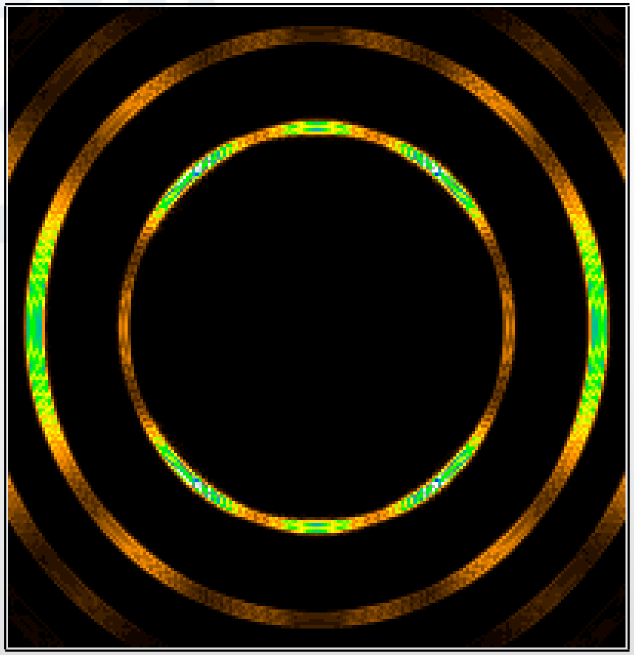
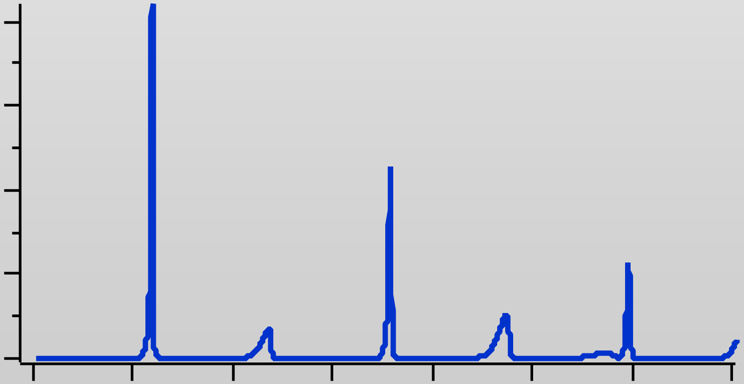
Physics of undulators

J. Chavanne
Insertion Device Group
ASD



-Introductory remarks
-Basis of undulator radiation
-Spectral properties
-Source size
-Present technology
-Summary

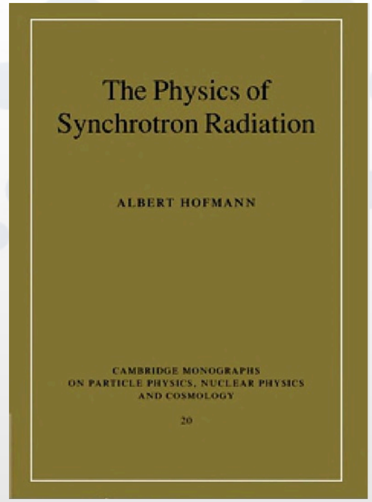


Books

Undulators, wigglers
And their applications

H. Onuki, P.Elleaume

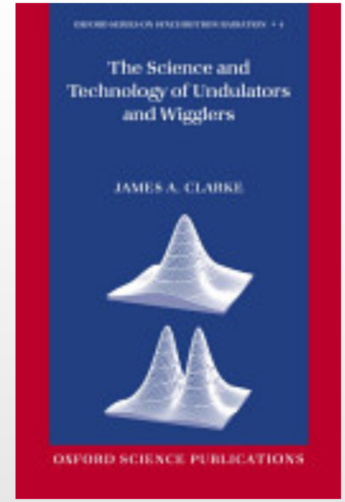
Modern theory of undulator radiations
Several ESRF authors



The physics of
Synchrotron radiation

A. Hofman

Very accessible



The science and Technology
of Undulators and Wigglers

J. A Clarke

Very clear approach

Few preliminary remarks

Many software simulations are used for undulator radiations:

All simulations done using

SRW Synchrotron Radiation Workshop (O. Chubar, P.Elleaume)

- wavefront propagation
- near & far field
- will evolve in near future

B2E (B to E) also ESRF tool

- field measurement analysis
- undulator spectrum with field errors

Unfortunately very few topics in undulator physics will be presented

Electrons in the framework of relativity

Any particle with non zero mass cannot exceed speed of light

Electron energy: $E = \gamma mc^2 = \gamma E_0$

E_0 is the electron energy at rest =0.511 MeV

γ is the relativistic **Lorentz factor** also defined as $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ Speed of electron

ESRF: E= 6.04 GeV so $\gamma = E / E_0 = 11820$ $v/c = \beta_e = \sqrt{1-1/\gamma^2} \approx 1 - \frac{1}{2\gamma^2}$

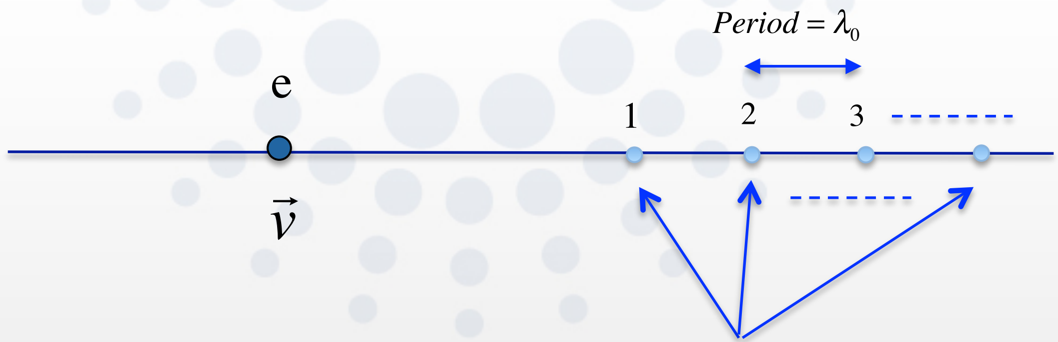
Electron

Mass: m=9.10938 e^{-31} Kg
Charge: e=-1.60218 e^{-19} C

Speed of light in vacuum: c = 299792.45 m/s

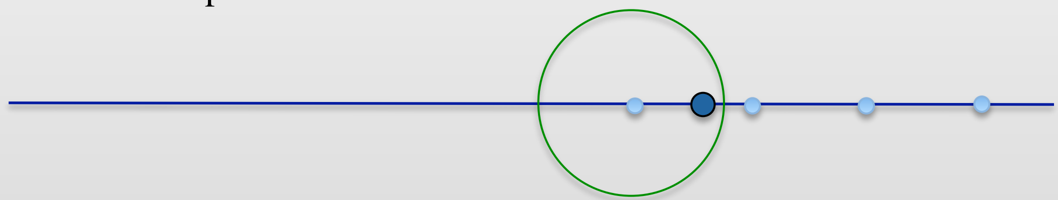
Energy E	v/c
1 MeV	0.869
100 MeV	0.9999869
1 GeV	0.999999869
6 GeV	0.9999999964

