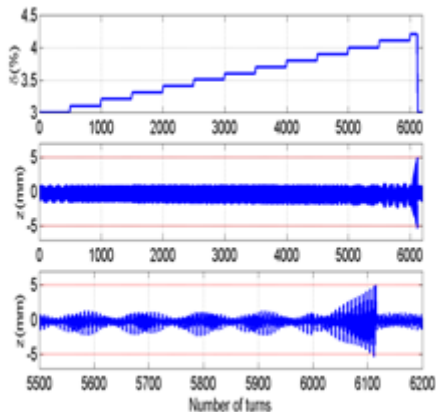
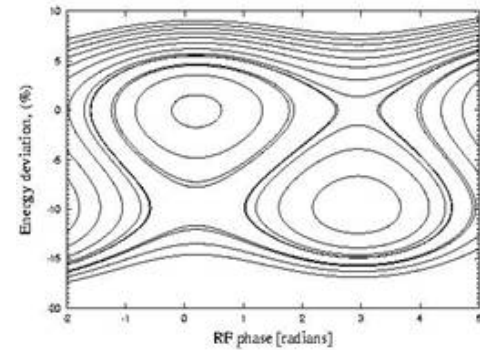
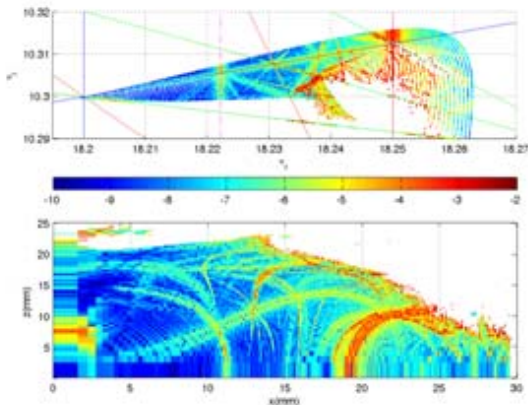
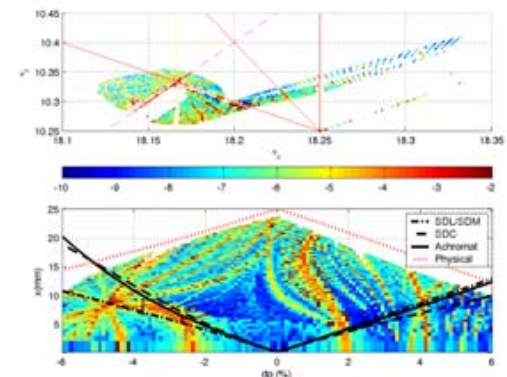


# Tools for Modeling Dynamics Apertures, Touschek Lifetime and Insertion Devices at SOLEIL

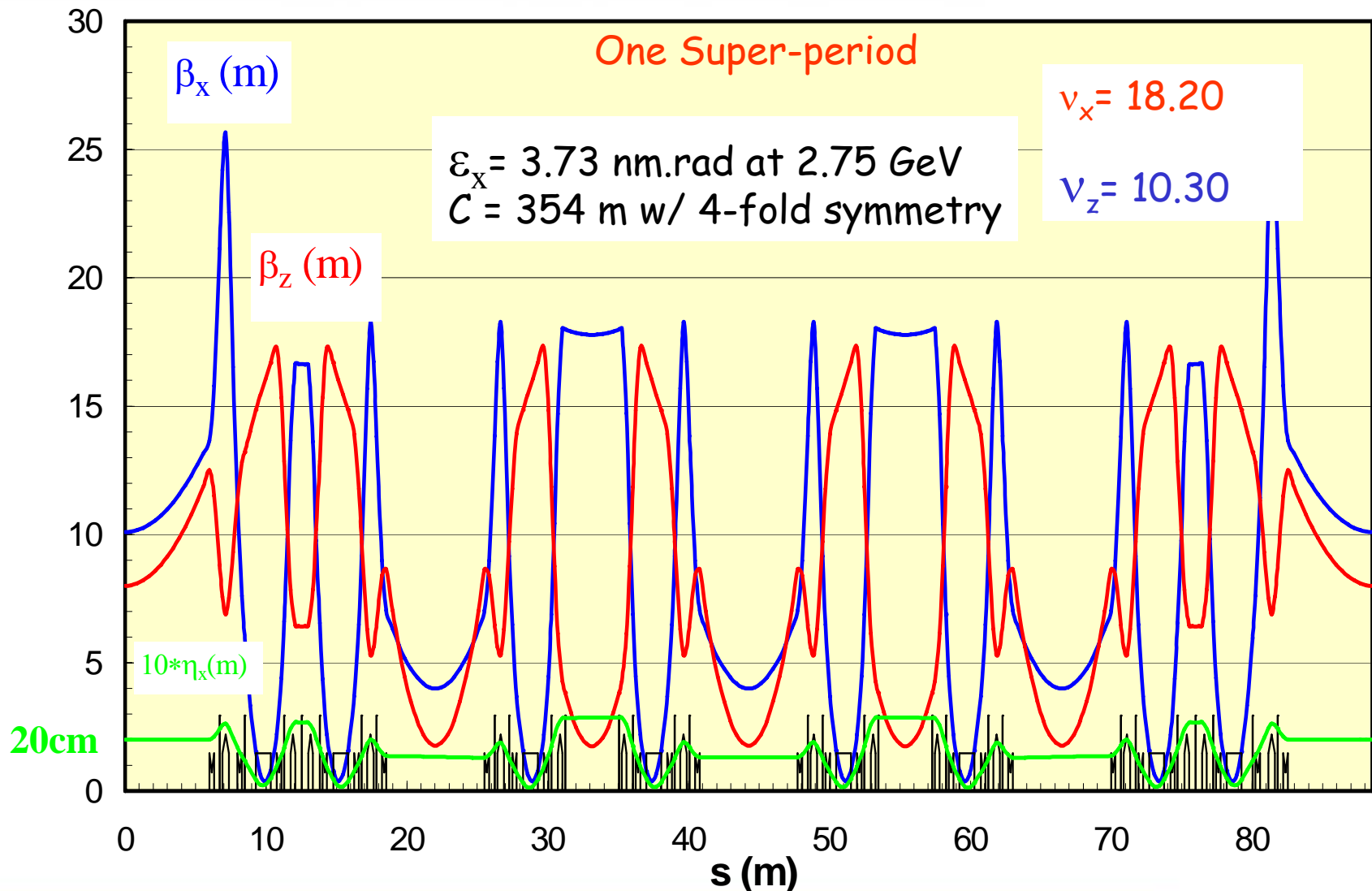


Laurent S. Nadolski  
On behalf of the  
Accelerator Physics Group  
Synchrotron SOLEIL



- **Non-linear dynamics optimization**
  - Criteria for optimization
  - Modeling
  - Philosophy adopted at SOLEIL
- **Numerical Tools through examples**
  - Tracking codes
  - Frequency map analysis
  - Off momentum dynamics
  - Touschek lifetime computation
  - Insertion device modeling
- **Conclusion**

# Linear Optics based on DBA lattice with distributed dispersion



# Criteria for optimization non-linear dynamics

- **Large on momentum dynamic aperture (DA)**
  - Ensure 100% injection efficiency (beam stay clear, top-up operation)
  - Large enough (safe margin, robustness)
    - To anticipate small energy mismatch, orbit errors
    - To anticipate non-linearities from multipoles, insertion devices which will reduce furthermore the DA
- **Large off-momentum dynamic aperture**
  - Ensure large Touschek lifetime (33h @ 500 mA, 4 MV,  $K = 1\%$ , multibunch)  
→ Ensure stability for off-momentum particle (up to  $\pm 6\%$ )
  - Margin of stability for ID effects
- **Tune footprint in an area almost free of low order resonance**
  - Tune shifts with amplitude constrained between resonance lines
  - Tune shifts with energy constrained between resonance lines
  - Minimizing the first order amplitude distortions of sextupolar resonances
    - Small enough sextupole strength
    - Based of analytical formulae where only first order quantities are involved.
- **Linear and non-linear dynamics are strongly entangled (back & forth)**
  - No use of automatic procedure for full optimization



# Optimization Method

- Tune shift w/ amplitude
- Tune shift w/ energy
- Robustness to errors  
multipoles  
coupling  
IDs

- 4D tracking
- 6D tracking

- (x-z) fmap → Injection eff.
- (x-δ) fmap → Lifetime
- Touschek computation

Resonance identification

Good Working Point

No

Improvement  
Needed?

Yes

Lattice design  
Fine tuning

Tracking  
NAFF

Dynamics analysis

NAFF suggestions

Number of Knobs:  
10 quadrupole families  
10 sextupole families

$$f = p_1 |\lambda_D|^2 + p_2 |\lambda_F|^2 + p_3 |\lambda_G|^2 + p_4 |\lambda_H|^2 + p_5 |\lambda_L|^2 + p_6 \left( \frac{\Delta U_x}{\epsilon_x / \pi} \right)^2 + p_7 \left( \frac{\Delta U_x}{\epsilon_z / \pi} \right)^2 + p_8 \left( \frac{\Delta U_z}{\epsilon_z / \pi} \right)^2 + p_9 \xi_x^2 + p_{10} \xi_z^2$$

# What is included in the Model?

- **Systematic multipole errors**
  - large  $\delta$ -acceptance, large  $\eta$ -function  $\rightarrow$  high order required
  - Dipole: up to 14-poles
  - Quadrupoles: up to 28-poles
  - Sextupoles: up to 54-poles
  - Correctors (steerers): up to 22-poles
    - Secondary coils in sext.  $\rightarrow$  strong 10-pole term (large  $\delta$ )
    - Typical set of current for orbit correction (model/reality)
- **From magnetic measurements: anticipation of effects**
  - Add true m-poles (both systematic and non systematic)
  - Dipole: fringe field, gradient error, edge tilt errors
  - Quad.: fringe field (to be done), octupole (banana effect for QL)
- **Coupling errors** (random rotation of quadrupoles)
- **Insertion devices** (destroy often Acc. Physicists' work!)

# Codes used at SOLEIL

- **BETA-SOLEIL (fast, user friendly)**
  - Lattice design
  - Error settings and corrections (alignment, m-poles, ...)
  - Sextupole optimization
  - Tune shifts, on/-off DA (limitation for large amplitude of 2nd order matrix code)
  - 4+2D Touschek lifetime computation
- **Tracy II (AT) → long term tracking code**
  - Full symplectic code
    - 4th order Ruth and Forest integrator, Laskar's scheme integrator
    - Inclusion of arbitrary m-poles, validity for large off-momentum oscillation amplitudes
  - True 6D tracking code
  - Frequency map analysis (NAFF package)
  - 4D or 6D energy acceptance computation, Touschek lifetime
- **Turn number selections for DA, FMA: @ SOLEIL 1026 turns is enough**
  - Choice dictated by
    - A good convergence near resonances
    - Beam damping times
    - 4D/6D
  - Use of diffusion coefficient to extrapolate dynamics for high number of turns

# Mapped IDs *à la* ESRF

- IDs with complex EM-field (polarization, low gap, quasi-periodic mode)
  - Halbach formalism does not work!
- Nonlinear 2D maps of IDs are generated using the 3D RADIA code.
- BETA-SOLEIL and TRACYII have been modified in order to read the IDs maps.
- Thin lens model (2nd order integrator):
  - choice of number of lenses fixed by tune convergence
  - full ID, full ID + end poles
- Good agreements with e-beam meas.

See my second talk on Tuesday

- The angular kicks experienced by the particle are derived from the function:

$$\phi(x, z, s) = \left( \int_{-\infty}^s B_x ds' \right)^2 + \left( \int_{-\infty}^s B_z ds' \right)^2$$

- $\phi$  is integrated over 1 period resulting in a potential function  $U$ :

$$U(x, z) = \int_{1\text{period}} \phi(x, z, s) ds$$

- The angular kick experienced by a particle over the undulator period is:

$$\Delta x' = -\frac{1}{2(p/e)^2} \frac{\partial U}{\partial x}(x, z)$$

$$\Delta z' = -\frac{1}{2(p/e)^2} \frac{\partial U}{\partial z}(x, z)$$



# Frequency map analysis

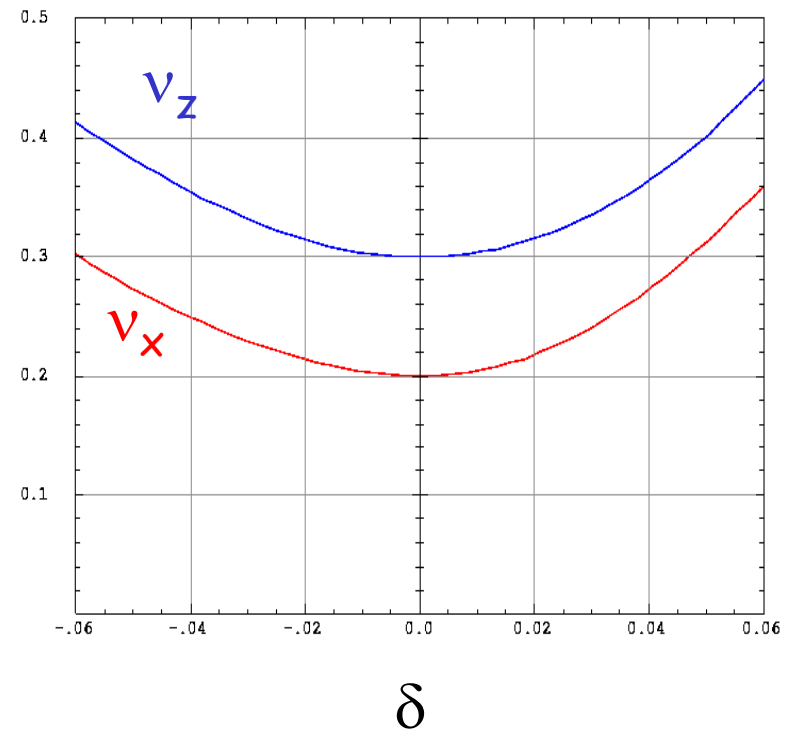
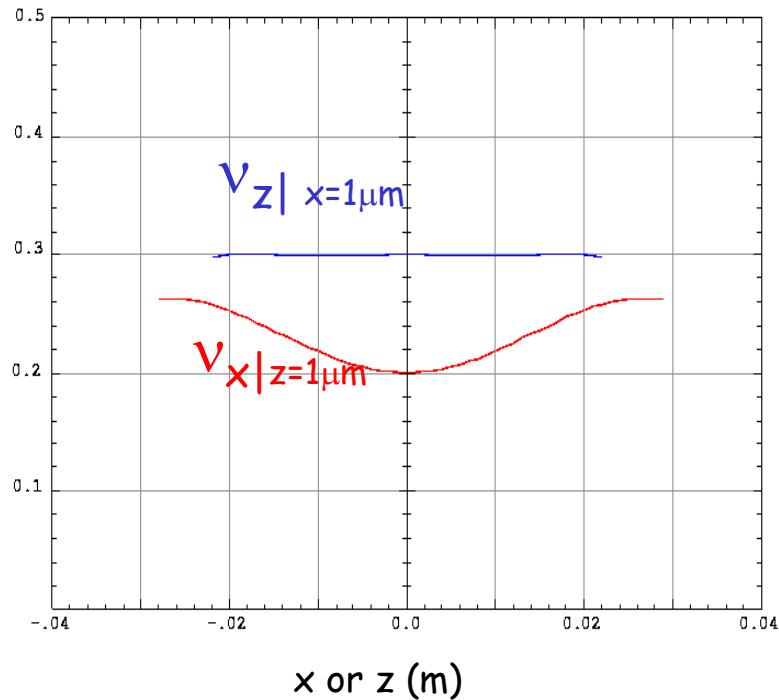
## FMA at design stage for the SOLEIL lattice

