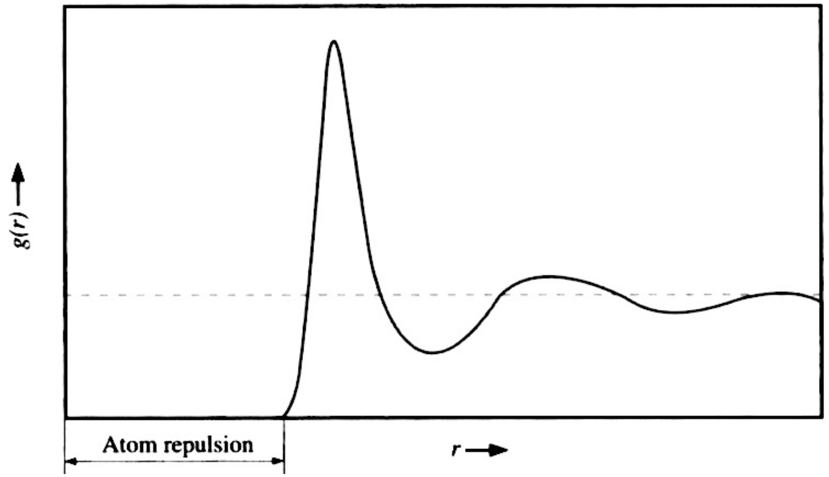
averaging over polar angles

$$\left\langle \exp\left\{ i\boldsymbol{K}\cdot(\mathbf{R}_i - \mathbf{R}_j)\right\}\right\rangle = \frac{\sin(Kr_{ij})}{Kr_{ij}}$$



replace sum over atoms by
radial distribution function

g(r) for monoatomic liquid



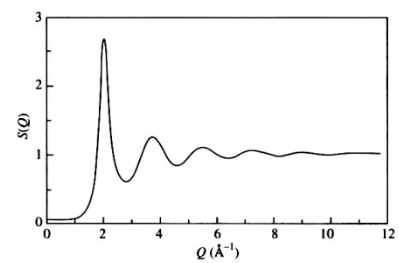
mono-atomic liquid

$$\frac{d\sigma}{d\Omega} = N\langle b^2\rangle + \sum_{i \neq j}^{N} \langle b_i\rangle\langle b_j\rangle e^{-i\mathbf{K}.(\mathbf{R}_i - \mathbf{R}_j)} = N\langle b^2\rangle + 4\pi\rho\langle b^2\rangle \int_0^\infty r^2 g(r)\frac{\sin(Kr)}{(Kr)}dr$$

note that $\int_0^\infty r^2 \frac{\sin(Kr)}{(Kr)}dr = 0$ unless $K = 0$

$$\frac{d\sigma(K \neq 0)}{d\Omega} = N\langle b^2\rangle S(K) \quad with = S(K) = 1 + \frac{4\pi\rho}{K}\int_0^\infty r\,[g(r) - 1]\sin(Kr)dr$$

S(K): structure factor



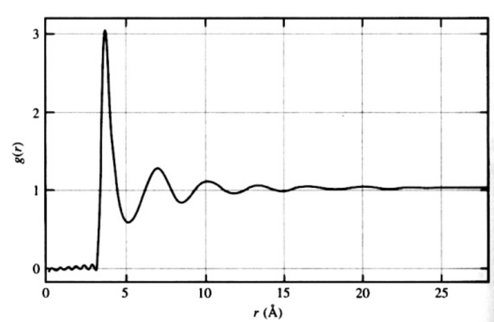
Liquid Argon

T= 85K

Experimental data

very different from $F_N(\boldsymbol{K})$ in crystalline materials

- relates to intensity (not amplitude)
- not well structured, only a few peaks



Inverted $\rho(r)$

(Fourier transform)

J.L. Yarnell et al.

PRA 7, 2130 (1973)

In n-component systems, there are n(n+1)/2 site-site radial distributions

To be measured, using isotopic substitution.

In liquids, strictly speaking there is no elastic neutron scattering:

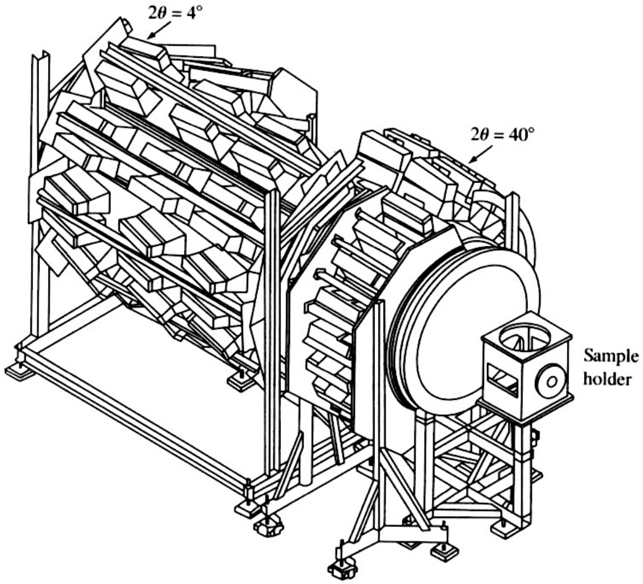
nuclei recoil under neutron impact

need to span all (***K***,ω) space

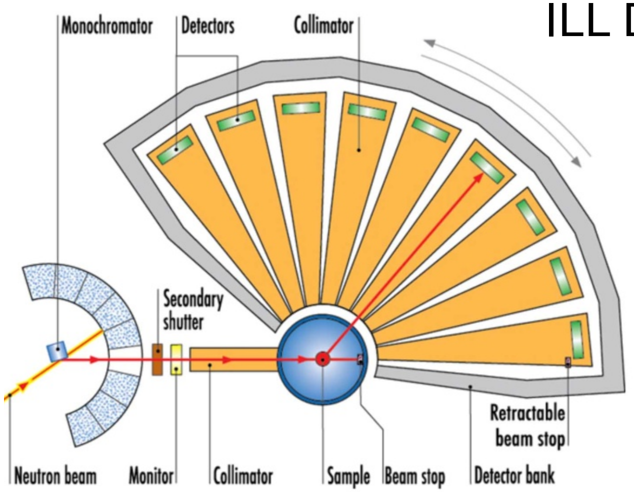
or apply inelasticity corrections Placzek PRB 86, 377 (1956)

Typical instruments

Sandals ISIS

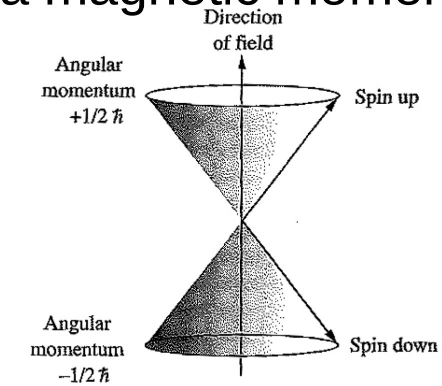


ILL D4



Neutrons have a spin ½ and therefore carry a magnetic moment

neutron beams can be polarised



$$\boldsymbol{\mu}_n = -\gamma \mu_N \boldsymbol{\sigma} \quad \text{where } \mu_N = \frac{eh}{2m_p} \quad \gamma = 1.913$$

For comparison, electrons have $\boldsymbol{\mu}_e = -2\mu_B \mathbf{s}$ where $\mu_B = \frac{eh}{2m_e}$

Neutron magnetic moments feel magnetic fields created in materials:

- electrons: dipole moments and currents
- nuclei: dipole moments (neglected here)

The potential *V* in the scattering cross section should include these effects

$$\left(\frac{d^2\sigma}{dE_f\, d\Omega}\right)_{\lambda_i \to \lambda_f} = \frac{k_f}{k_i}\left(\frac{m_n}{2\pi h^2}\right)^2 \left|\langle \mathbf{k}_f \lambda_f | V | \mathbf{k}_i \lambda_i \rangle\right|^2 \delta\left(E_i - E_f + E_{\lambda_i} - E_{\lambda_f}\right)$$

MAGNETISM!

Magnetic field created at distance ***R*** electron with momentum ***p***

$$\boldsymbol{B} = \boldsymbol{B}_S + \boldsymbol{B}_L = \frac{\mu_0}{4\pi}\left\{\mathrm{curl}\left(\frac{\boldsymbol{\mu}_e \times \mathbf{R}}{R^2}\right) - \frac{2\mu_B}{h}\frac{\mathbf{p}\times\mathbf{R}}{R^2}\right\}$$

Potential of a neutron in *B*

$$\boldsymbol{V}_m = -\boldsymbol{\mu}_n \cdot \boldsymbol{B} = -\frac{\mu_0}{4\pi}\gamma\mu_N 2\mu_B\left\{\mathrm{curl}\left(\frac{\boldsymbol{s}\times\mathbf{R}}{R^2}\right) + \frac{1}{h}\frac{\mathbf{p}\times\mathbf{R}}{R^2}\right\}$$

During scattering, neutron changes from state $\boldsymbol{k}_i,\sigma_i$ to $\boldsymbol{k}_f,\sigma_f$

$$\left(\frac{d^2\sigma}{dE_f\, d\Omega}\right)_{\sigma_i\lambda_i\to\sigma_f\lambda_f} = \frac{\boldsymbol{k}_f}{\boldsymbol{k}_i}\left(\frac{m_n}{2\pi\hbar^2}\right)^2 \left|\left\langle \mathbf{k}_f\sigma_f\lambda_f\left|\boldsymbol{V}_m\right|\mathbf{k}_i\sigma_i\lambda_i\right\rangle\right|^2 \delta\left(E_i - E_f + E_{\lambda_i} - E_{\lambda_f}\right)$$

Complex evaluation:

magnetic interaction is long range

magnetic forces are not central

MAGNETISM!