Simple periodic emitter

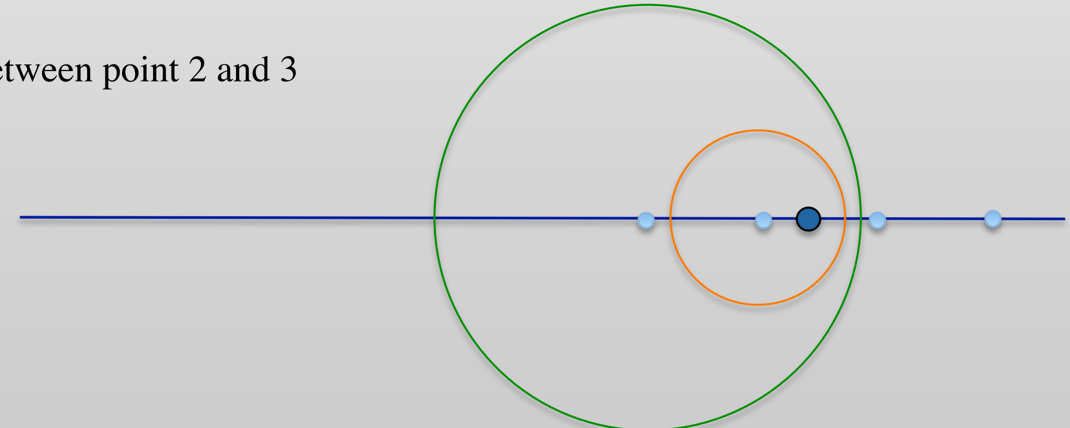


Points with wavefront emission

Somewhere between point 1 and 2

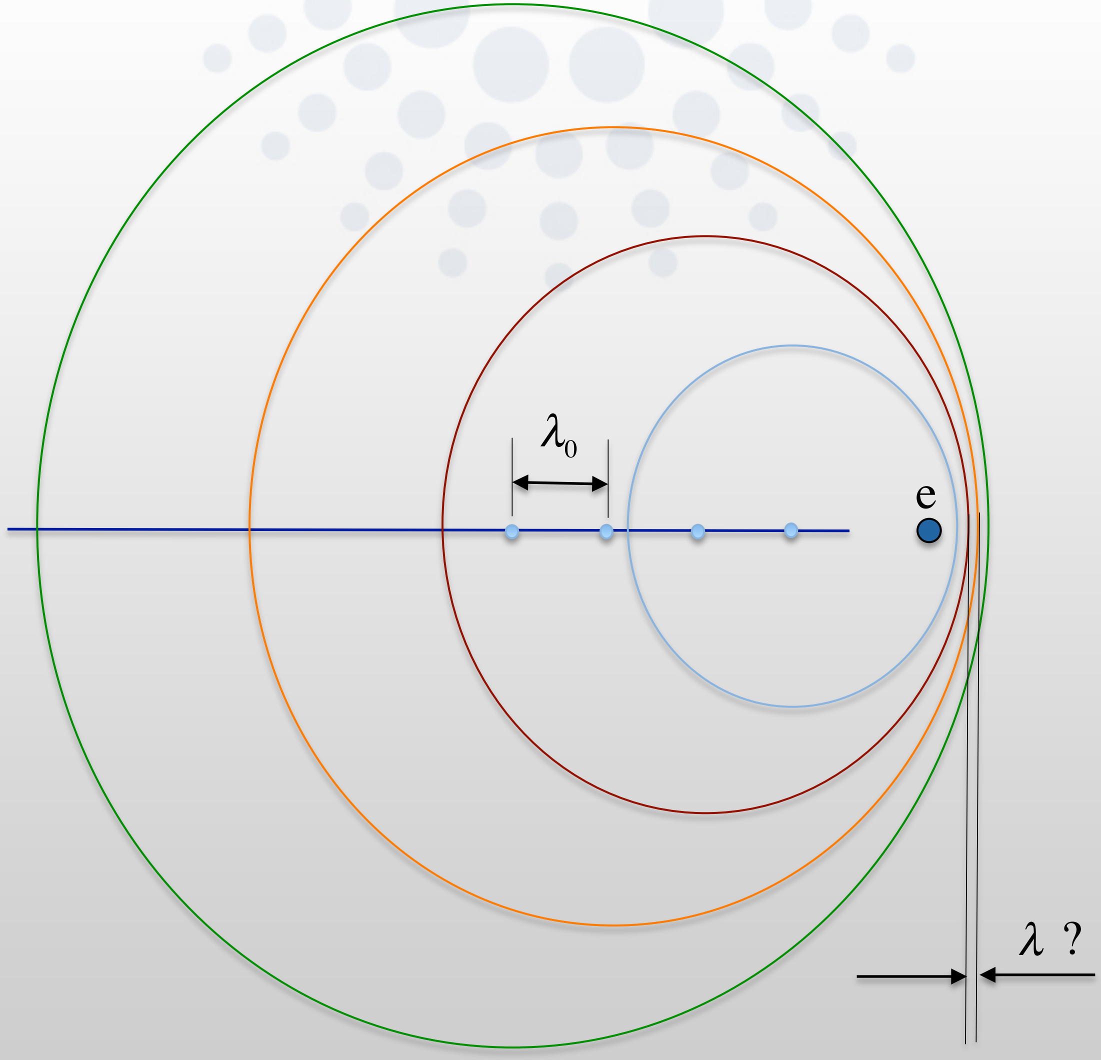


And between point 2 and 3

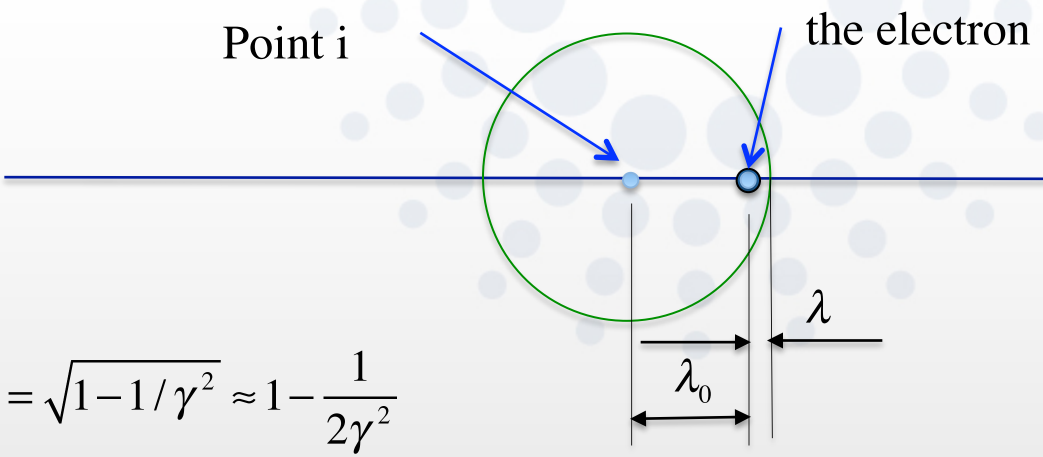


Simple approach

The question: what is the relation between λ_0 and λ ?



On axis observation



$$v/c = \beta_e = \sqrt{1 - 1/\gamma^2} \approx 1 - \frac{1}{2\gamma^2}$$

Time taken by the electron to move from point i to point i+1: $\Delta t = \dfrac{\lambda_0}{\beta_e c}$

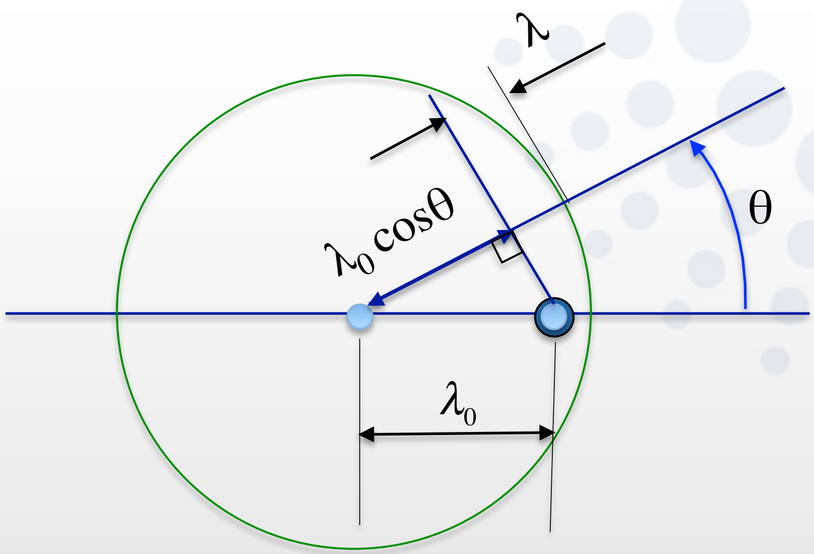
During this time the wavefront created at point i has expanded by $r = c\dfrac{\lambda_0}{\beta_e c} = \dfrac{\lambda_0}{\beta_e}$

Therefore we have: $$\boxed{\lambda = \frac{\lambda_0}{\beta_e} - \lambda_0 \approx \frac{\lambda_0}{2\gamma^2}}$$

Example: $\lambda_0 = 28mm$ we get $\lambda = 1\mathring{A}$ with the ESRF energy (γ=11820)

Remark: in the backward direction $\lambda \approx 2\lambda_0$

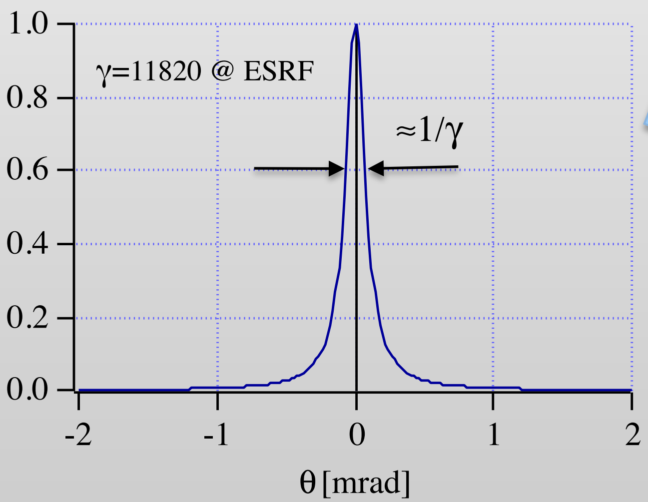
Off axis observation



$$\lambda(\theta) = \frac{\lambda_0}{\beta_e} - \lambda_0 \cos\theta \approx \lambda_0 \left(1 - \cos\theta + \frac{1}{2\gamma^2}\right)$$