- A numerical method based on a refined FFT (J. Laskar)
  - Convergence as  $1/N^4$  using a Hanning windows ( $1/N$  for FFT)
  - Use of diffusion index
    - Predict orbit diffusion
    - Identify resonance
    - Show stable to strongly non linear areas in dynamic aperture and tune space
- Gives us a global view of the dynamics (footprints, DA contents & limitations)
- Shows dynamics sensitivity to quadrupoles, sextupoles and insertion devices
- Reveals nicely effect of coupled resonances, specially cross term  $v_z(x)$
- Enables us to modify the working point to avoid resonances or regions in frequency space
- Importance of coupling correction to small values (below 1%)
- 4D/6D ...

# Nominal working point (18.2, 10.3)

No coupling resonance crossing

$$v_x - v_z = 8 \quad (\Delta v = 0.1).$$



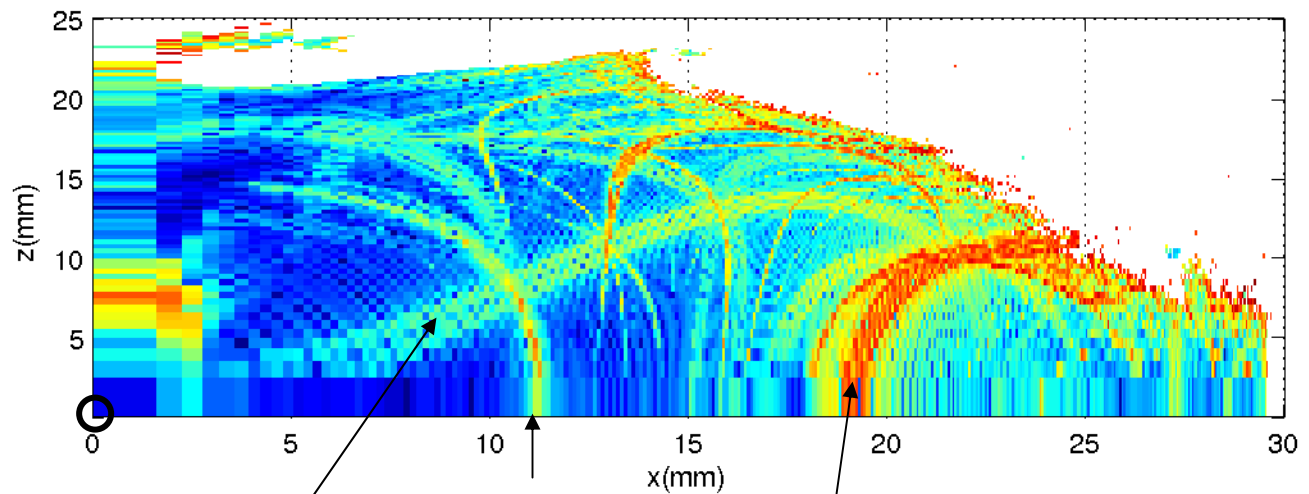
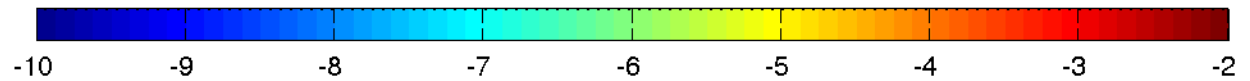
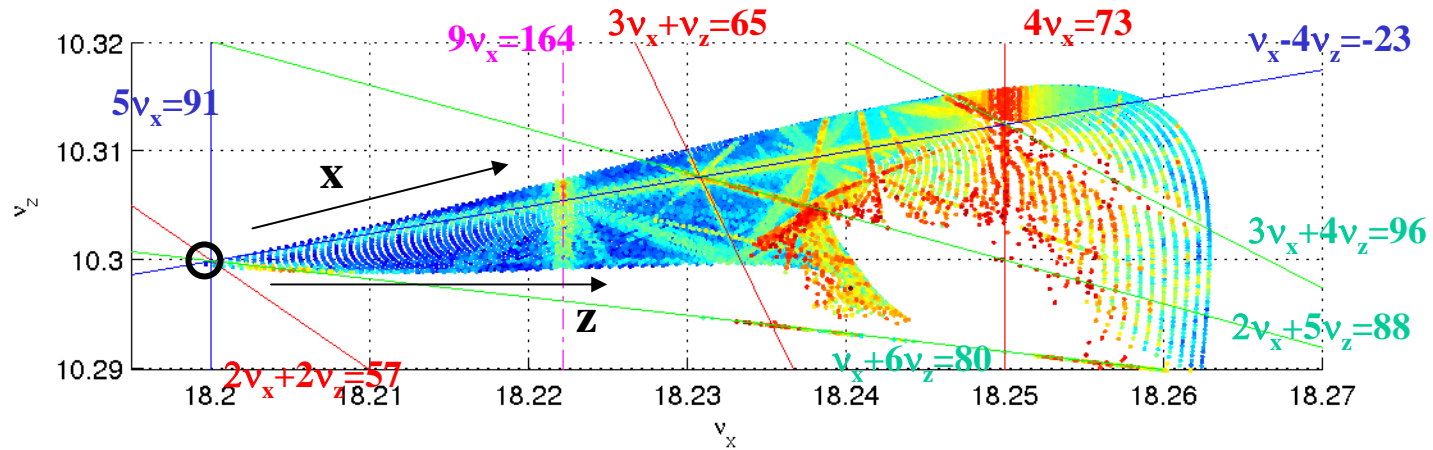
SDAC

SDAC

Just looking at these curves, dynamics seems very clean ...

# On-momentum Dynamics -- Working point: (18.2, 10.3)

Bare lattice  
(no errors)



$$v_x - 4v_z = -23$$

$$9v_x = 164$$

$$4v_x = 73$$

WP sitting on  
resonance node

$$v_x + 6v_z = 80$$

$$5v_x = 91$$

$$v_x - 4v_z = -23$$

$$2v_x + 2v_z = 57$$

Ok if low amplitude

Beware of tune shifts  
from IDs!

# On-momentum dynamics with 1.9% coupling

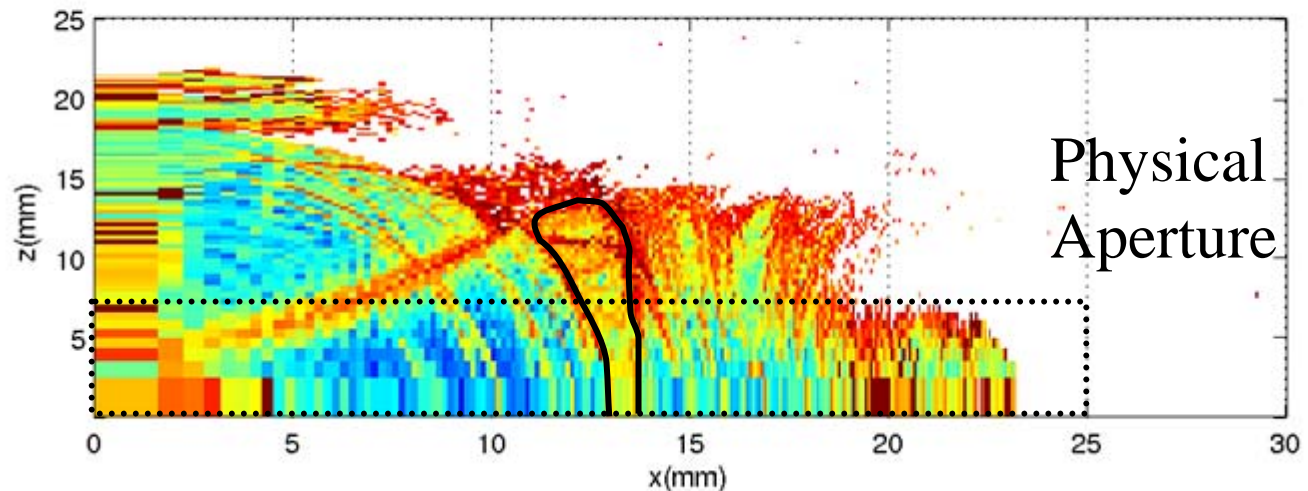
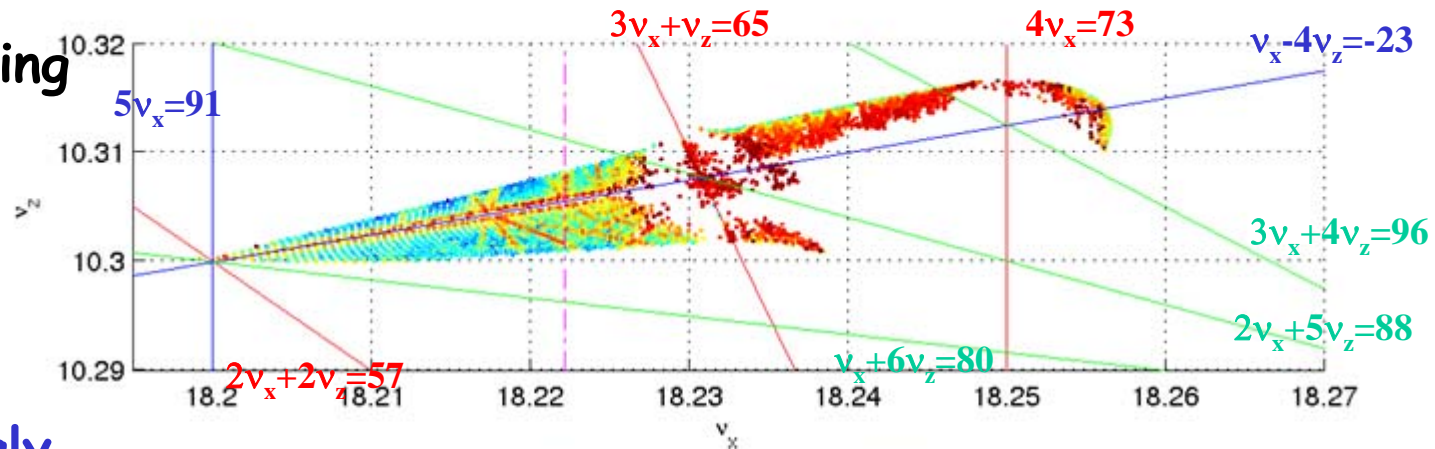
Randomly rotating  
160 Quads