After long calculations

$$\left(\frac{d^2\sigma}{dE_f\, d\Omega}\right)_{\sigma_i\lambda_i\to\sigma_f\lambda_f} = \frac{k_f}{k_i}(\gamma r_0)^2 \left|\langle\sigma_f\lambda_f|\sigma\cdot \boldsymbol{Q}_\perp|\sigma_i\lambda_i\rangle\right|^2 \delta\left(E_i - E_f + E_{\lambda_i} - E_{\lambda_f}\right)$$

r_0: classical radius of electron 2.810^{-13} cm

'strength' of scattering ~ $(\gamma r_0)^2$ of the order of 0.3 barn

$$\boldsymbol{Q}_\perp = \sum_{\text{electrons } x} \exp\left(i\boldsymbol{K}\cdot\mathbf{r}_x\right)\left\{\hat{\boldsymbol{K}}\times\left(\boldsymbol{s}_x\times\hat{\boldsymbol{K}}\right)+\frac{i}{hK}\left(\boldsymbol{p}_x\times\hat{\boldsymbol{K}}\right)\right\}$$

whereas for nuclear scattering $\sum_j b_j \exp\left(i\boldsymbol{K}\cdot\mathbf{R}_j\right)$

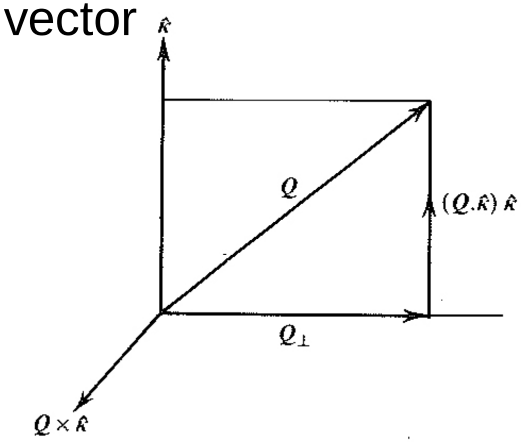
The $\boldsymbol{Q}_\perp$ operator is related to the magnetisation of the scattering system

separating spin and orbital contributions

$$\mathbf{Q}_\perp(\boldsymbol{K}) = \hat{\mathbf{K}}\times\left(\mathbf{Q}(\boldsymbol{K})\times\hat{\mathbf{K}}\right)$$ where $\mathbf{Q}(\boldsymbol{K}) = -\frac{1}{2\mu_B}\mathbf{M}(\boldsymbol{K})$ $\hat{\mathbf{K}}$ is a unit vector

M(**K**) is the Fourier transform of ***M***(***r***)

$\boldsymbol{Q}_\perp$ is the vector projection of **Q** on to the plane perpendicular to ***K***



KEY MESSAGES

- ‘nuclear’ and ‘magnetic’ interactions have similar strengths

- interactions with electrons connect to magnetisation densities

- ‘magnetic’ scattered intensities are proportional to space and time Fourier transforms of site correlation functions for magnetic moments

MAGNETISM!

Similarly to nuclear scattering, magnetic neutron scattering probes 'correlations'

$$\left(\frac{d^2\sigma}{dE_f\, d\boldsymbol{\Omega}}\right) = \frac{k_f}{k_i}\frac{(\gamma r_0)^2}{2\pi\hbar}\sum_{\alpha\beta}\left(\delta_{\alpha\beta} - \hat{K}_\alpha \hat{K}_\beta\right)\int \langle Q_\alpha(-\boldsymbol{K},0) Q_\beta(\boldsymbol{K},t)\rangle \exp(-i\omega t)\, dt$$

$$Q_\beta(\boldsymbol{K},t) = \exp(iHt/\hbar) Q_\beta(\boldsymbol{K}) \exp(-iHt/\hbar)$$

geometrical factor

equivalent to nuclear scattering

Elastic scattering – thermal average at infinite time

$$\left(\frac{d\sigma}{d\boldsymbol{\Omega}}\right)_{el} = (\gamma r_0)^2 \sum_{\alpha\beta}\left(\delta_{\alpha\beta} - \hat{K}_\alpha \hat{K}_\beta\right)\langle Q_\alpha(-\boldsymbol{K}) Q_\beta(\boldsymbol{K})\rangle$$

or $\left(\frac{d\sigma}{d\boldsymbol{\Omega}}\right)_{el} = \left(\frac{\gamma r_0}{2\mu_B}\right)^2 \left|\hat{\boldsymbol{K}} \times \left\{\langle \mathbf{M}(\boldsymbol{K})\rangle \times \hat{\boldsymbol{K}}\right\}\right|^2$ with $\mathbf{Q}(\boldsymbol{K}) = -\frac{1}{2\mu_B}\mathbf{M}(\boldsymbol{K})$

$\mathbf{M}(\boldsymbol{K})$ contains all information on magnetic arrangements

symmetry, periodicity, moments, …..