For small angles : $\cos\theta \approx 1 - \frac{\theta^2}{2}$

$$\lambda(\theta) \approx \frac{\lambda_0}{2\gamma^2}(1 + \gamma^2\theta^2)$$



Interesting to look at $\frac{\lambda(0)}{\lambda(\theta)} = \frac{1}{1 + \gamma^2\theta^2}$

Photon energy: $E_p = h\frac{c}{\lambda}$ $\qquad \frac{\lambda(0)}{\lambda(\theta)} = \frac{E_p(\theta)}{E_p(0)}$

The radiated energy is concentrated in a narrow cone of typical angle 1/γ

Important remarks

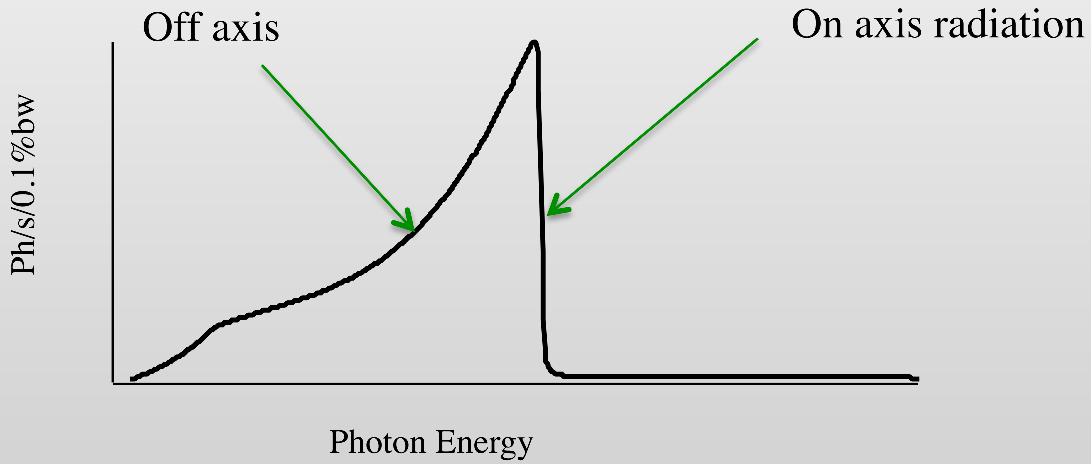
From our simple "periodic emitter " we have seen :

- Radiations at wavelength of ~ 1 Å can be produced with a spatial wavelength of few centimeters and few GeV electron beam

- Emitted radiations are highly collimated (~ 1/γ)

The angular dependence of emitted wavelength has a direct consequence on associated spectrum

$$\lambda(\theta) \approx \frac{\lambda_0}{2\gamma^2}(1+\gamma^2\theta^2)$$



Presence of "tails" at low energy side on harmonics with non zero angular acceptance

Other approaches

Lorentz transform + Doppler shift:

Electron frame

spatial period $\lambda_1 = \lambda_0/\gamma$

emitted wavelength: λ_1

Lorentz transform (Observer frame → Electron frame)

angle dependent Doppler shift (Electron frame → Observer frame)

Observer frame

spatial period λ_0

$$\lambda(\theta) = \lambda_1 \gamma \left(1 - \beta_e \cos\theta\right)$$

$$\lambda(\theta) \approx \frac{\lambda_0}{2\gamma^2}(1 + \gamma^2\theta^2)$$

example:

undulator with λ_0=28 mm has λ_1= 2.36 μm ,

1.6 m long undulator has a length of 0.135 mm in electron frame

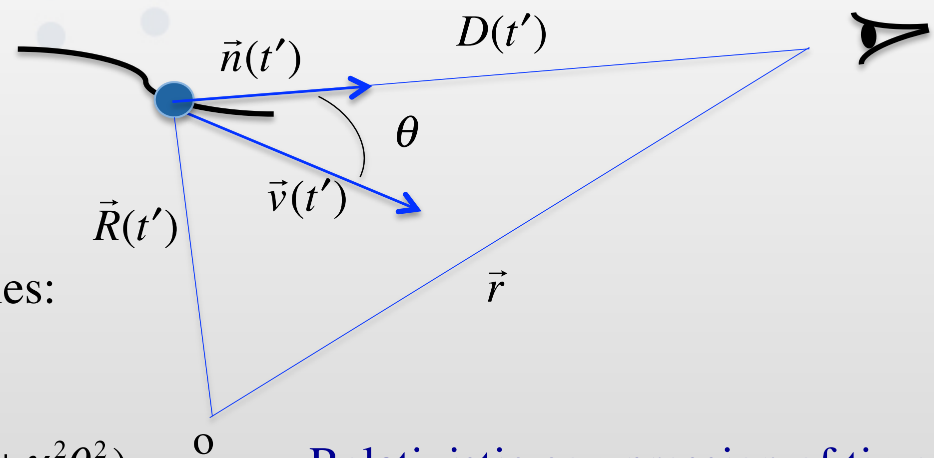
Electron and observer times

Electron moving with speed $\vec{v}(t') = c\vec{\beta}_e(t')$

Wave emitted at time t' by electron received at time t by observer

$$t = t' + \frac{D(t')}{c}$$

$$\frac{dt}{dt'} = 1 - \vec{n}(t')\vec{\beta}_e(t')$$



For ultra-relativistic electron and small angles:

$$\frac{dt}{dt'} = 1 - \beta_e \cos\theta = 1 - \sqrt{1 - 1/\gamma^2}\cos\theta \approx \frac{1}{2\gamma^2}(1 + \gamma^2\theta^2)$$

Relativistic compression of time:

Observer time evolves several orders of magnitude slower than electron time

Basis for “retarded potentials” or Lienard-Wiechert potentials

Improving oscillator model

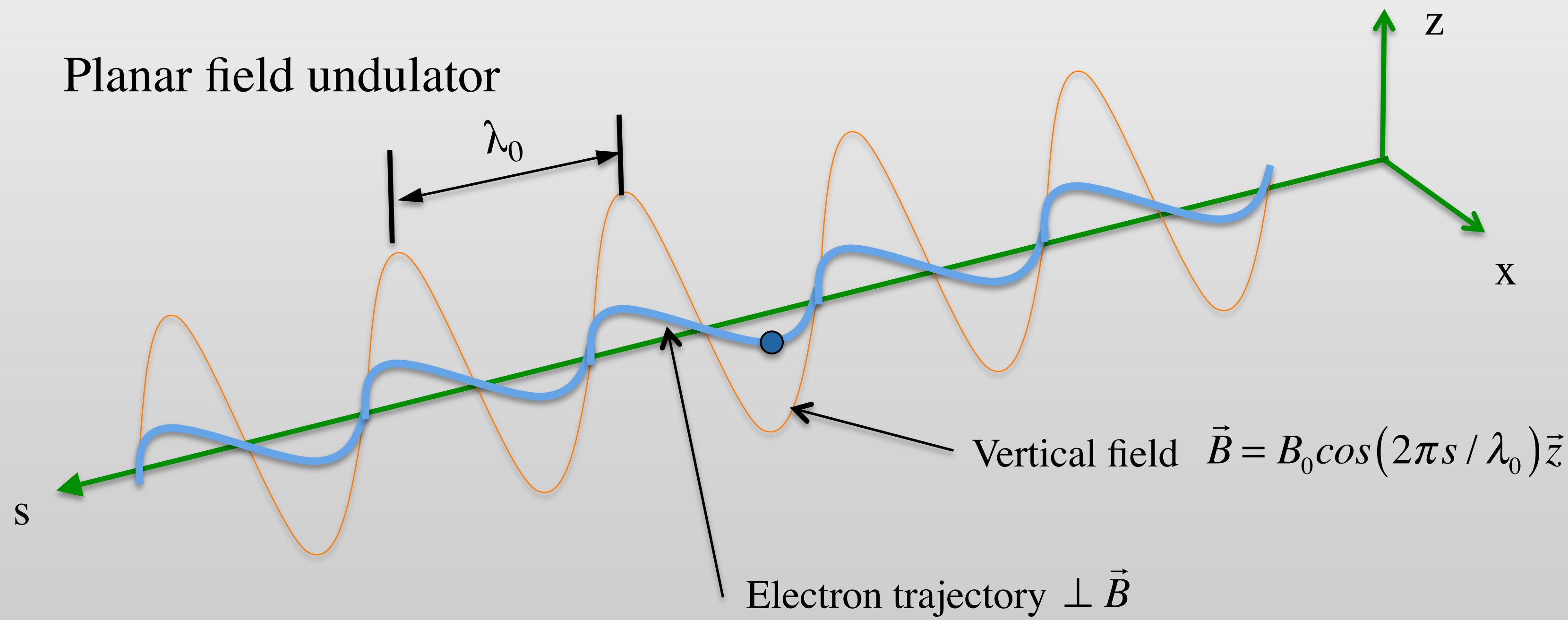
How to get periodic emission from an electron ?
Apply periodic force on electron: $\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B})$