• Map fold  
Destroyed

• Coupling strongly  
impacts

$3v_x + v_z = 65$

• Resonance node  
excited



Resonance island

$3v_x + v_z = 65$

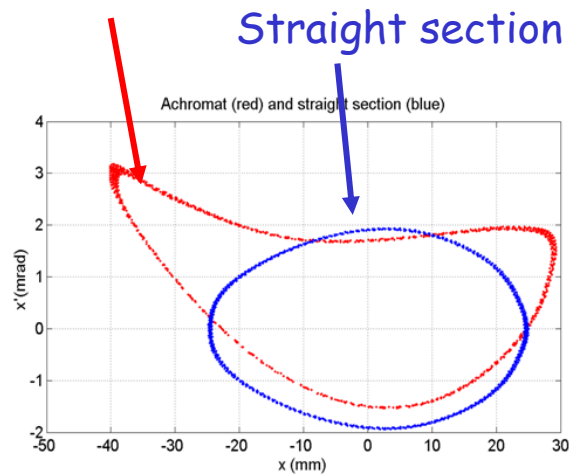


# Physical Aperture limitations Included at early stage

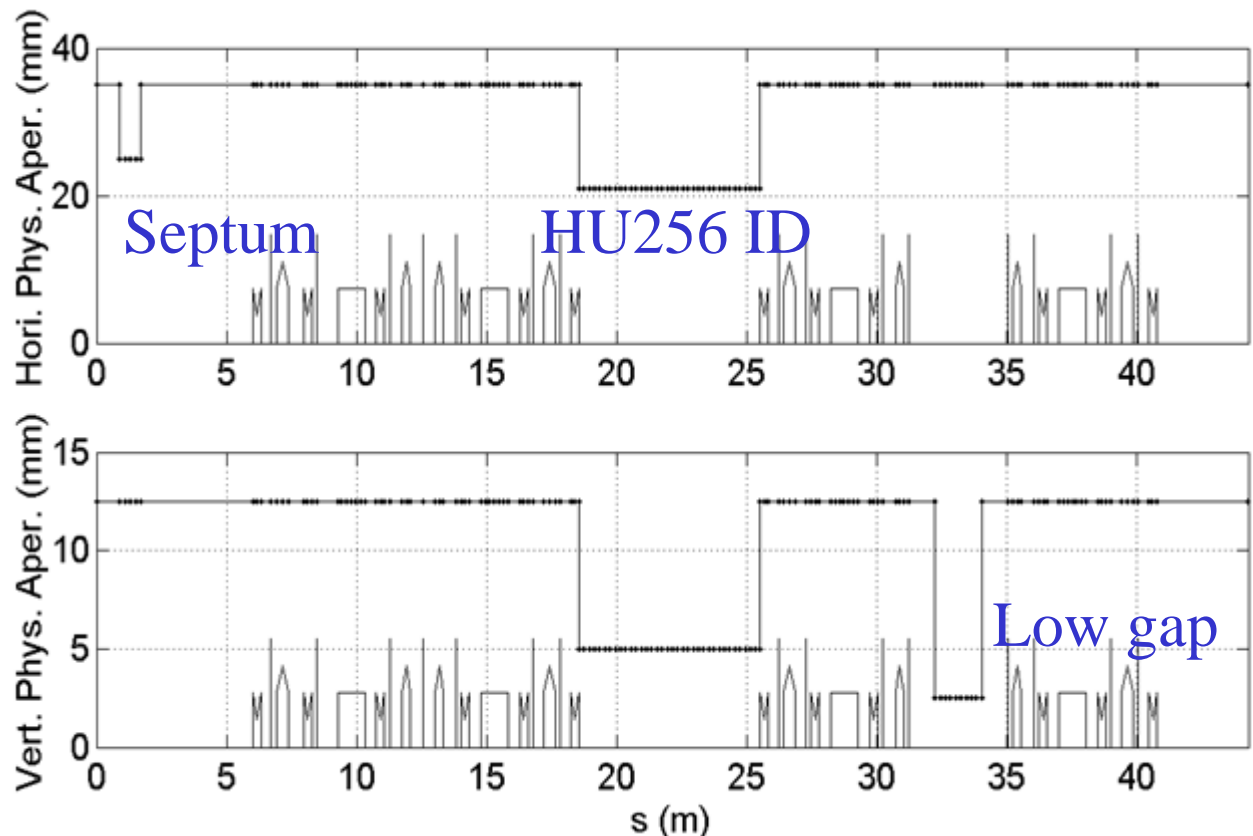
Need to introduce into the code real vacuum chamber dimensions all around the ring (no sufficient to check only at a single  $s$ -position: cf phase space distortions)

- H-plane: Absorbers, septum, etc...
- V-plane: Absorbers, small gaps, etc...

## Achromat

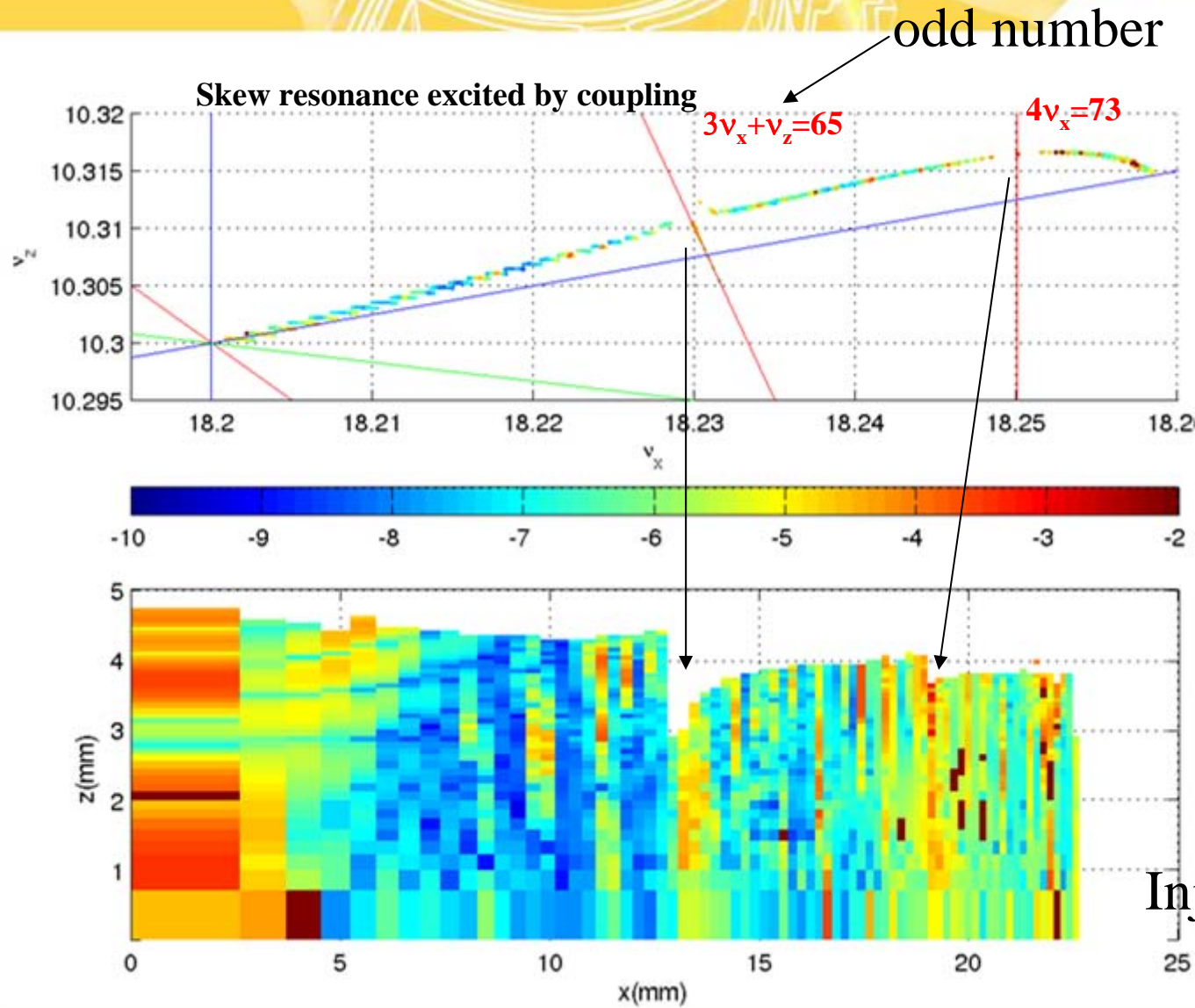


H-phase space





# Importance of including Vacuum Chamber Entanglement with Beam Dynamics



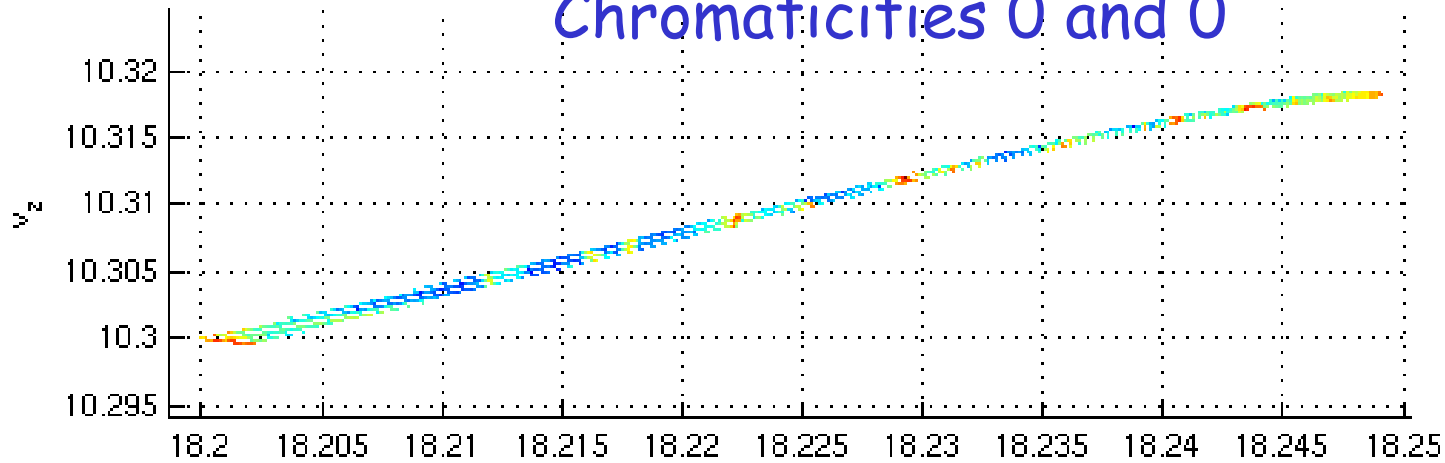
1% linear coupling

Injection @ 12mm

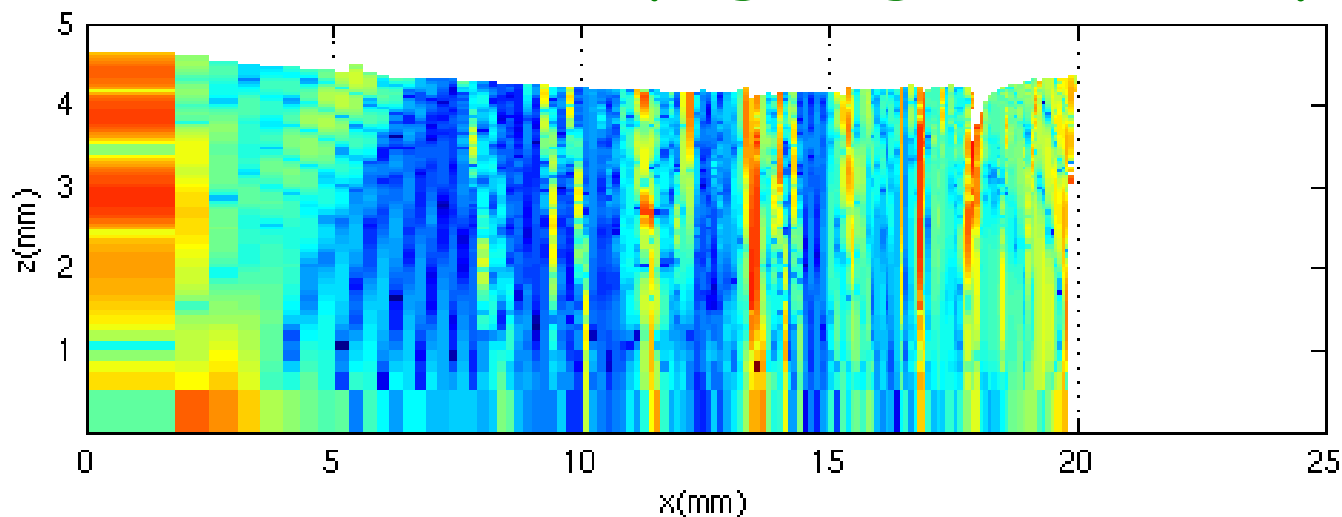
# FMA 18.20 10.30

## couplage 0.5%

Chromaticities 0 and 0



Trying to go to low coupling value



# Effect of the H-Corrector Decapolar Component (1)

- Reduction of Touschek lifetime from 35 h to 25-30h
- Correction of H&V closed orbit and coupling.
- ❖ The 3D calculations of the dipole field of the correctors located in the sextupoles indicated that the best field generates a decapolar component of:

$$\frac{\Delta B_4}{B_0} = 0.43 \quad @ \quad x = 35mm$$

- ❖ We get the following integrated decapolar strength:

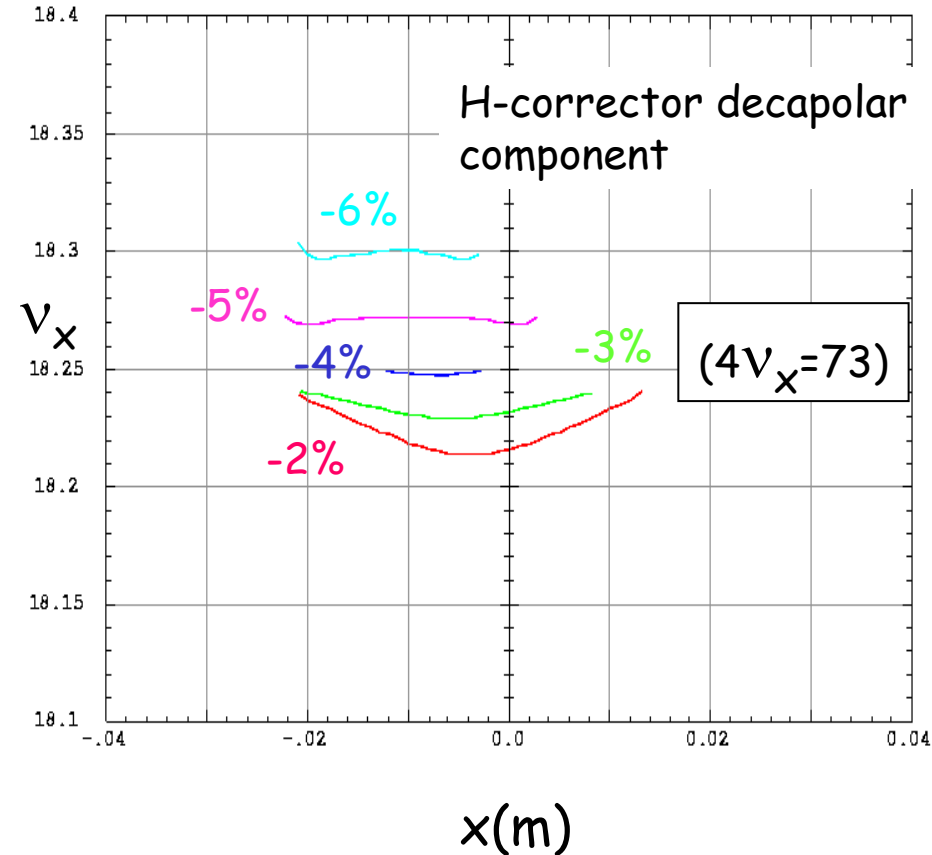
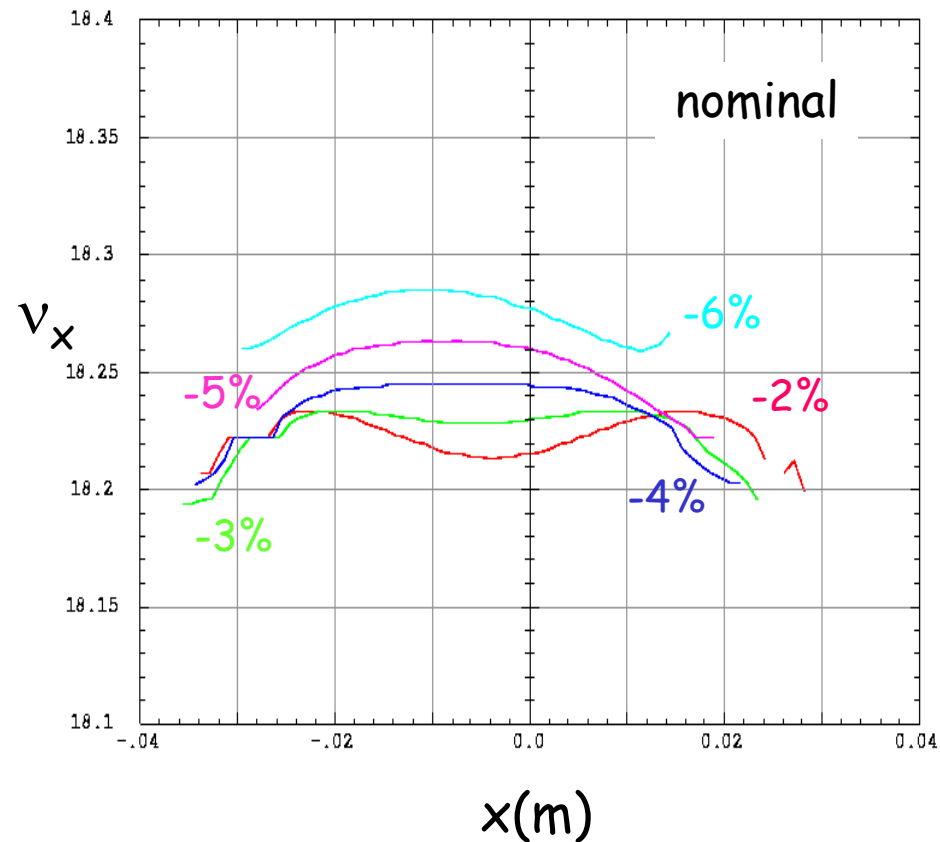
$$D_4 = \Theta \times 0.43 \times (0.035)^{-4} \quad \Theta \text{ is the kick given by the correctors}$$