MAGNETIC DIFFRACTION

Ferromagnets

localised magnetic system has the same periodicity as lattice

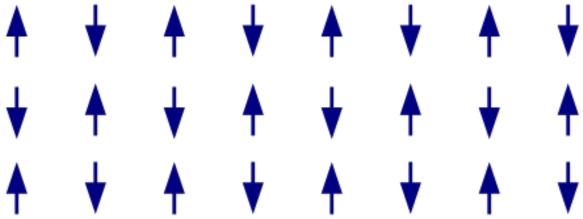
$$\left(\frac{d\sigma}{d\Omega}\right)_{el} = (\gamma r_0)^2 N \frac{(2\pi)^3}{v_0}\langle S^\eta\rangle^2 \quad \sum_\tau \left\{\frac{1}{2} g F(\tau)\right\}^2 \exp(-2W) \times \left\{1-(\hat{\tau}\cdot \hat{\eta})_{Aver}^{\;2}\right\}\delta(\boldsymbol{K}-\tau)$$

- magnetic intensity on top of nuclear intensity

- ‘magnetic form factor’ not constant as b –

spatial distribution of ‘magnetic’ electrons

- Measurements of intensities give *F*(τ) which allow M(r) to be calculated



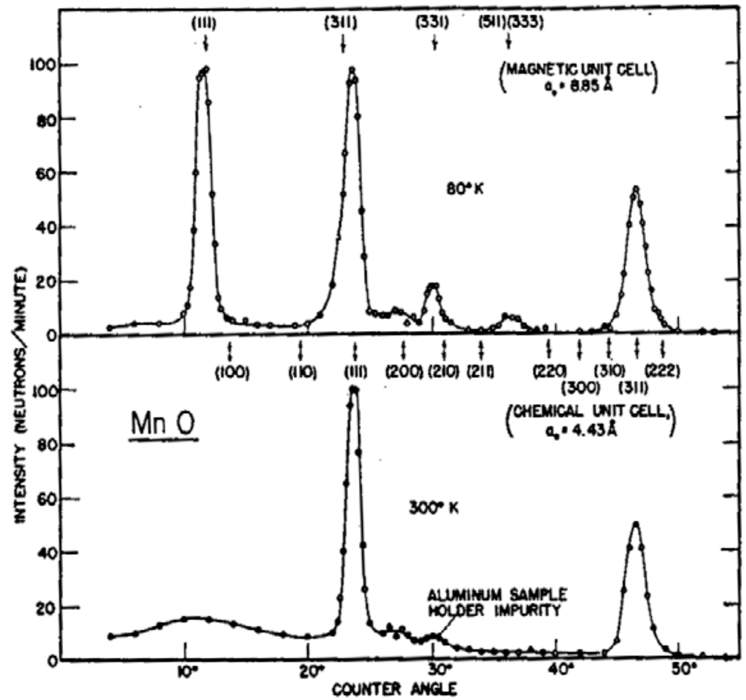
Non-ferromagnets

new periodicity in space leads to new Bragg peaks

Non-ferromagnets

neutrons allow to probe local magnetic order

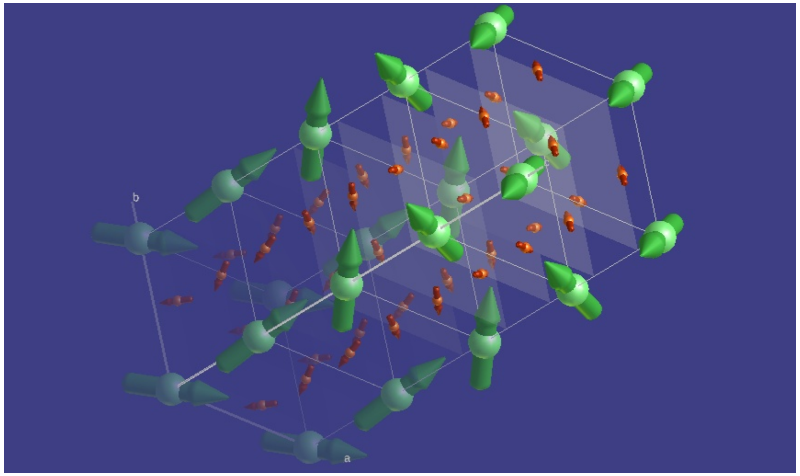
C. Shull et al. 1949



powder samples or single crystals

'easy' and routine experiments!