~ 10 MV/m with conventional technology

B= 1 T
$V \approx c$
} 300 MV/m

Best option

Planar field undulator



Undulator equation (1)

$$\frac{d\vec{P}}{dt} = -e(\vec{v} \times \vec{B}) \qquad \vec{P} = \gamma m \vec{v} \qquad \vec{B} = B_0 \cos(2\pi s / \lambda_0)\vec{z}$$

Assumptions: γ constant , $\beta_x = v_x/c, << 1$, $\beta_z = v_z/c << 1$

Angular motion

$$\beta_x(s) = \frac{e}{2\pi\gamma mc} B_0 \lambda_0 \sin(\frac{2\pi s}{\lambda_0}) = \frac{K}{\gamma}\sin(\frac{2\pi s}{\lambda_0})$$

$$K = \frac{e}{2\pi mc} B_0 \lambda_0 = 0.9336 B_0[T]\lambda_0[cm]$$

$$\beta_z(s) = cst = 0$$

Deflection parameter

Electron trajectory

$$x(s) = -\frac{K\lambda_0}{2\pi\gamma}\cos(\frac{2\pi s}{\lambda_0}) = -x_0 \cos(\frac{2\pi s}{\lambda_0})$$

$$z(s) = cst = 0$$

γ	λ_0[cm]	B[T]	K	$x_o[\mu m]$
11820	2	1	1.87	0.5

Undulator equation (2)

We need to know the longitudinal motion β_s of the electron in the undulator to bring more consistence to our initial "naïve" device:

Since γ is constant so is $\beta_e^2 = \beta_x^2 + \beta_s^2 = 1 - \frac{1}{\gamma^2}$ $\quad (\beta_x(s) = \frac{K}{\gamma}\sin(\frac{2\pi s}{\lambda_0}))$

$$\beta_s(s) \approx 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2} + \frac{K^2}{4\gamma^2}\cos(\frac{4\pi s}{\lambda_0})$$

Average longitudinal relative velocity: $\hat{\beta}_s \approx 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2}$

Angle dependent emitted wavelength: $\lambda(\theta) \approx \frac{\lambda_0}{2\gamma^2}(1 + \frac{K^2}{2} + \gamma^2\theta^2)$

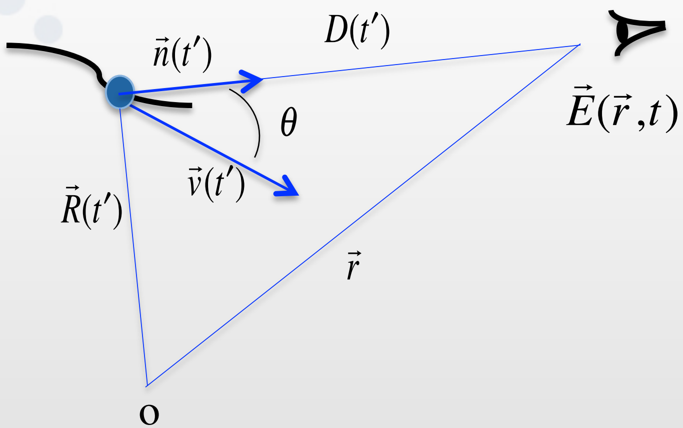
γ	λ_0[cm]	B[T]	K	λ(0)
11820	2	1	1.87	1.96 Å
11820	2	0.1	0.187	0.72 Å

We have now a field dependent wavelength

The electric field

The electric field $\vec{E}(\vec{r},t)$ seen by an observer is the relevant quantity to determine

Has always B field "companion": $\vec{B}(\vec{r},t) = \frac{\vec{n}(t')}{c} \times \vec{E}(\vec{r},t)$



Moving charge along arbitrary motion:

Electric field includes two terms

$$\vec{E}(\vec{r},t) = \vec{E}_1(\vec{n}(t'),\vec{v}(t'),D(t')) + \vec{E}_2(\vec{n}(t'),\vec{v}(t'),D(t'))$$

Velocity field or
Coulomb field
Decays as $1/D^2$

Acceleration field
Decays as 1/D

Needs to find t'(t) to evaluate $\vec{E}(\vec{r},t)$

Far field approximation: drop velocity field and $\vec{n}(t')$ constant

Frequency domain

$$\vec{E}(\vec{r},\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\vec{E}(\vec{r},t)e^{i\omega t}dt$$

Electric field in time domain

Electric field in frequency domain
Complex quantity

Wavefront propagation

Phase ?

Coherence

Etc..

$$N(\vec{r},\omega)=\frac{\alpha\left|\vec{E}(\vec{r},\omega)\right|^{2}}{\hbar\omega}$$

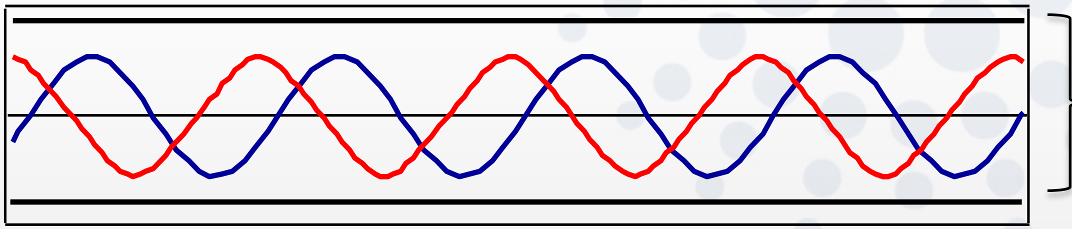
Number of photons at ω

Importance of deflection parameter

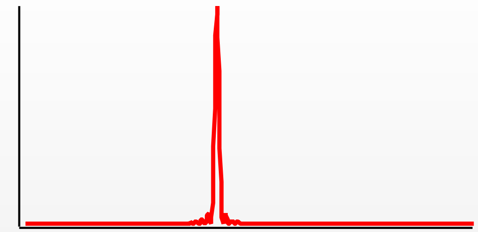
Peak angular deflection $\frac{K}{\gamma}$