$$B = D_4 x_0^4 + 4 D_4 \eta \delta x_0^3 + 6 D_4 \eta^2 \delta^2 x_0^2 + 4 D_4 \eta^3 \delta^3 x_0 + D_4 \eta^4 \delta^4$$

Feed-down effect: Off-momentum particles see octupole field  $4\nu_x$  resonance



# Effect of the H-Corrector Decapolar component (2)

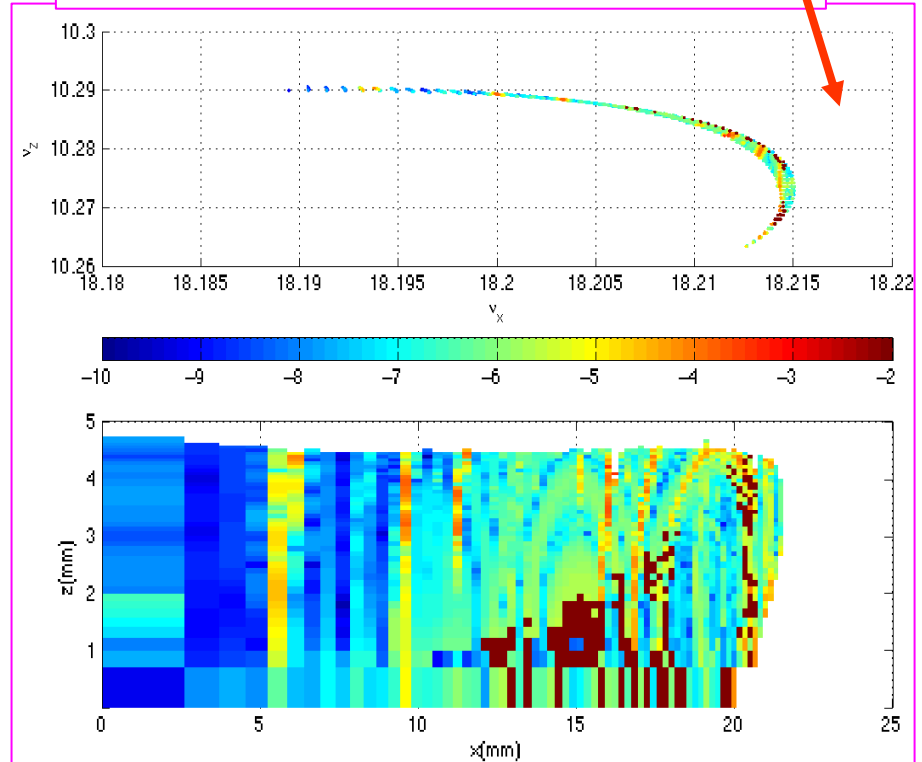
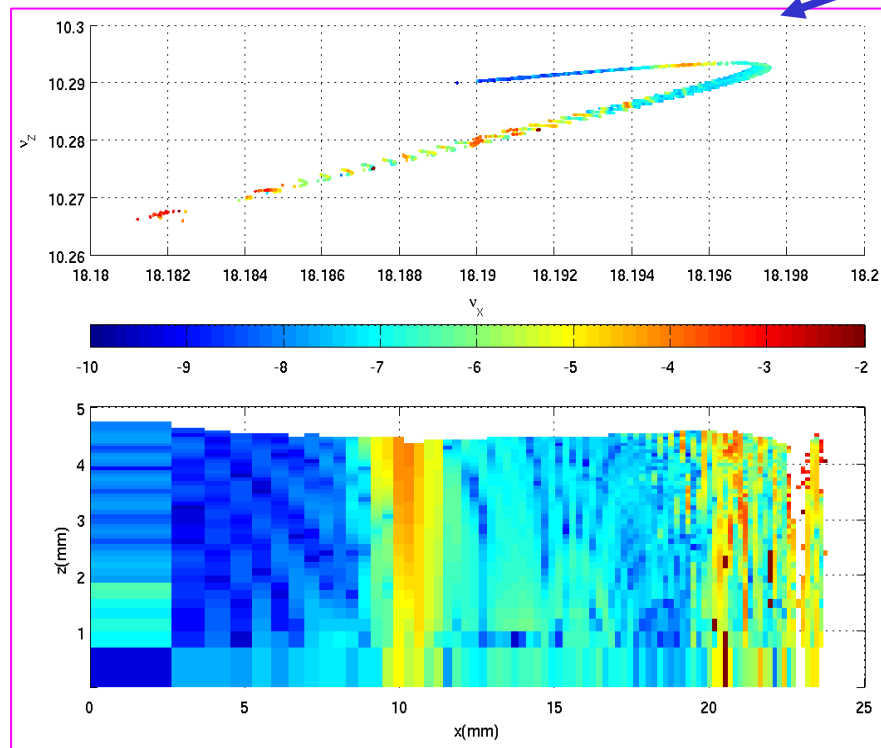
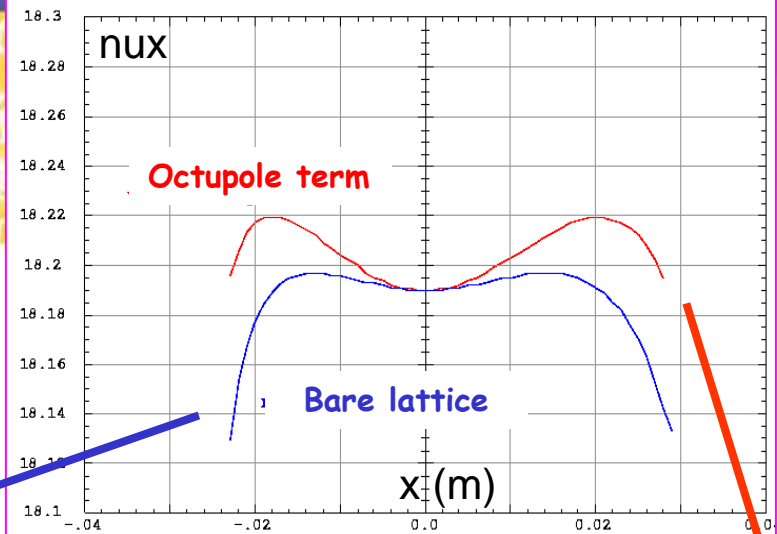


Dependent amplitude off-momentum tunes

# Effect of large non systematic octupole in Long Quadrupoles

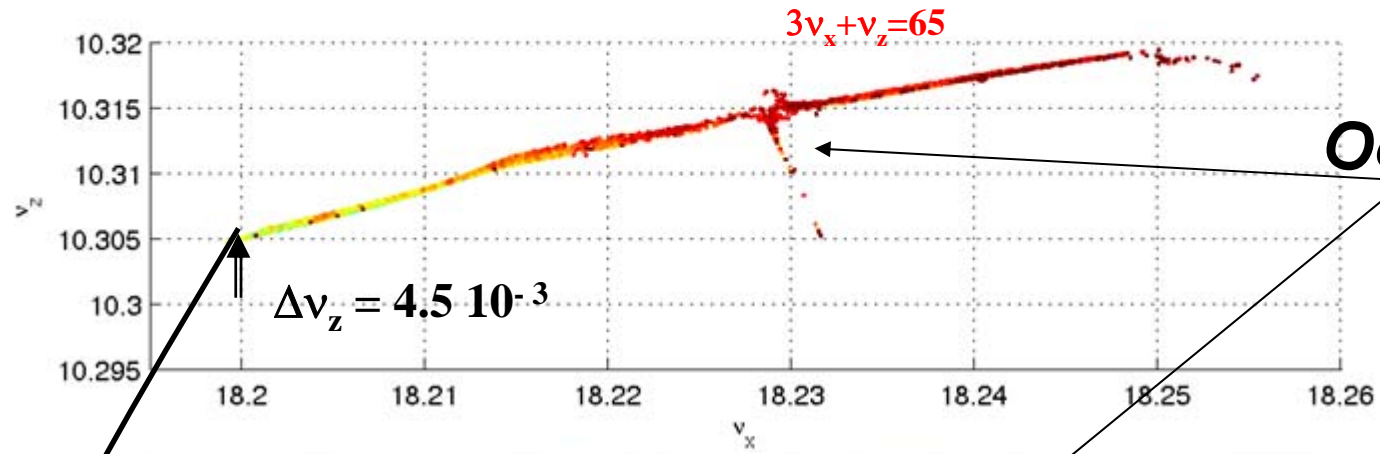
## Example of WP 18.19 / 10.29

→ retrofit with sextupoles

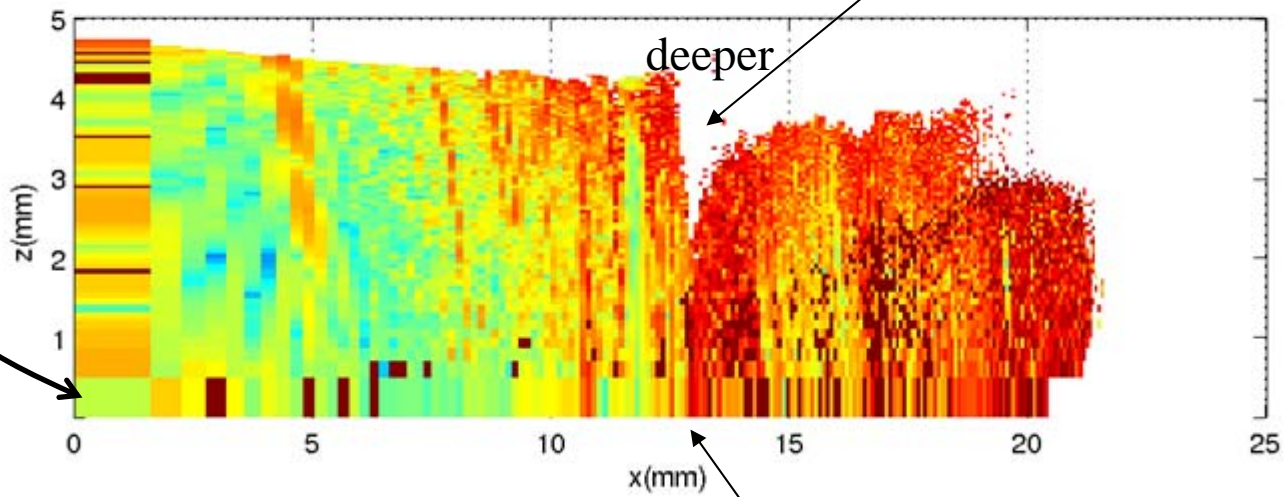
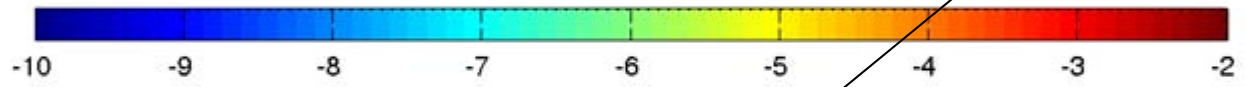