One of the very strong points for neutrons



More complex materials

Important for new devices

KEY MESSAGE

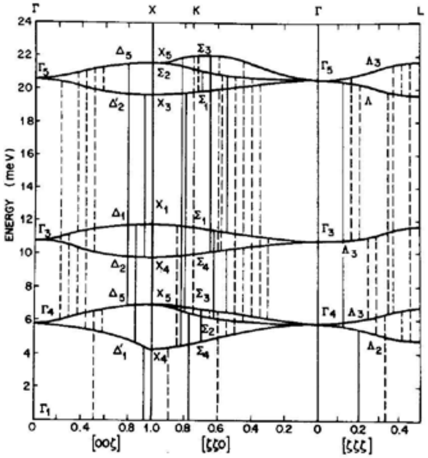
• neutron diffraction (powder) is the the method of choice to determine magnetic structures (if not the only one …)

INELASTIC MAGNETIC SCATTERING OF NEUTRONS

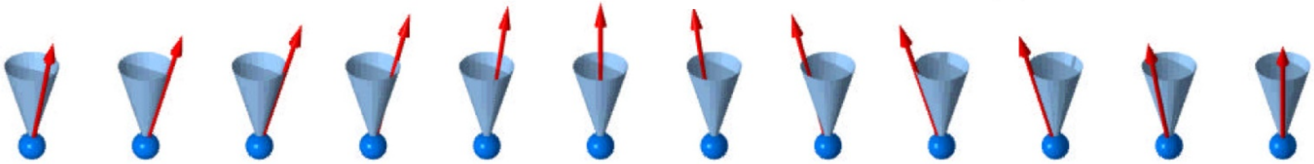
Inelastic magnetic neutron scattering probes magnetic ‘correlations’

simple localised ‘magnetic’ excitations

(crystal field levels)

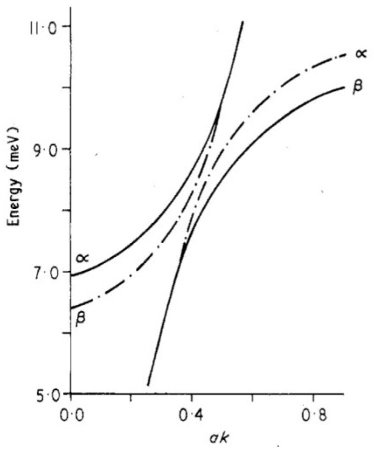


spin waves



fluctuations (spin density fluctuations)

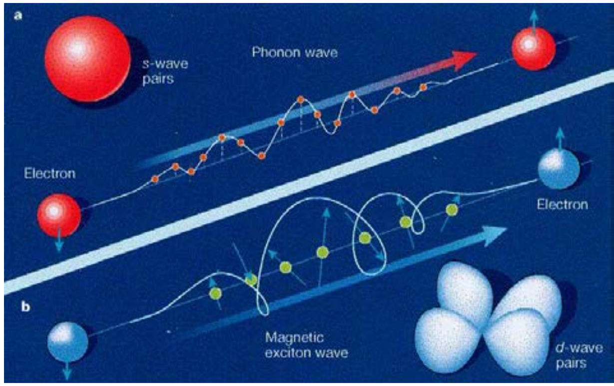
interactions between lattice and magnetism



very powerful experimental method

investigating pairing mechanisms in superconductors

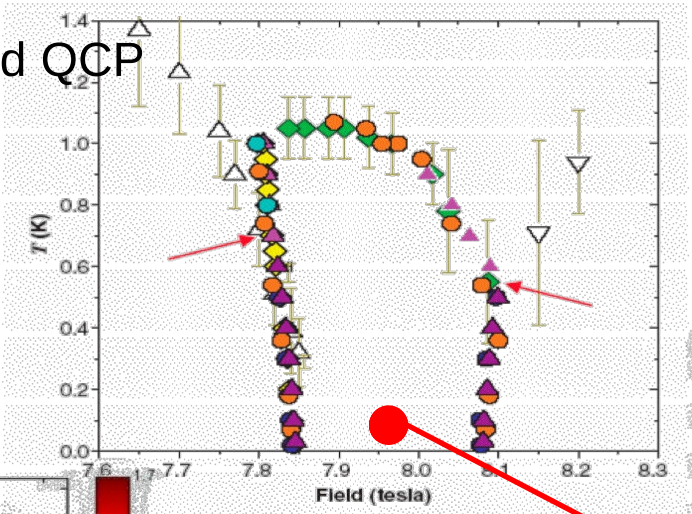
phonon-like or magnetic?