$K \ll 1$

— angle
— electric field



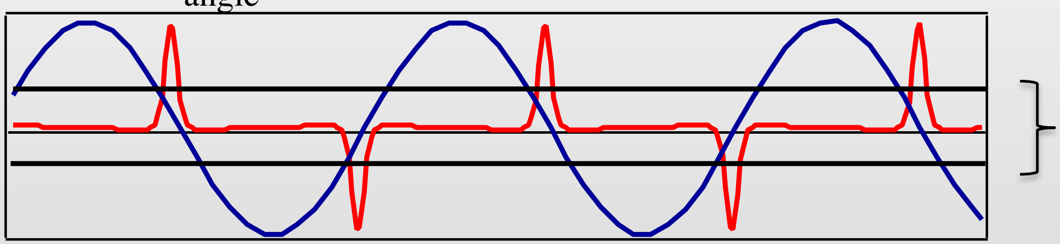
time



Frequency

$K > 1$

— Electric field
— angle



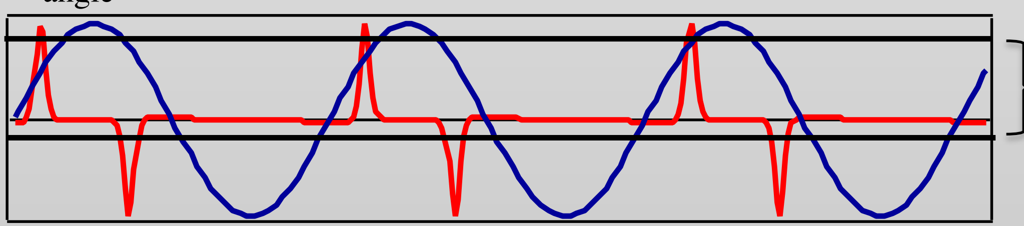
time

Only odd harmonics

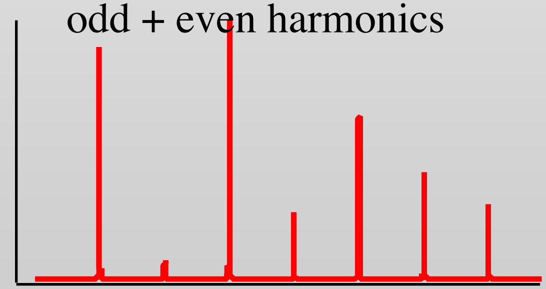
Frequency

Off axis observation with angle θ

— Electric field
— angle



odd + even harmonics



Frequency

On axis angular spectral flux

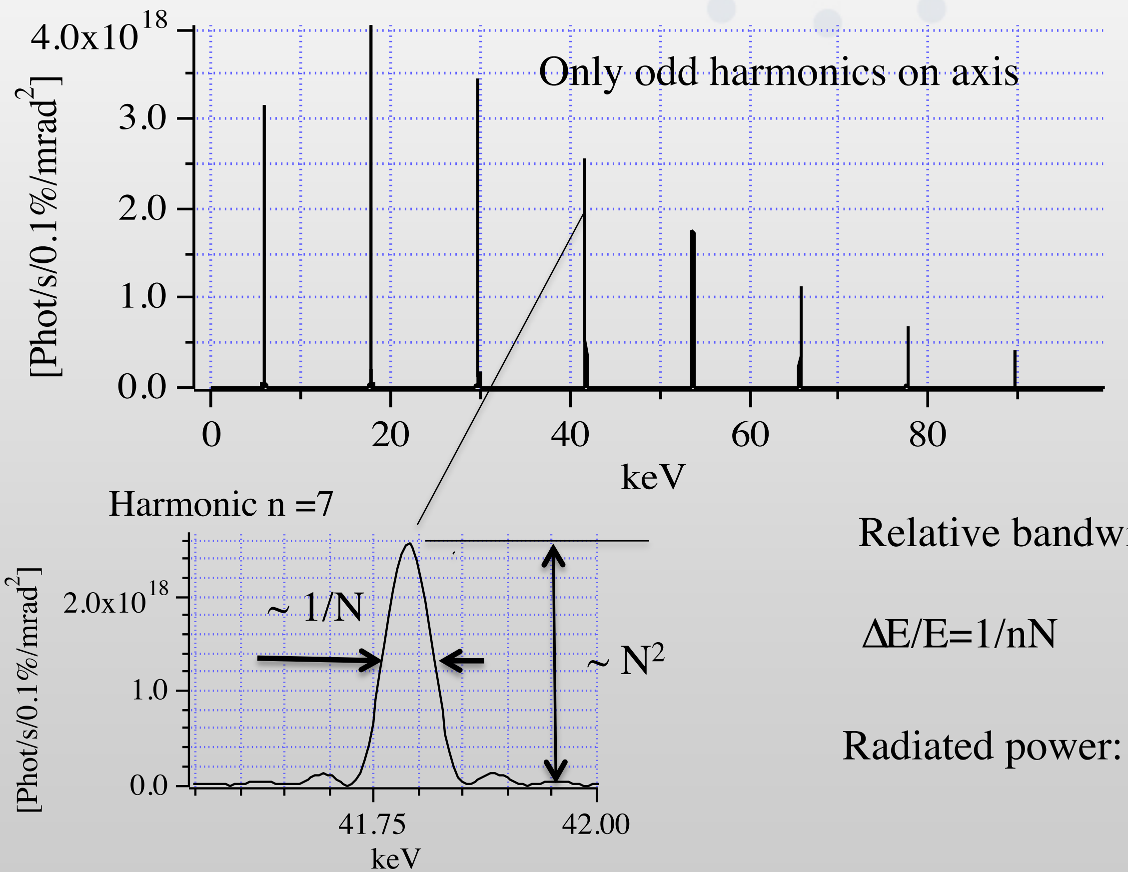
Spectral photon flux units: Watts/eV can be translated into photons/sec/relative bandwidth

Ex: 1 phot/s/0.1%bw= 1.602e-16 W/eV

Angular spectral flux: photon flux/unit solid angle

Usual unit is phot/sec/0.1%/mrad^2:

Ideal on axis angular spectral flux with filament electron beam (zero emittance)



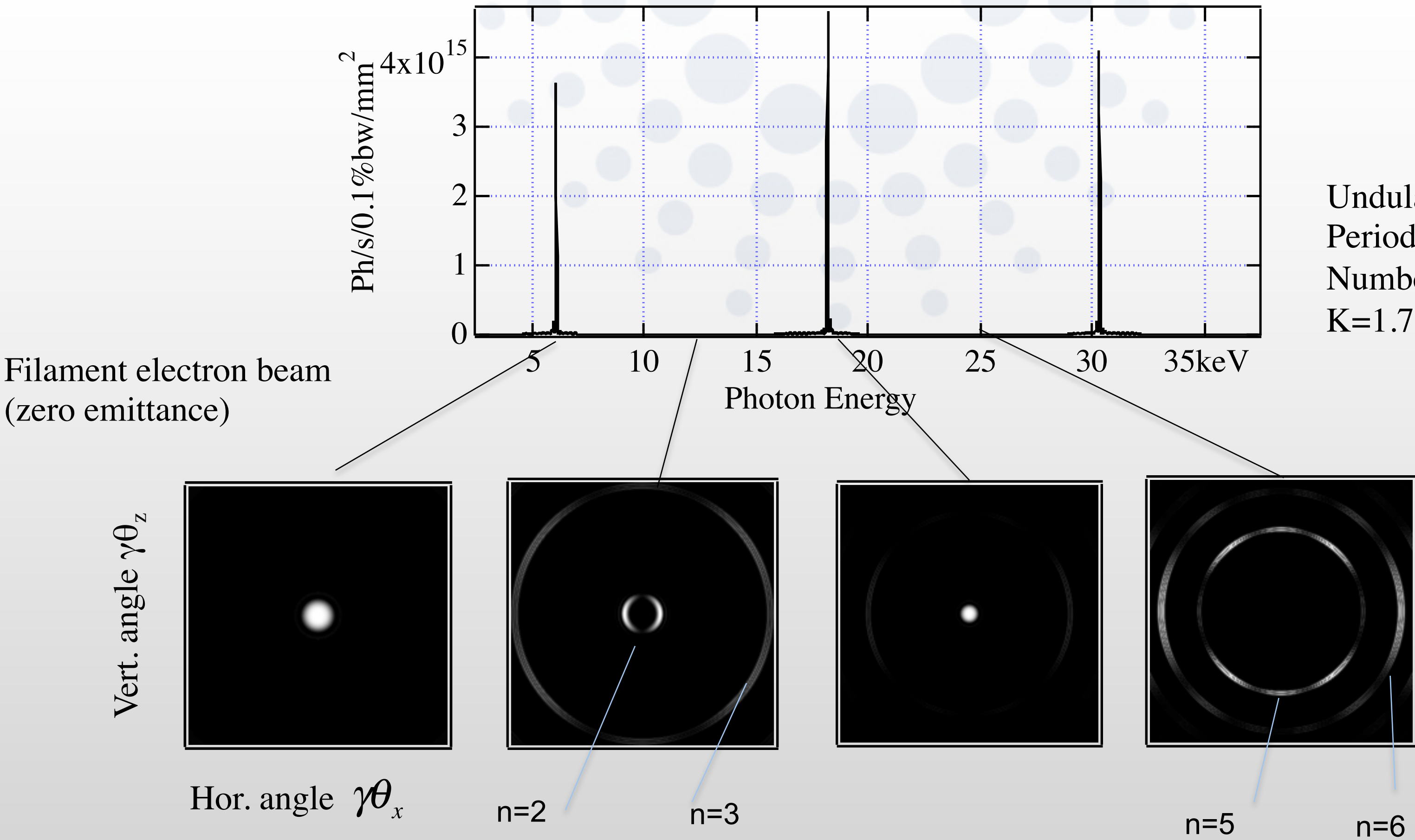
Undulator:
Period λ_0= 22 mm
Number of period **N**=90
K=1.79

Relative bandwidth at harmonic n:

$\Delta E/E = 1/nN$

Radiated power: ~ $N^2/N=N$ proportional to N

Off axis radiation



Undulator:
Period λ_0= 22 mm
Number of period **N**=90
K=1.79

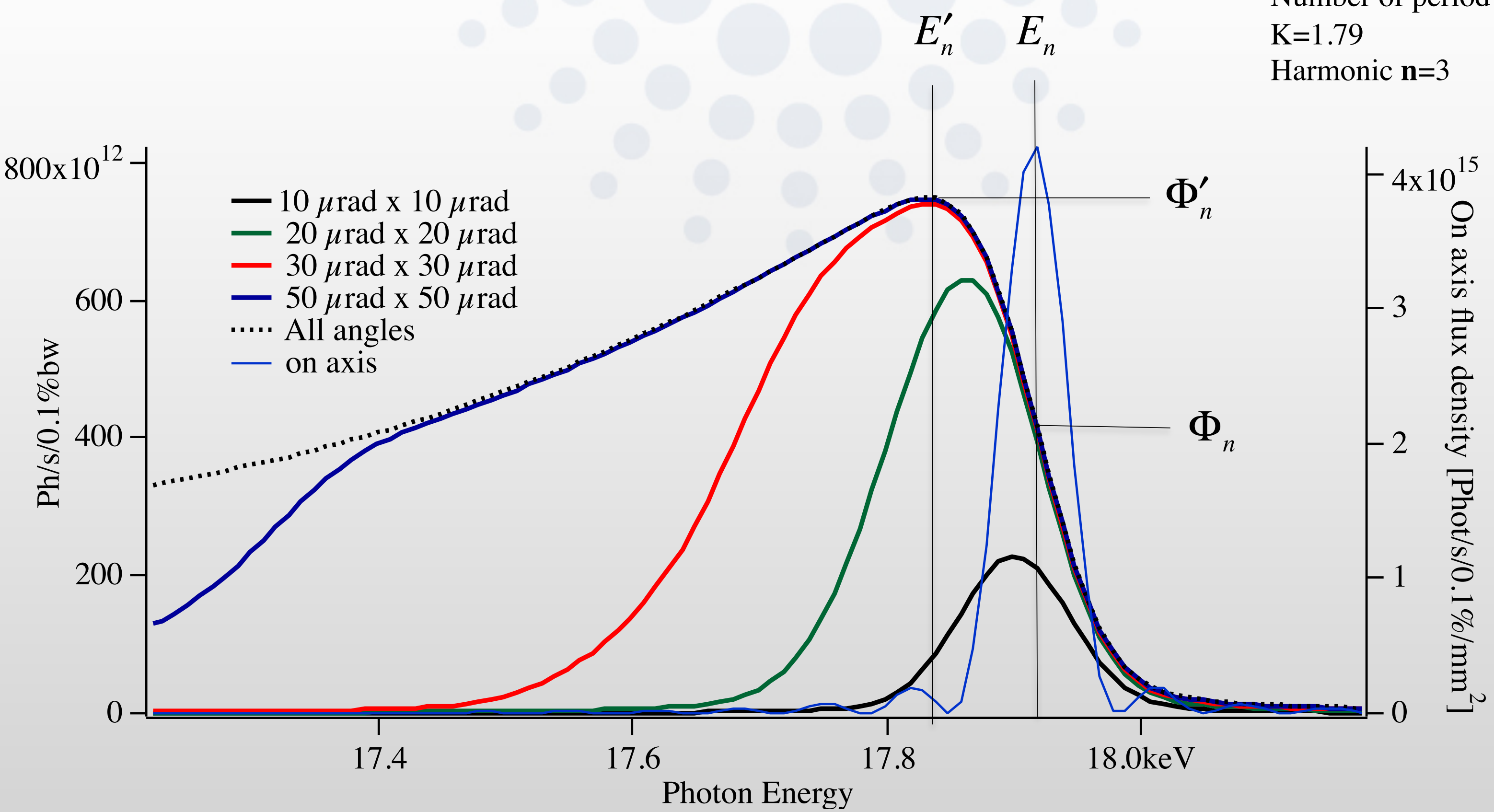
Wavelength of harmonic n

$$\lambda_n(\theta) = \frac{\lambda_0}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right) \qquad \theta^2 = \theta_x^2 + \theta_z^2$$

Angle integrated flux

Ideal filament electron beam

Undulator:
Period λ_0= 22 mm
Number of period **N**=90
K=1.79
Harmonic **n**=3



E_n energy of on axis resonance

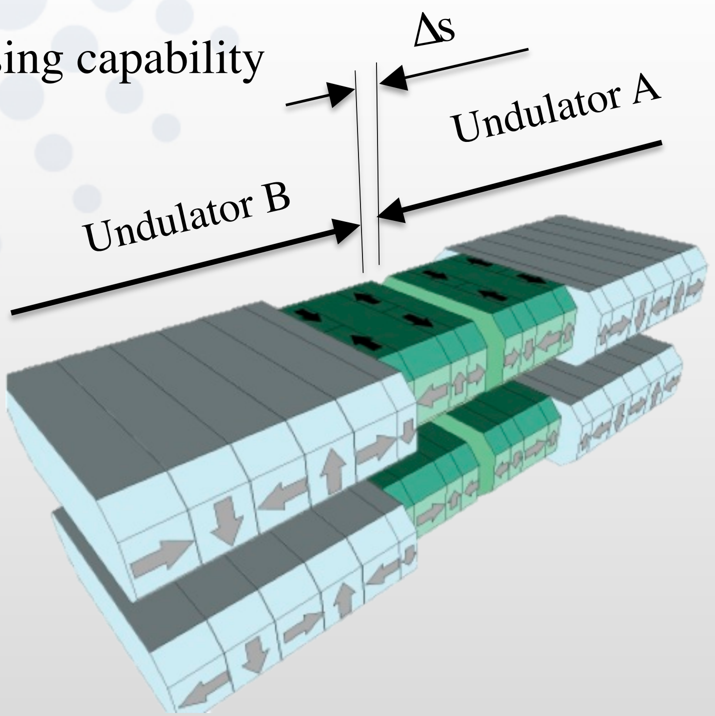
$$\Phi'_n \approx 2\Phi_n \qquad E'_n = E_n\left(1-\frac{1}{nN}\right)$$

$$E_n(\theta)=\frac{2hc\gamma^2}{\lambda_0\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)}=\frac{0.95E^2[GeV]}{\lambda_0[cm]\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)}$$

Angle integrated flux (remark)

A unique specificity of ESRF:

Segmented independent undulators with passive phasing capability
~ all in-air segments



For a fixed energy and collecting aperture
Undulator gaps are optimized for maximum flux

One undulator

$$E'_n = E_n\left(1-\frac{\alpha}{nN}\right) \qquad 0 \le \alpha \le 1$$

Two undulators

$$E'_n = E_n\left(1-\frac{\alpha}{2nN}\right)$$

The optimum gap depends on the length of undulator

Δs depends on period
2.5 mm for λ_0=18 mm
5 mm for λ_0=35 mm

Radiated power

Radiated power & power density can be an issue for ESRF beamlines

Total power emitted by an Insertion device: (only a fraction is generally taken by a beamline)

$$P[kW] = 1.266 E^2[GeV] I[A] \int_{-\infty}^{\infty} (B_x^2[T] + B_z^2[T]) ds$$

ID with arbitrary field

$$P[kW] = 0.633 E^2[Gev]\, B_0^2[T]\, I[A]\, L[m]$$

Undulator length

Planar sinusoidal field undulator
B_0: peak field

On axis power density:

$$dP / d\Omega[W / mrad^2] = 10.84\, B_0[T] E^4[Gev] I[A] N$$

N: number of periods, K>1

Ex: ESRF 6 .04 Gev with I=0.2 A

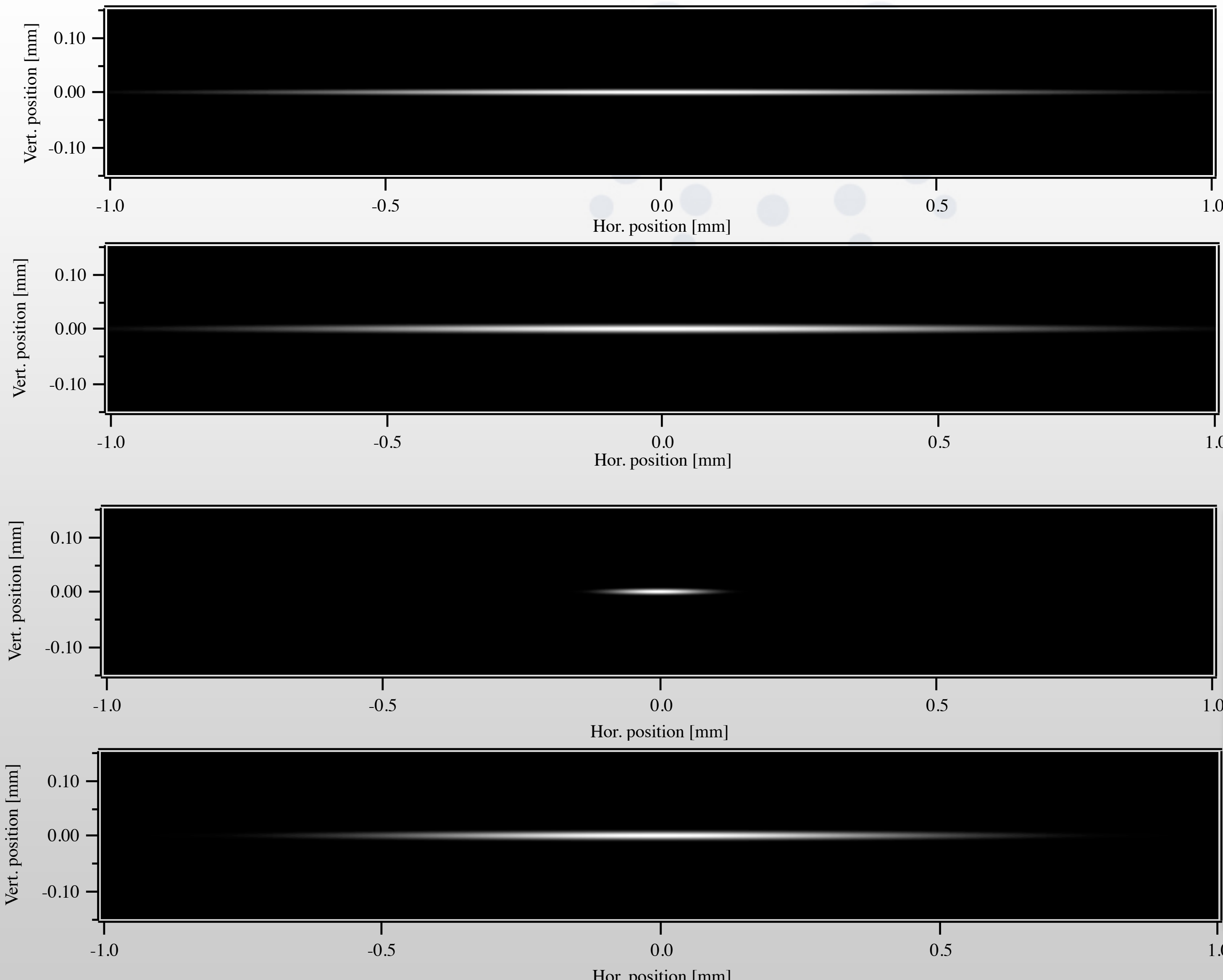
Period[mm]	**L[m]**	**N**	**$B_{0[T]}$**	**P[kW]**	**Dp/dΩ[kW/mrad²]**
22	2	90	0.87	7	260
27	5	185	0.52	6.7	277

With ~ all ESRF IDs at minimum gap: the total radiated power is ~ 300 kW (0.2 A, 6.04 Gev)
(to be compared to ~ 1 MW for all dipoles)

Electron beam size

Geometrical size of ESRF electron beam (assuming Gaussian beam)

$\varepsilon_x = 4nm \quad \varepsilon_z = 3pm$



High beta straight

middle

@ 3 m from middle

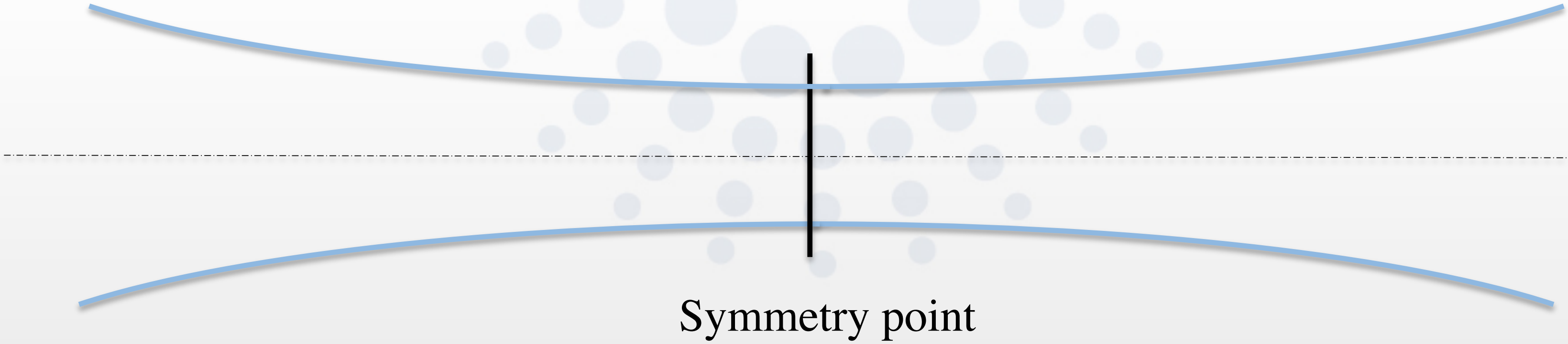
Low beta straight

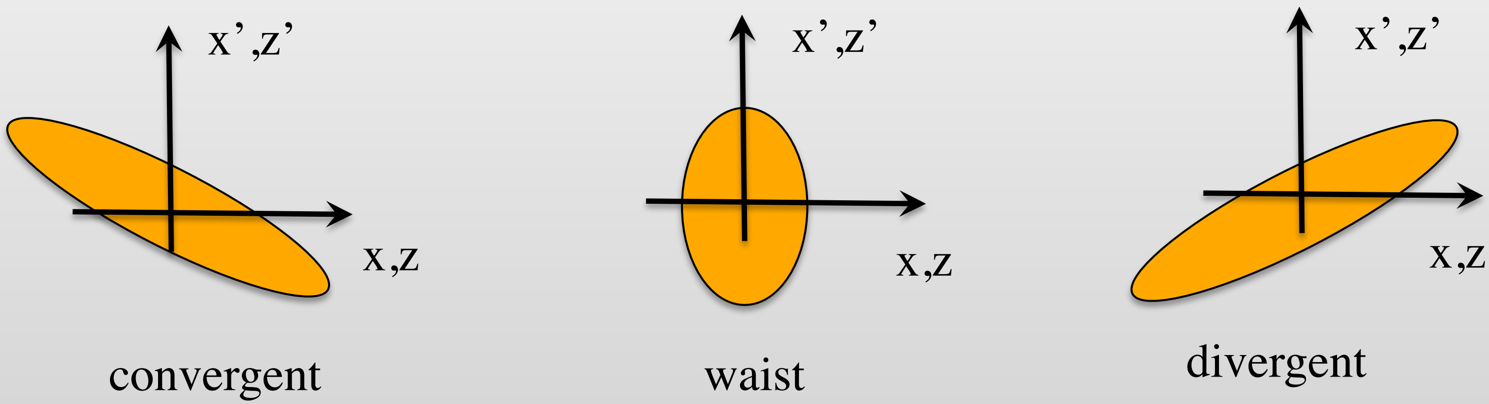
middle

@ 3 m from middle

Phase space

Electron beam envelope along ID straight section



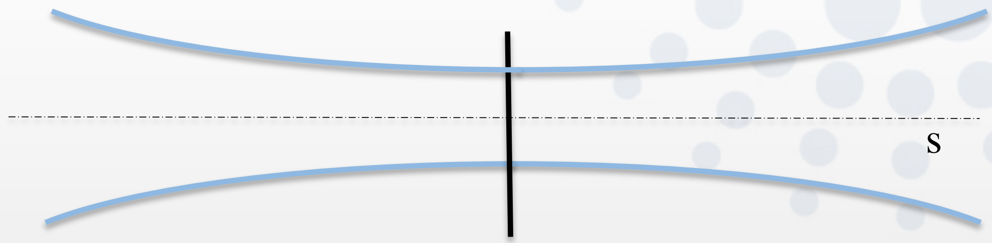


(rms) beam occupancy in horizontal & vertical phase space

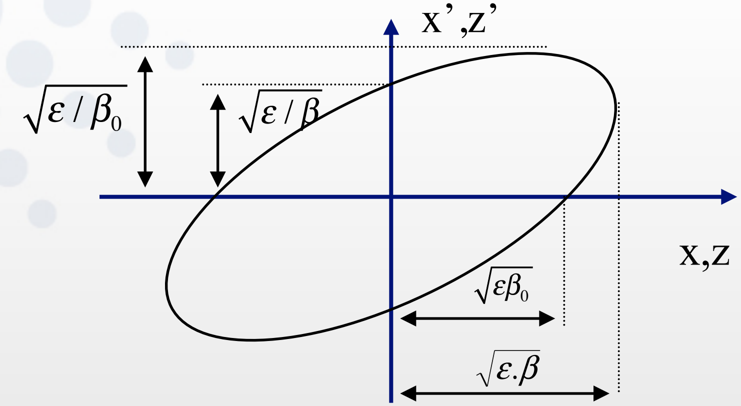
Ellipse of constant area= $\pi\varepsilon$ (ε : emittance)

Electron beam in ID straight

Beam size and divergence are derived from the knowledge of beta $\beta_{x,z}(s)$ functions and emittance $\varepsilon_{x,z}$