# Adding effect of 3 in-vacuum IDs

Note: 1 single U20 → negligible effect



ID excites Octupole term



Injection trouble if stronger

# Off-momentum dynamics exploration

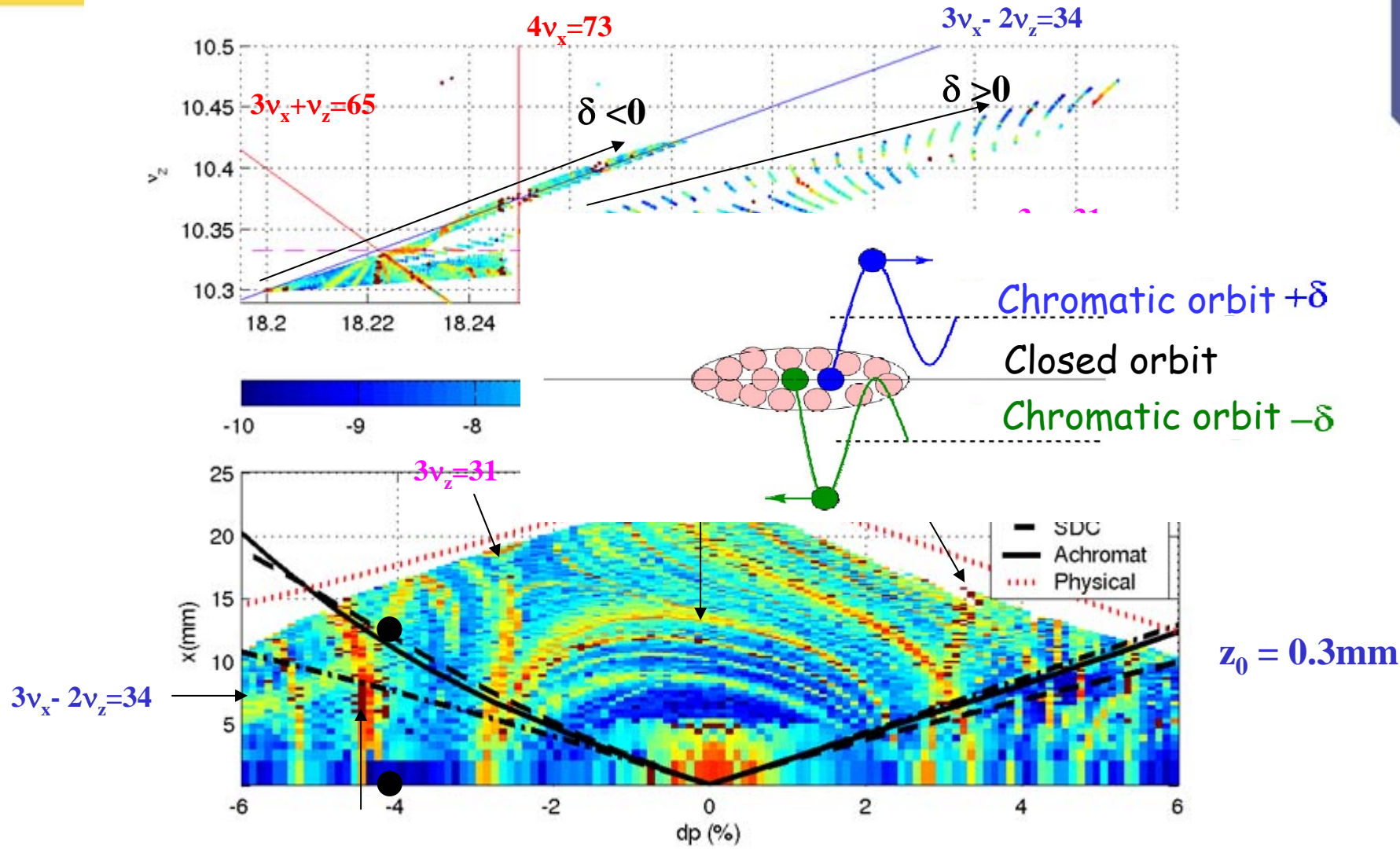
## Several approaches:

- Off-momentum frequency maps
- Energy/betatron-amplitude frequency maps
  - Very rich and concise
  - Static and dynamic information at the same time
- Touschek lifetime - momentum acceptance
  - 4D tracking
  - 6D tracking