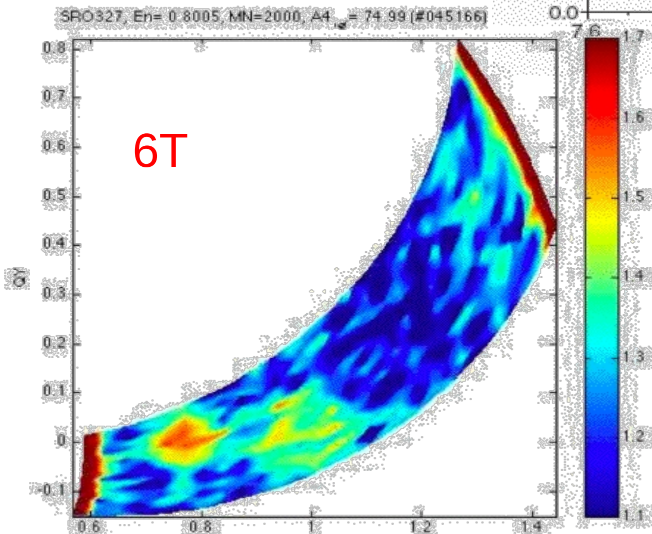


$Sr_3Ru_2O_7$ field-induced QCP

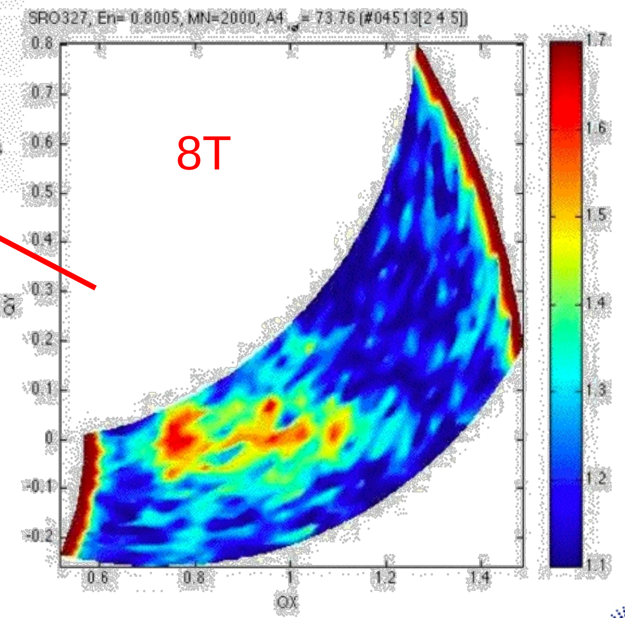
phase diagram:

low T and high H





T=40 mK
E=0.8 meV

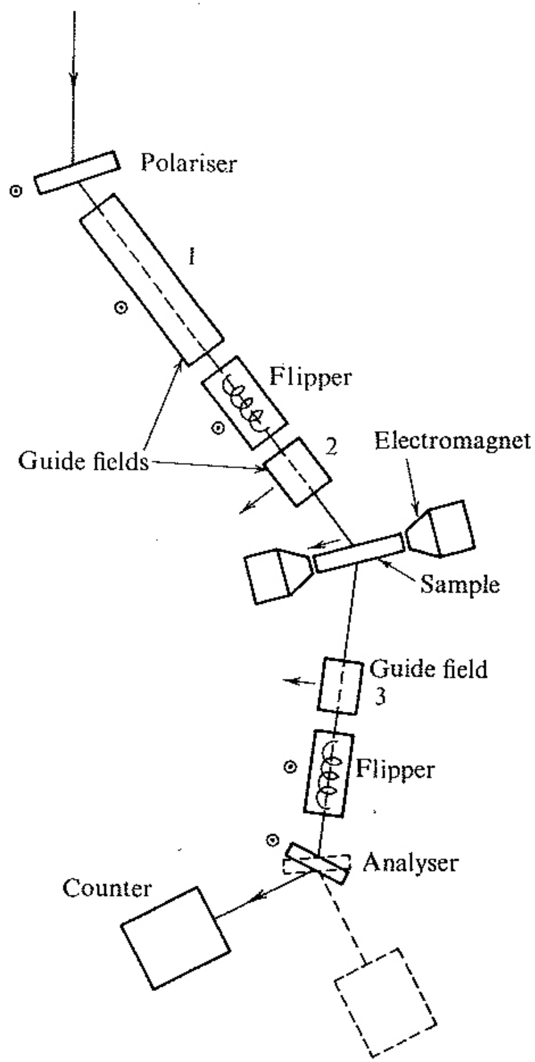


NEUTRON POLARISATION

scattering cross section involves neutron spin states

another neutron degree of freedom

- use of polarised neutrons
- use of polarisation
- use of neutron spin precession in fields
  - Larmor precession
  - spin manipulation
    - spin filters
    - NSE methods