# Off momentum dynamics w/o IDs



# Off momentum dynamics w/ $3 \times U20$

## What's about Effect of synchrotron radiation and damping?

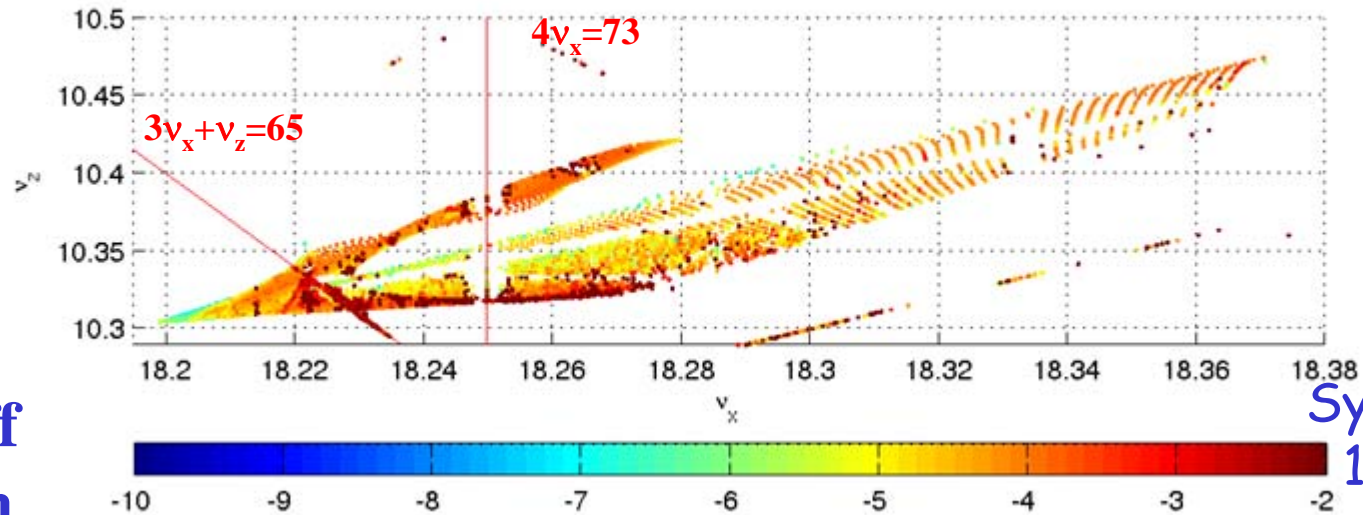
U20

B-Roll off  
 $\beta_x = 18 \text{ m}$   
 $g = 5 \text{ mm}$

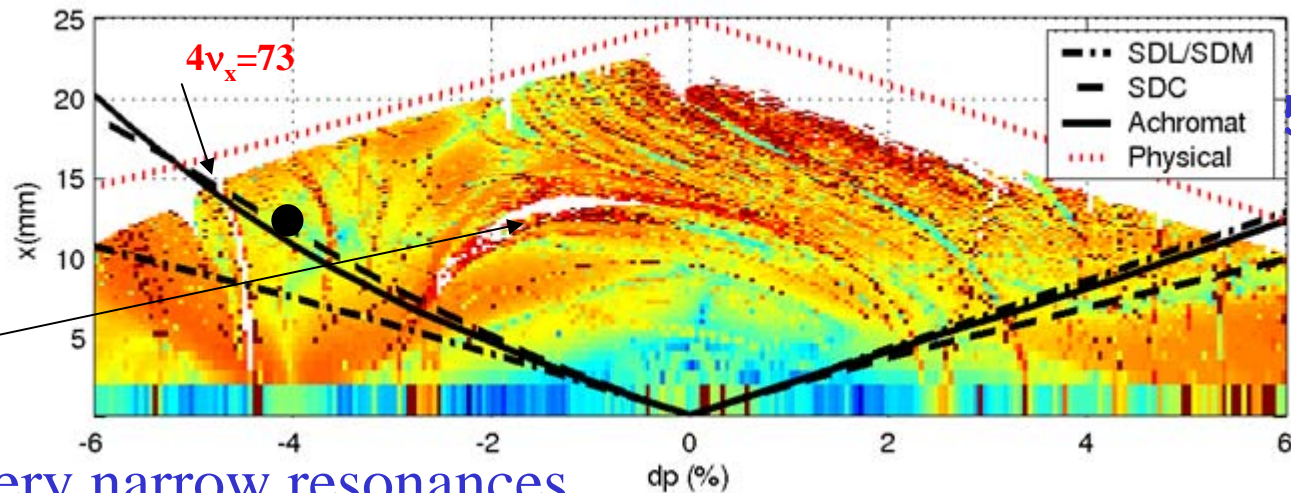
$3\nu_x + \nu_z = 65$

Loss over  
 $>400$  turns

Stable in 6D



Synchrotron  
 140 turns



Damping  
 5600 turns

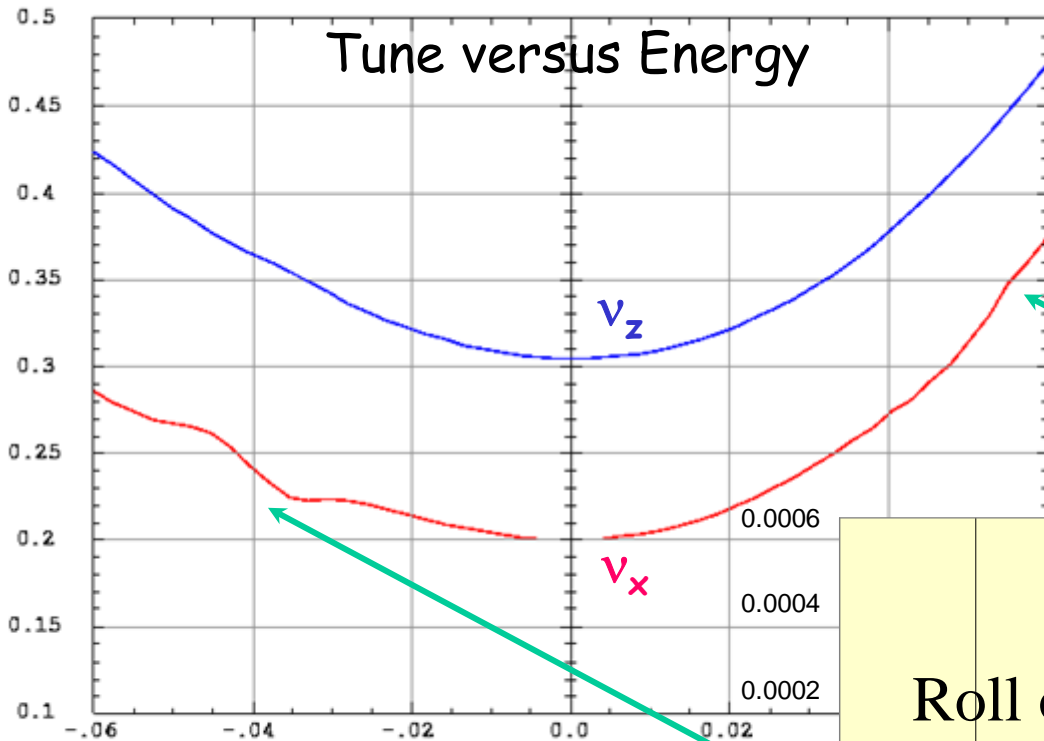
Very narrow resonances

# Tune Shift with Energy

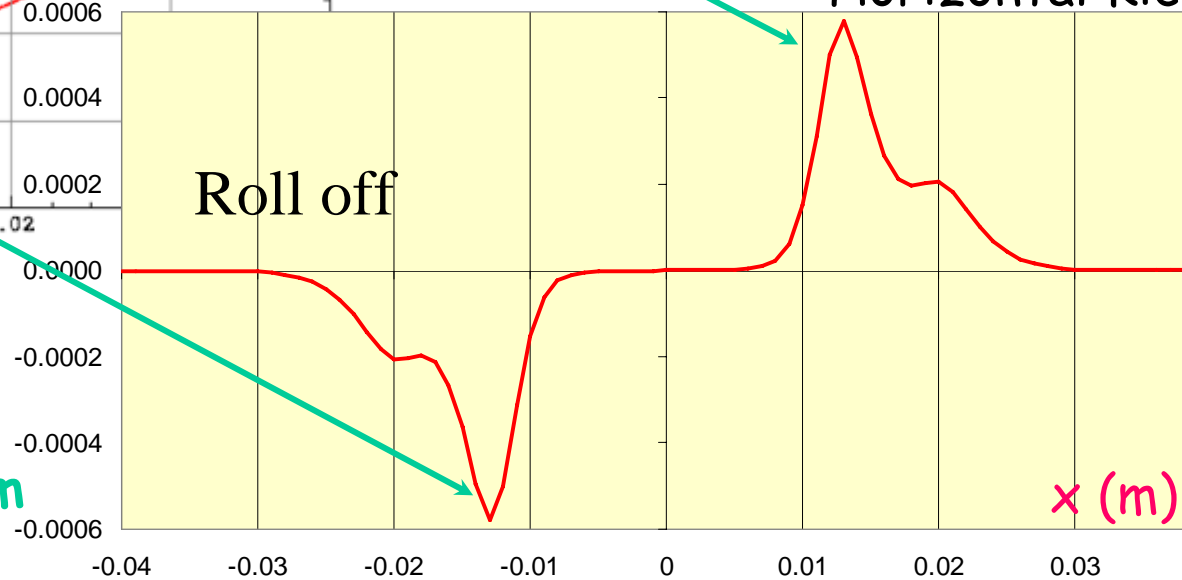
$3 \times U20$

$dp/p = + 5 \%$   
corresponds to  
Nonlinear Xchrom = + 12 mm

Tune versus Energy



Horizontal Kick



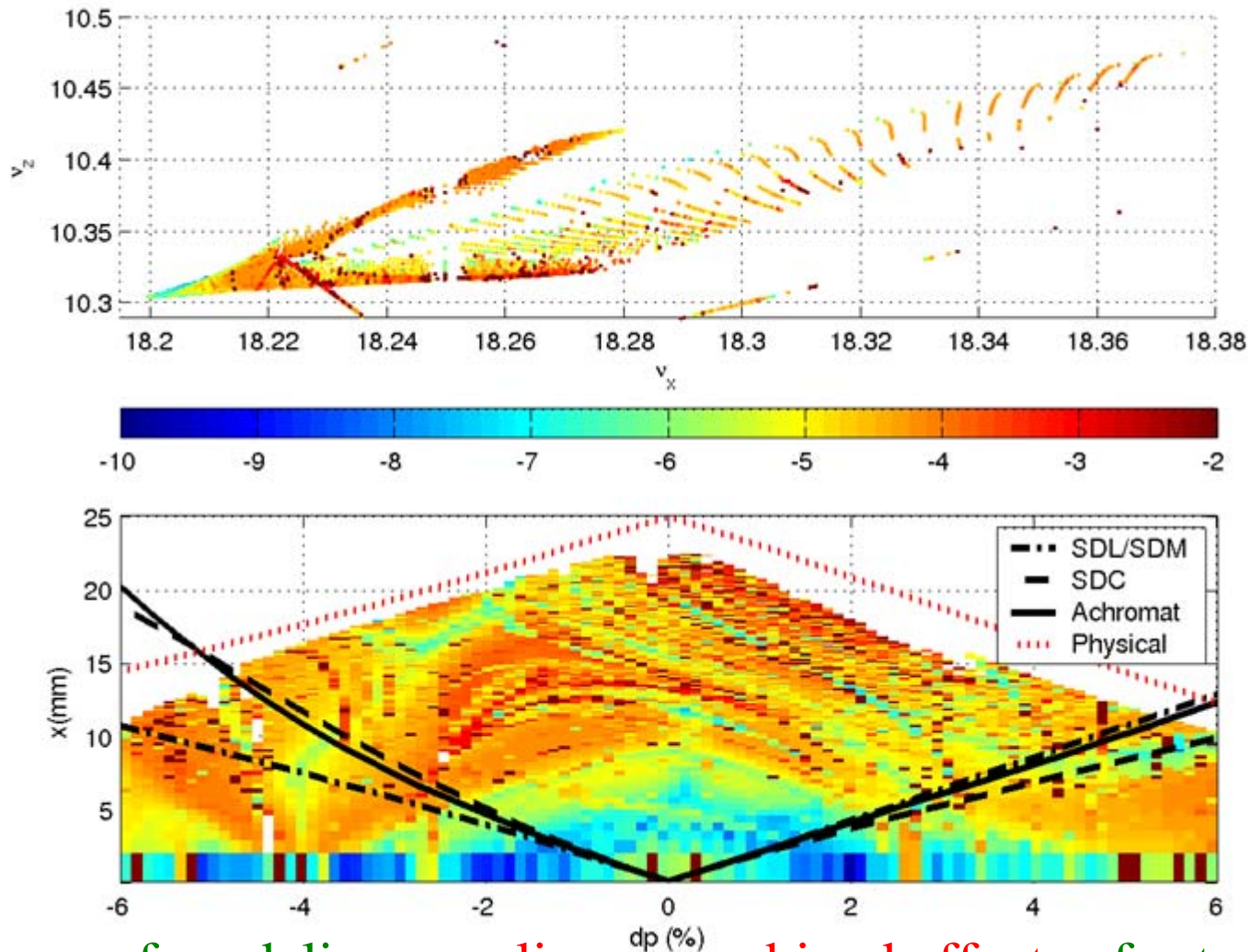
$dp/p = - 4 \%$   
corresponds to

Nonlinear Xchrom = - 12 mm



# Coupling reduction by a factor 2 with 3 x U20

Usefulness to go to low coupling value



Usefulness of modeling **non-linear** combined effects of set of IDs



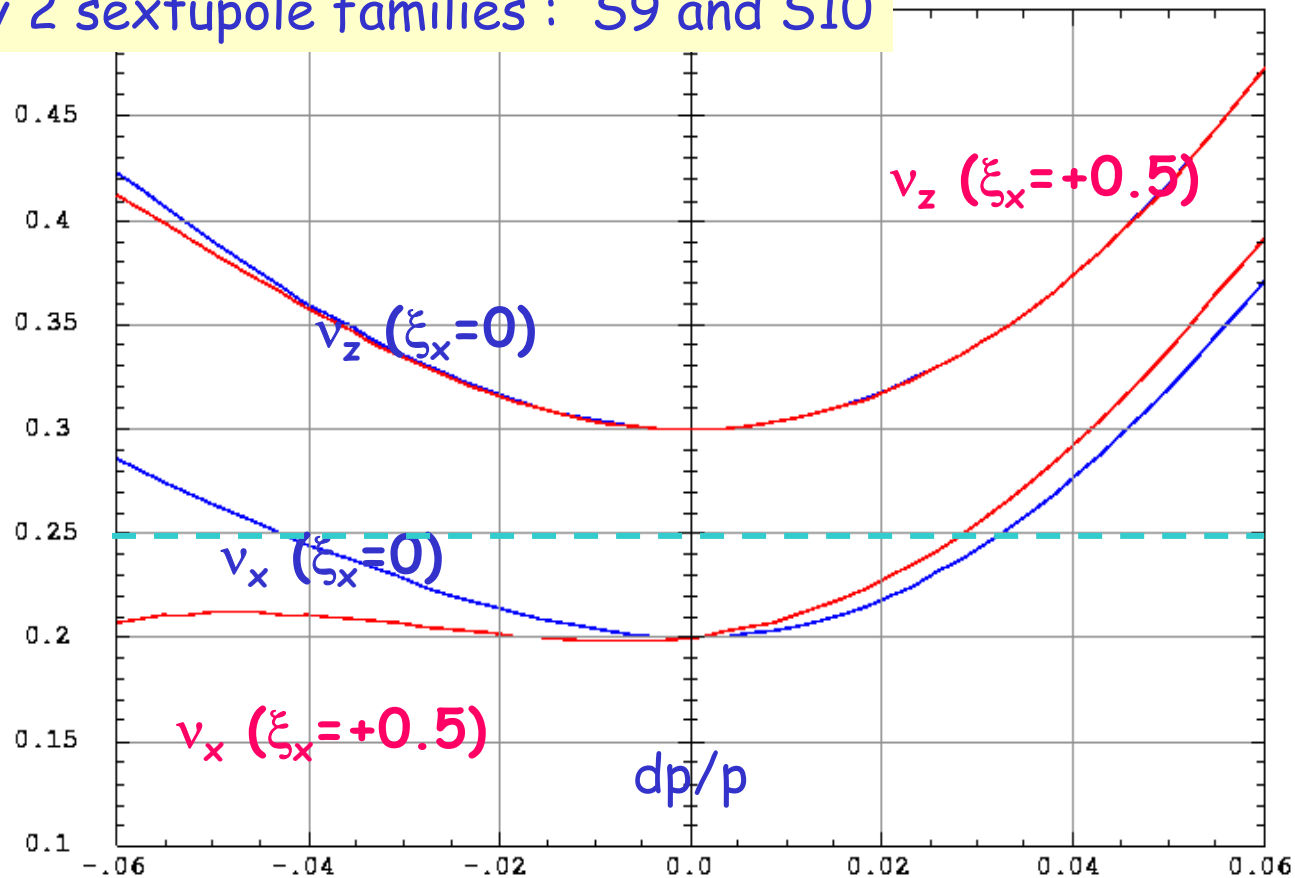
# Optimization of a New Point Enhanced philosophy

- On momentum
  - $3 \nu_x + \nu_z = 65$  to be avoided (not shown w/o FMA)
  - WP to be shifted from resonance node
  - Control of tune shift with amplitude using sextupole knobs
    - $\nu_x(J_x, J_z) = a J_x + b J_z$
    - $\nu_z(J_x, J_z) = b J_x + c J_z$
- Off momentum  $\nu_x(\delta)$ 
  - Large energy acceptance
  - Control of the tune shift with energy using sextupoles
  - The  $4 \nu_x = 73$  resonance has to be avoided for insertion devices

~~Going to 100 mm magnet pole width?~~

One solution :  $\xi_x = +0.5$  to avoid  
 $4\nu_x = 73$  at  $dp/p = -4.4\%$

Using only 2 sextupole families : S9 and S10

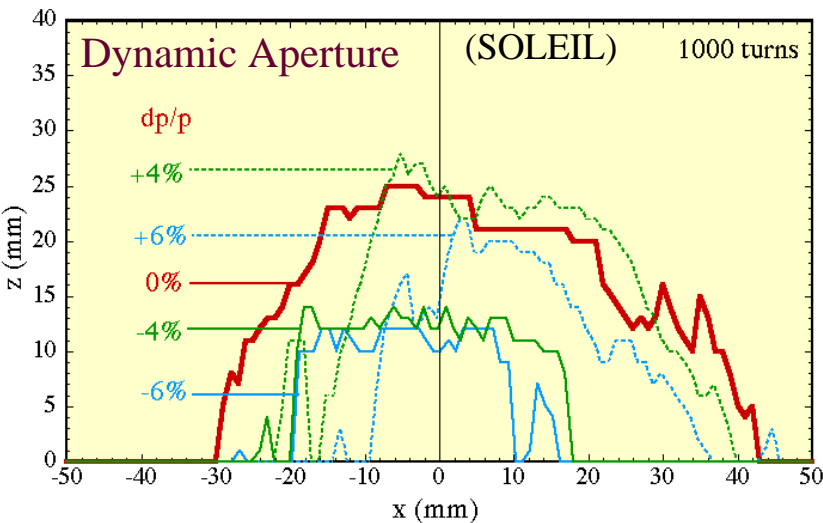
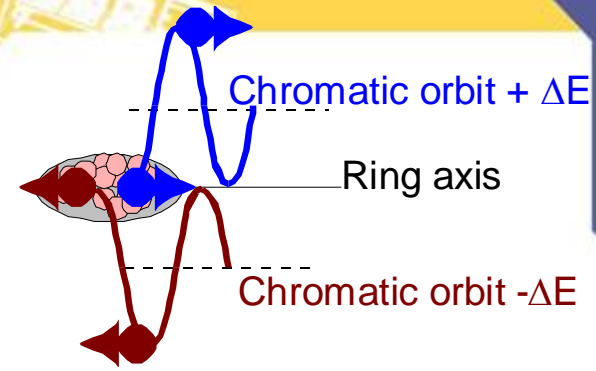


Tune shift w/ energy optimized with sextupoles to avoid in addition the  $4\nu_x = 73$  resonance for negative energy offsets

# Touschek Beam Lifetime

**Touschek Effect:** scattering within the bunch

$$\frac{1}{\tau_{T\frac{1}{2}}} = \left( \frac{r_e^2 c N}{8\pi \gamma^3 \sigma_l} \right) \cdot \frac{1}{L} \int_0^L \frac{C \left[ \left( \frac{\mathcal{E}_{acc}(s)}{\gamma \sigma'_x(s)} \right)^2 \right]}{\sigma_x(s) \sigma_z(s) \sigma'_x(s) \mathcal{E}_{acc}^2(s)} ds$$



High density-Touschek scattering of particles - large longitudinal transfers of energy - loss unless large acceptance: RF acceptance, physical aperture, dynamic aperture for large energy deviations

$$\tau_T \propto \mathcal{E}_{acc}^{>2} \quad (\text{SOLEIL: } \mathcal{E}_{acc} = 4 \text{ to } 6\%)$$

Induced amplitude in a non zero dispersion location:

$$x = \sqrt{A_x \beta_{x1}} + \eta_1 \delta = (\sqrt{H_0 \beta_{x1}} + \eta_1) \cdot \delta \quad A_x = \gamma_{x0} (\eta_0 \delta)^2 + 2\alpha_{x0} (\eta_0 \delta) (\eta'_0 \delta) + \beta_{x0} (\eta'_0 \delta)^2 = H_0 \delta^2$$

# Transverse Energy Acceptance

❖ Some other effects have to be considered:

⇒ **non-linear betatron motion**, i.e., transverse phase space distortion,

⇒ **non-linear synchrotron motion**, i.e., effects of higher order chromaticities and higher order momentum compaction factors (already partly implemented in BETA),

⇒ **synchrotron radiation**: to follow the amplitude variation of the particles during the damping process (diffusion, resonance crossing),

⇒ **coupling from horizontal to vertical plane**:

- induces also a vertical betatron amplitude from Touschek scattering:

⇒  $\varepsilon_{\text{acc}}$  limitations from small vertical gaps,

- possible diffusion process in the vicinity of skew resonances

⇒ Vertical amplitude growing,

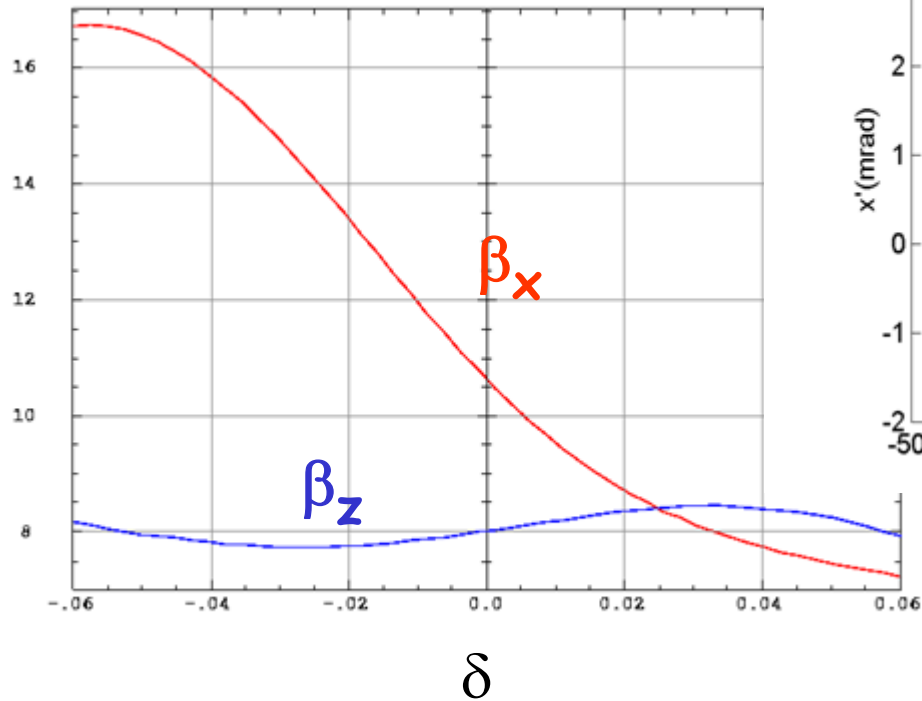
⇒ **higher order multipole effects.**



# Twiss function variations

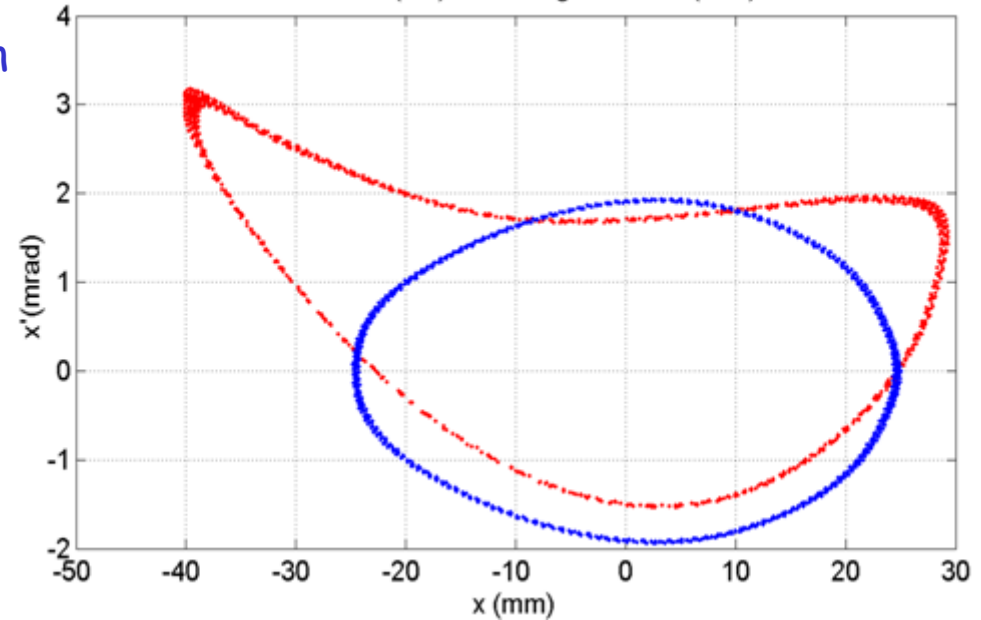
## Betatron function variations with energy

Center of the long straight section



## Non-linear betatron motion with amplitude

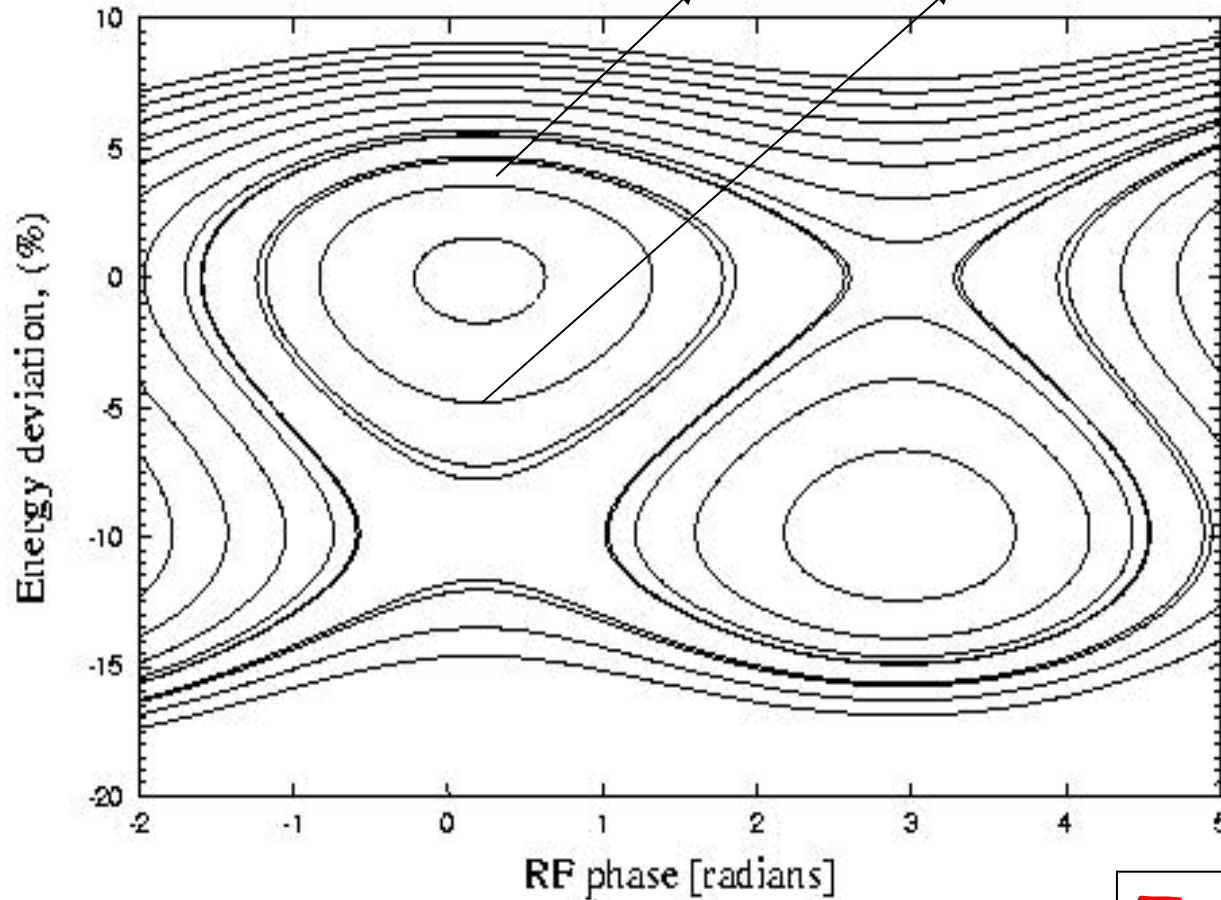
Achromat (red) and straight section (blue)



NB: Other reason for having the vacuum vessel defined everywhere for the tracking

# Non-linear synchrotron motion

+3.8% ↔ -6%



$$\alpha_1 = 4.38 \cdot 10^{-4}$$

$$\alpha_2 = 4.49 \cdot 10^{-3}$$

$$\alpha_1 = \oint \frac{\eta_1}{\rho} ds$$

$$\alpha_2 = \oint \left( \frac{\eta_1'^2}{2} + \frac{\eta_2}{\rho} \right) ds$$

**Tracking 6D required**

# Touschek Tracking

- ❖ One way to take into account these effects all together is to perform a 6D tracking.
- ❖ The energy acceptance calculation problem is then reduced to the very simple question:

Is the particle with starting coordinates  $(0,0,0,0, \pm\delta,0)$  stable or not?

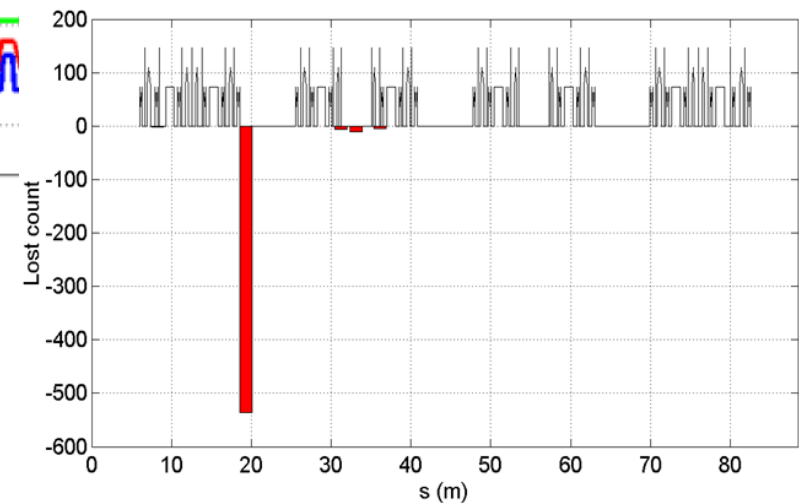
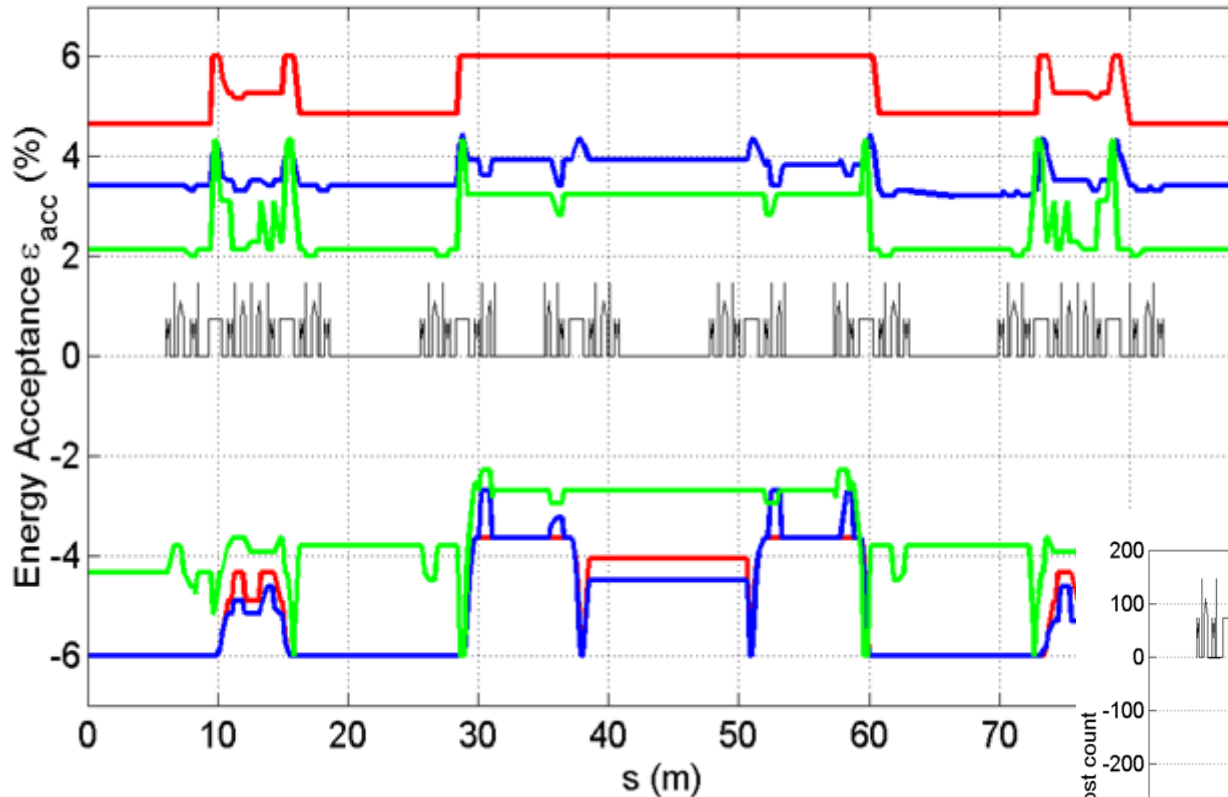
⇒ Particles tracking using the TRACY II code— Ruth and Forest's fourth order symplectic integrator.

Particles are tracked:

- ✓ over 1026 turns
- ✓ with a starting vertical amplitude of 0.3 mm
- ✓ with a given energy deviation  $\delta$ : from -6% to 6% by steps of 0.1%.
- ✓ Understand where (s-location, plane) and why particles are lost?  
→ Provide hints for improving/changing the working point

# Example of working point 18.30 & 10.27

TRACY: 4D (red), 6D (blue), and 6D w/ 1% coupling (green)