NEUTRON POLARISATION

polarised neutron beams, *up (u)* and *down (v)* states $P = \dfrac{n_+ - n_-}{n_+ + n_-}$

previous cross-sections gives rise to 4 cross-sections $u \to u \quad v \to v \quad u \to v \quad v \to u$

coherent nuclear scattering

$$\left.\begin{matrix} u \to u \\ v \to v \end{matrix}\right\} \quad \bar{b} = \left\langle \frac{(I+1)b^+ + I\,b^-}{2I+1} \right\rangle_{isotopes}$$

$$\left.\begin{matrix} u \to v \\ v \to u \end{matrix}\right\} \quad \bar{b} = 0$$

incoherent nuclear scattering

$$\left.\begin{matrix} u \to u \\ v \to v \end{matrix}\right\} \quad \langle b^2 \rangle - \langle b \rangle^2 = \left\langle \left(\frac{(I+1)b^+ + I\,b^-}{2I+1} \right)^2 \right\rangle_{isotopes} - \left\langle \frac{(I+1)b^+ + I\,b^-}{2I+1} \right\rangle^2_{isotopes} + \frac{1}{3} \left\langle \left(\frac{b^+ - b^-}{2I+1} \right)^2 I(I+1) \right\rangle_{isotopes}$$

$$\left.\begin{matrix} u \to v \\ v \to u \end{matrix}\right\} \quad \langle b^2 \rangle - \langle b \rangle^2 = \frac{2}{3} \left\langle \left(\frac{b^+ - b^-}{2I+1} \right)^2 I(I+1) \right\rangle_{isotopes}$$

particular cases: unpolarised neutrons

Ni: all isotopes with I=0

Vanadium: only one isotope

Polarisation 'induces' interference between nuclear and magnetic scattering

$$\frac{d\sigma}{d\Omega} = N\frac{(2\pi)^3}{v_0}\left\{\left|F_N(\boldsymbol{K})\right|^2 + 2\left(\hat{P}\cdot\hat{\mu}\right)\left|F_N(\boldsymbol{K})\right|\left|F_M(\boldsymbol{K})\right| + \left|F_M(\boldsymbol{K})\right|^2\right\}$$

In ferromagnets, $\left|F_N(\boldsymbol{K})\right|$ and $\left|F_M(\boldsymbol{K})\right|$ are non-zero for the same *K* vectors

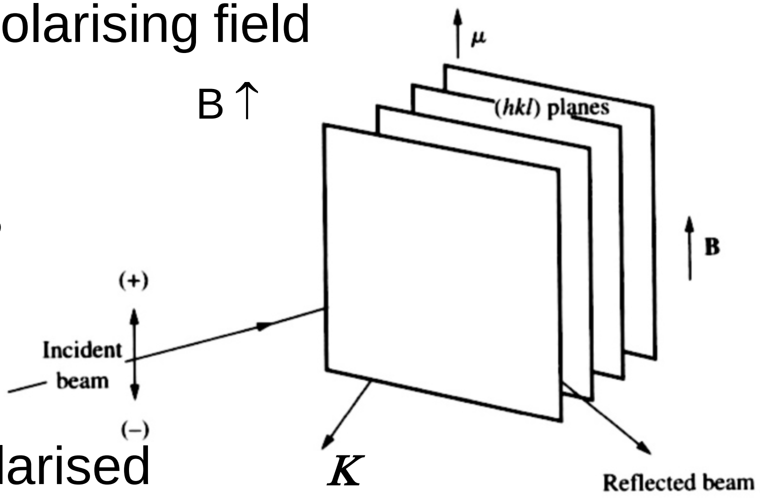
polarising field

application: polarisation devices

B ↑

$\left(\hat{P}\cdot\hat{\mu}\right) = \pm 1$ for neutrons (anti-)parallel to B

$$\frac{d\sigma}{d\Omega} = N\frac{(2\pi)^3}{v_0}\left|F_N(\boldsymbol{K}) \pm F_M(\boldsymbol{K})\right|^2$$



if matching F_N and F_M reflected beam is polarised

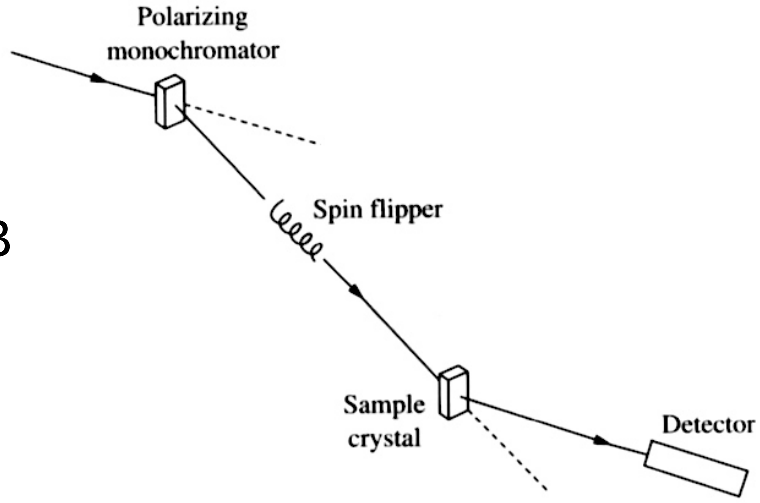
guide fields

Similar effects/applications in reflectometry – 'magnetic' optical index

polarising neutron guides

NEUTRON POLARISATION

application: precise measurement of weak magnetic signals



apply B perpendicular to ***K***

moments are aligned parallel to B

$$\frac{d\sigma}{d\Omega} = N\frac{(2\pi)^3}{v_0}\left|F_N(\boldsymbol{K}) \pm F_M(\boldsymbol{K})\right|^2$$

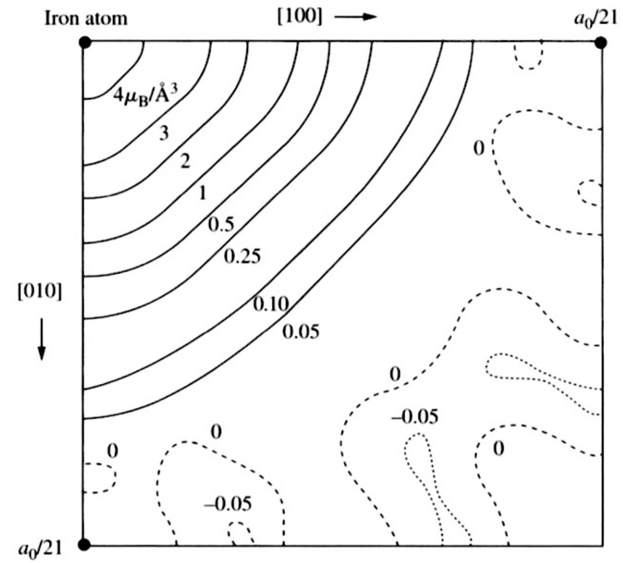
measure flipping ratio *R*

$$R = \left.\frac{d\sigma}{d\Omega}\right)^{+} / \left.\frac{d\sigma}{d\Omega}\right)^{-} = \left(\frac{1-\gamma}{1+\gamma}\right)^2 \text{ with } \gamma = F_M(\boldsymbol{K})/F_N(\boldsymbol{K})$$

if γ is small, R~1-4γ

allows to measure spin densities

Iron C.G Shull et al. J.Phys.Soc.Japan 17,1 (1962)

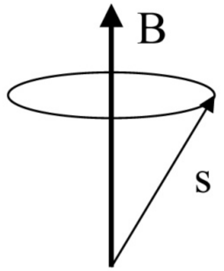


SO MANY OTHER FIELDS

neutron spin-echo: use of Larmor precession of neutron's spin

time evolution of s=1/2 in magnetic field B

$$\frac{d\vec{s}}{dt} = \gamma \vec{s} \times \vec{B} \quad \omega_L = |\gamma| B \quad \text{with } \gamma = -2913 * 2\pi \text{ Gauss}^{-1} \cdot \text{s}^{-1}$$

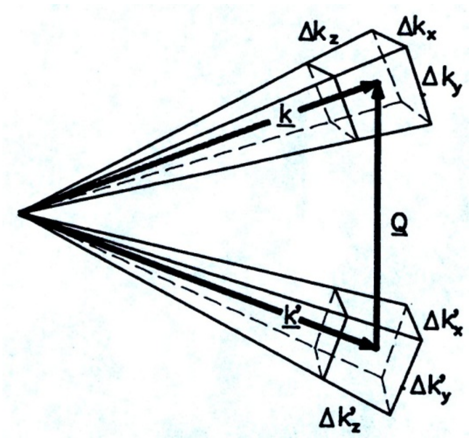


total precession angle $\phi = \omega_L t = \gamma \, \text{B} \; \text{d}/\text{v}$ depends neutron's velocity

with B=10 Gauss ~29 turns/m for 4Å neutrons

neutron spin-echo encodes neutron velocity – quite high resolution

without loss in intensity



NSE breaks the awkward relationship between intensity and resolution: the better the resolution, the smaller the resolution volume and the lower the count rate!

neutron reflectometry, SANS, …

SUMMARY OF KEY MESSAGES

- neutrons have no charge – low absorption
- ‘nuclear’ and ‘magnetic’ interactions have similar strengths
- interaction with nuclei very short range

  isotropy, isotope variation and contrast

- interactions with electrons lead to magnetisation densities

  neutron diffraction the method of choice to determine magnetic structures

- scattered intensities are proportional to space and time Fourier transforms of site correlation functions (positions and magnetic moments)
- accessible time and space domains cover a wide range of applications
- caveat: neutron sources are not very efficient …..

FURTHER READING

Introduction to the Theory of Thermal Neutron Scattering

G.L. Squires Reprint edition (1997) Dover publications ISBN 04869447

Experimental Neutron Scattering

B.T.M. Willis & C.J. Carlile (2009) Oxford University Press ISBN 978-0-19-851970-6

Neutron Applications in Earth, Energy and Environmental Sciences

L. Liang, R. Rinaldi & H. Schober Eds Springer (2009) ISBN 978-0-387-09416-8

Methods in Molecular Biophysiscs

I.N. Serdyuk, N. R. Zaccai & J. Zaccai Cambridge University Press (2007) ISBN 978-0-521-81524-6

Thermal Neutron Scattering

P.A. Egelstaff ed. Academic Press (1965)