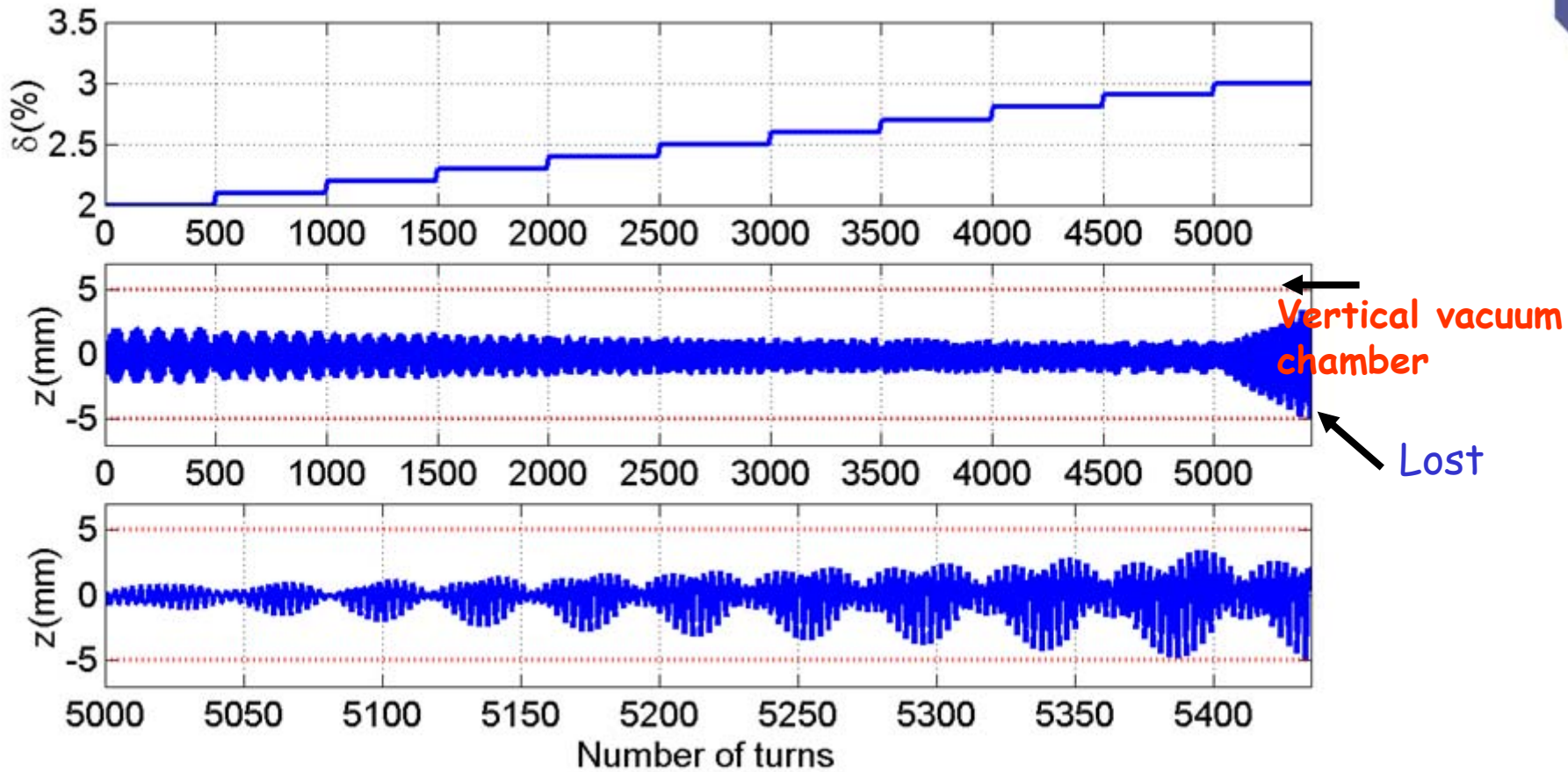


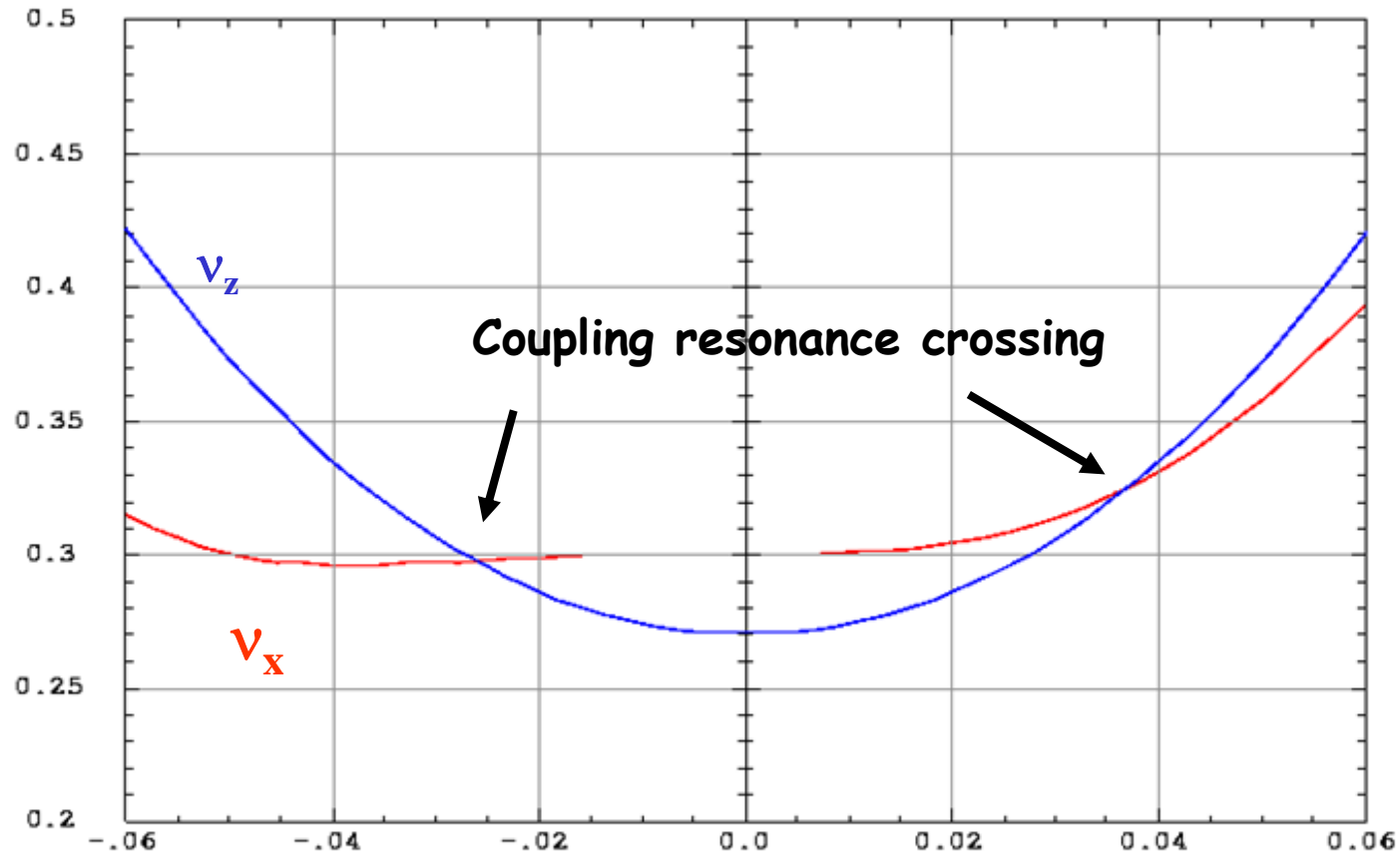
Particles lost in the vertical plane



# Hitting the vertical chamber by diffusion

Understanding the physics of losses ...



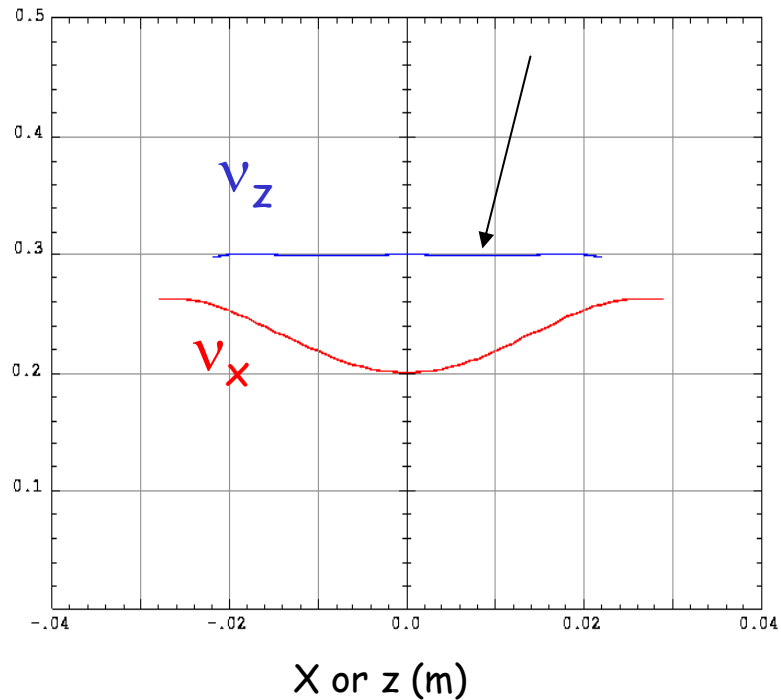


Linear coupling induced non-linear effects

Ensure that the linear coupling resonance is not crossed off-momentum

# Nominal working point (18.2, 10.3)

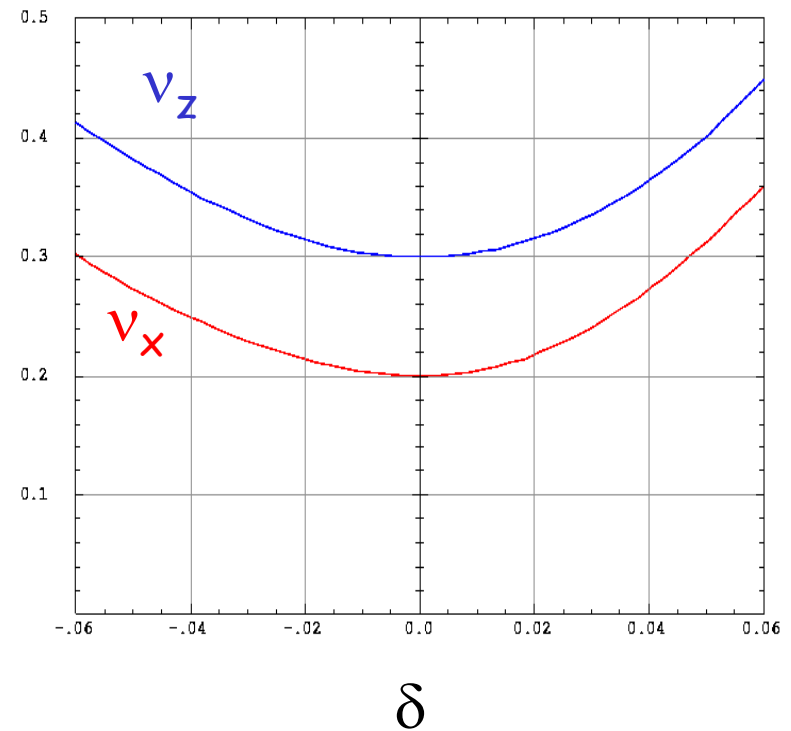
Flat vertical tune



SDAC

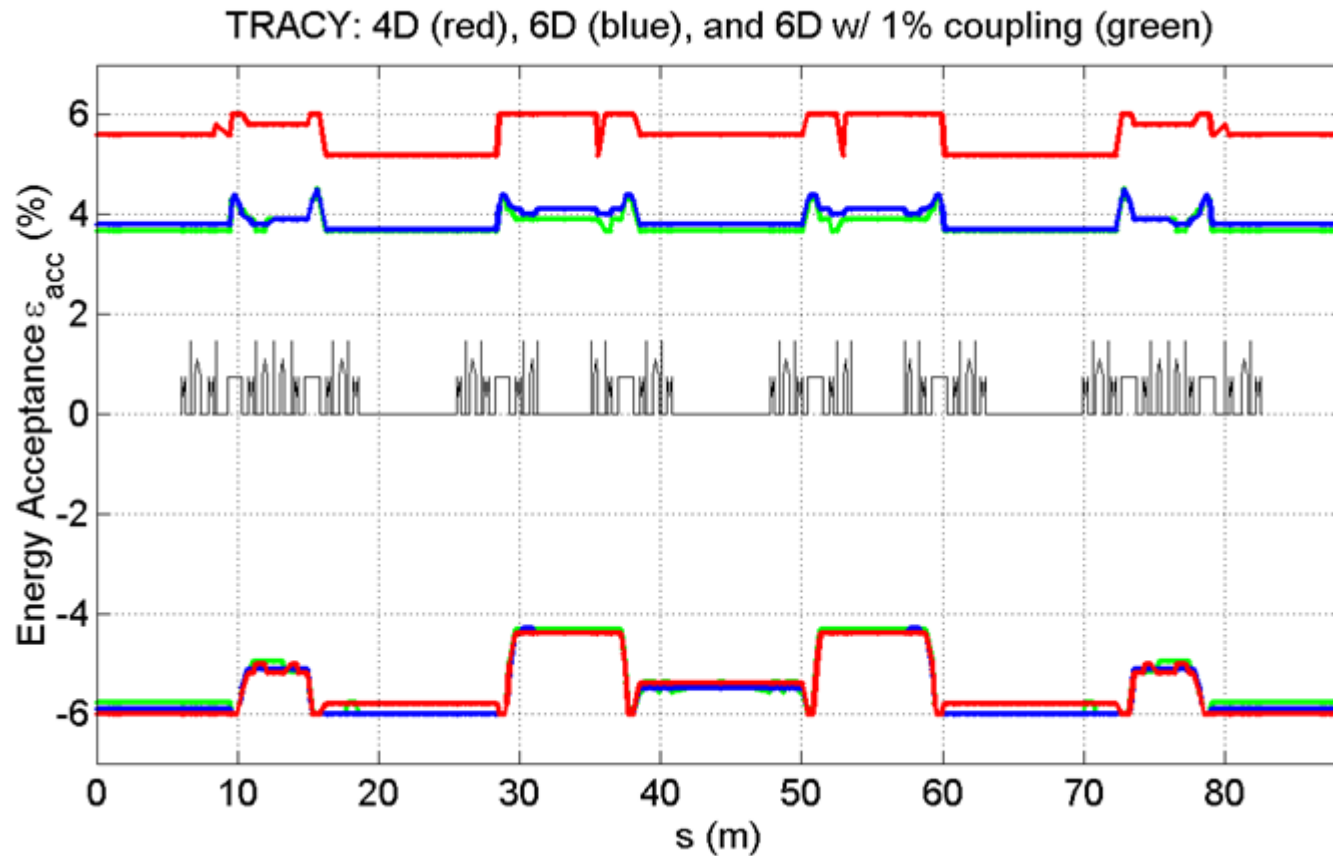
Non coupling resonance crossing

$$v_x - v_z = 8 \quad (\Delta v = 0.1).$$



SDAC

# Nominal working point 18.20 & 10.30 ( $\Delta v=0.1$ )





$$\frac{1}{\tau_{T1/2}(s)} = \frac{1}{2} \left[ \frac{1}{\tau_{T1/2}^+(\mathcal{E}_{acc}^+(s))} + \frac{1}{\tau_{T1/2}^-(\mathcal{E}_{acc}^-(s))} \right]$$

- ❖ Optics with 18.2 and 10.3:
- ❖ 6D Tracking with 1% coupling, mini gap ( $\pm 2.5$  mm in short straight section)
- ❖ natural bunch length (500 mA in 416 bunches, 4MV)

Example of local energy acceptance

$$\tau_{T1/2}^- = 66h \quad \tau_{T1/2}^+ = 22h$$

Long Straight section: +3.8% -5.8%

Short Straight section: +4.0% -4.3%

$$\tau_{T1/2} = 33h$$

- ❖ Combined with a gaz lifetime of 24h, this gives a total beam lifetime of approximately 16h.

# Conclusion

- Model used for giving magnet multipole tolerances, for validating the design of the IDs. **With modern tools: realistic estimations**
- **Modeling improvement**
  - Fringe field quadrupole
  - Full 6D ID tracking + radiation
  - Retrofit from real IDs
  - Robustness for non zero chromaticity (few bunch operation, TFB)
- **Sextupole families**
  - 2 families for chromaticity correction
  - n families: chromaticity + nonlinearity compensation
  - Individual sextupoles:
    - Increase number of families: 13 at SOLEIL enable us **to reduce  $\alpha_2$**
    - Flexibility for local compensation (slicing exp., local focusing for BLs)
- **See measurements with e-beam and how is the agreement with the model (Tomorrow's talk) .**

# References

## • Codes

- BETA (Loulergue -- SOLEIL)
- Tracy II (Nadolski -- SOLEIL, Boege - SLS, J. Bengtsson)
- AT (Terebilo <http://www.slac.stanford.edu/~terebilo/at/>)
- RADIA (O. Chubar, <http://www.esrf.fr/Accelerators/Groups/InsertionDevices/Software>)

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