S=0 at middle of straight section



For each plane

$$\beta(s) = \beta_0\left(1+\frac{s^2}{\beta_0}\right)$$

Rms size & divergence

$$\sigma(s) = \sqrt{\varepsilon\beta(s)+\eta^2\sigma_\gamma^2}$$

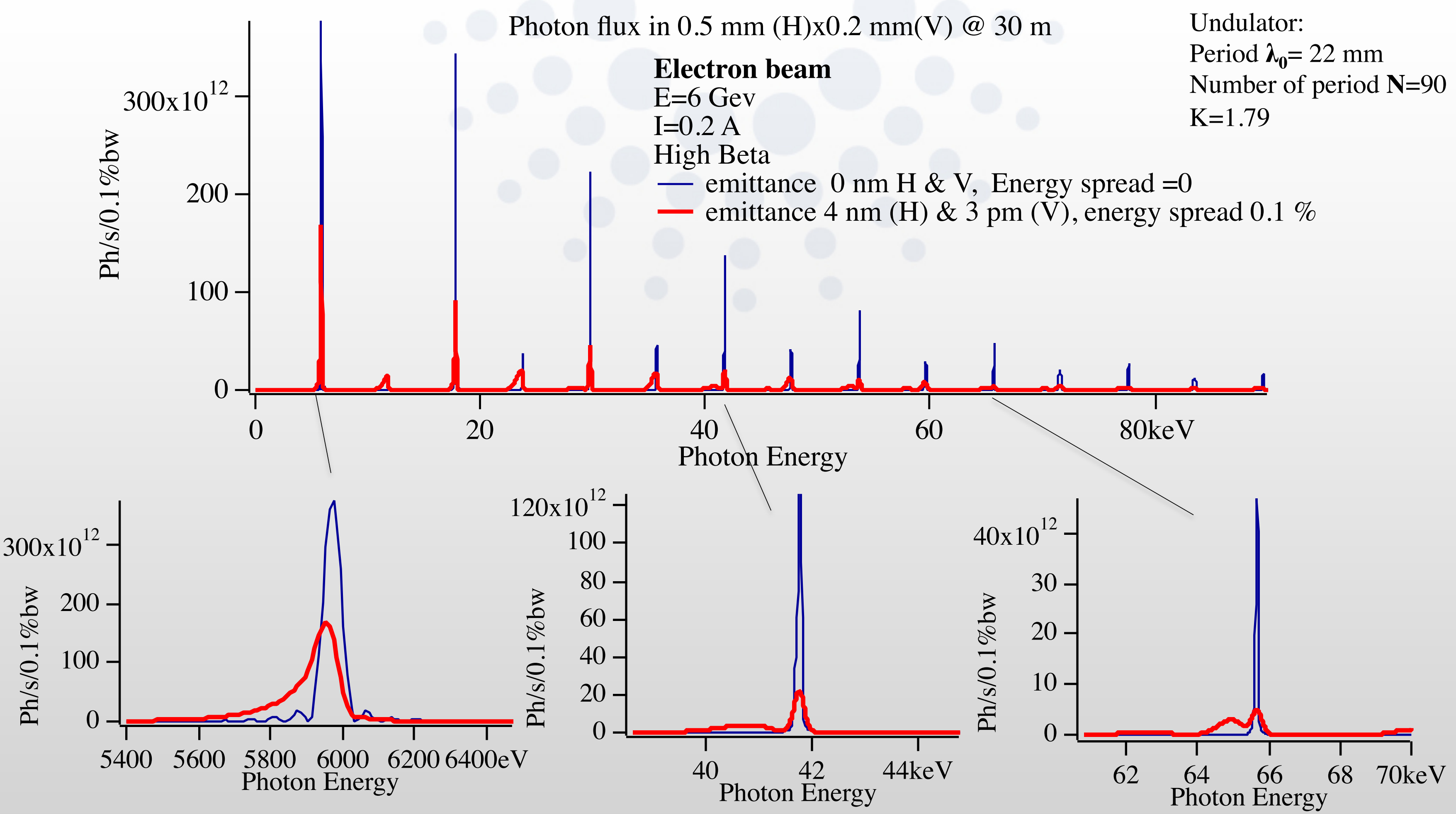
$$\sigma'(s) = \sqrt{\frac{\varepsilon}{\beta_0}} = cst$$

High beta	β_0[m]	η	ε [nm]	σ(0) [μm]	σ'[μrad]
horizontal	37.5	0.13	4	409	10.3
Vertical	3	0	0.003	3	1

Low beta	β_0[m]	η	ε [nm]	σ(0) [μm]	σ'[μrad]
horizontal	0.37	0.03	4	49	104
Vertical	3	0	0.003	3	1

η: dispersion
σ_γ relative rms energy spread: 0.1% @ ESRF

Undulator spectra with actual beam



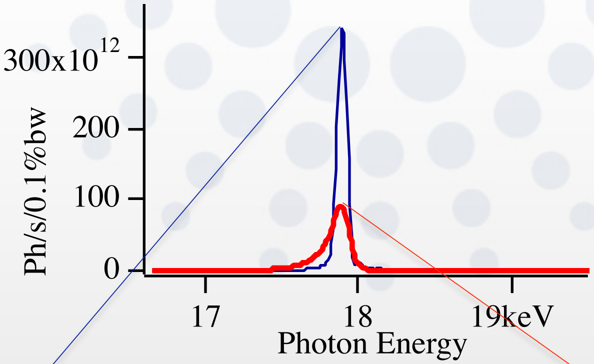
Spectral performances dominated by horizontal emittance and energy spread at high harmonics

~ additional off axis contribution due to electron beam size and divergence $\left(\lambda_n(\theta) = \frac{\lambda_0}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)\right)$

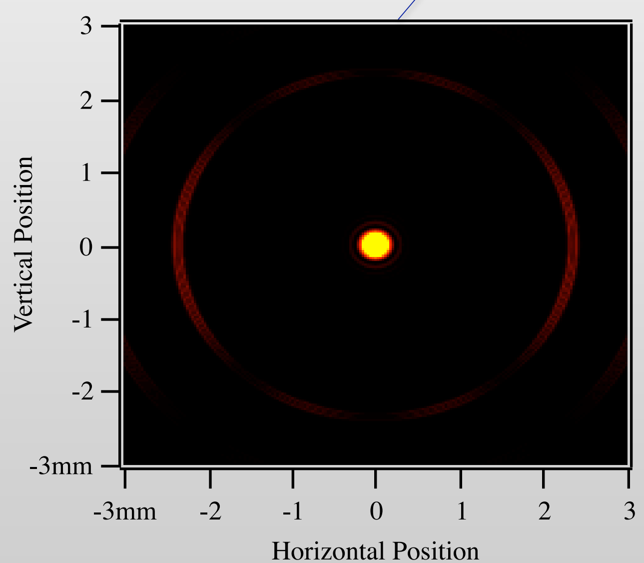
Beam size at beamline

Photon beam size @ 30 m from source

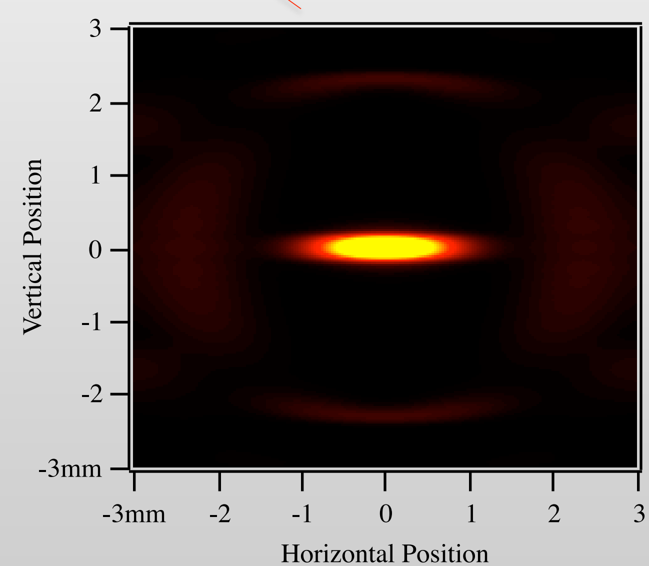
Undulator:
Period λ_0= 22 mm
Number of period **N**=90
K=1.79
Harmonic **n**=3



Electron beam:
Emittance;
Horizontal: 4nm
Vertical: 3 pm
Energy spread: 0.1 %



Ideal electron beam



Finite emittance, High Beta

Source size & divergence

Rms source size and divergence can be well evaluated using:

Electron beam

$$\Sigma_{x,z} = \sqrt{\sigma_n^2 + \sigma_{x,z}^2} \qquad \Sigma'_{x,z} = \sqrt{\sigma_n'^2 + \sigma_{x,z}'^2}$$

"natural" undulator emission
(single electron of filament electron beam)

Various expressions for σ_n and σ'_n found in literature
generally assuming Gaussian photon beam for "natural" size & divergence

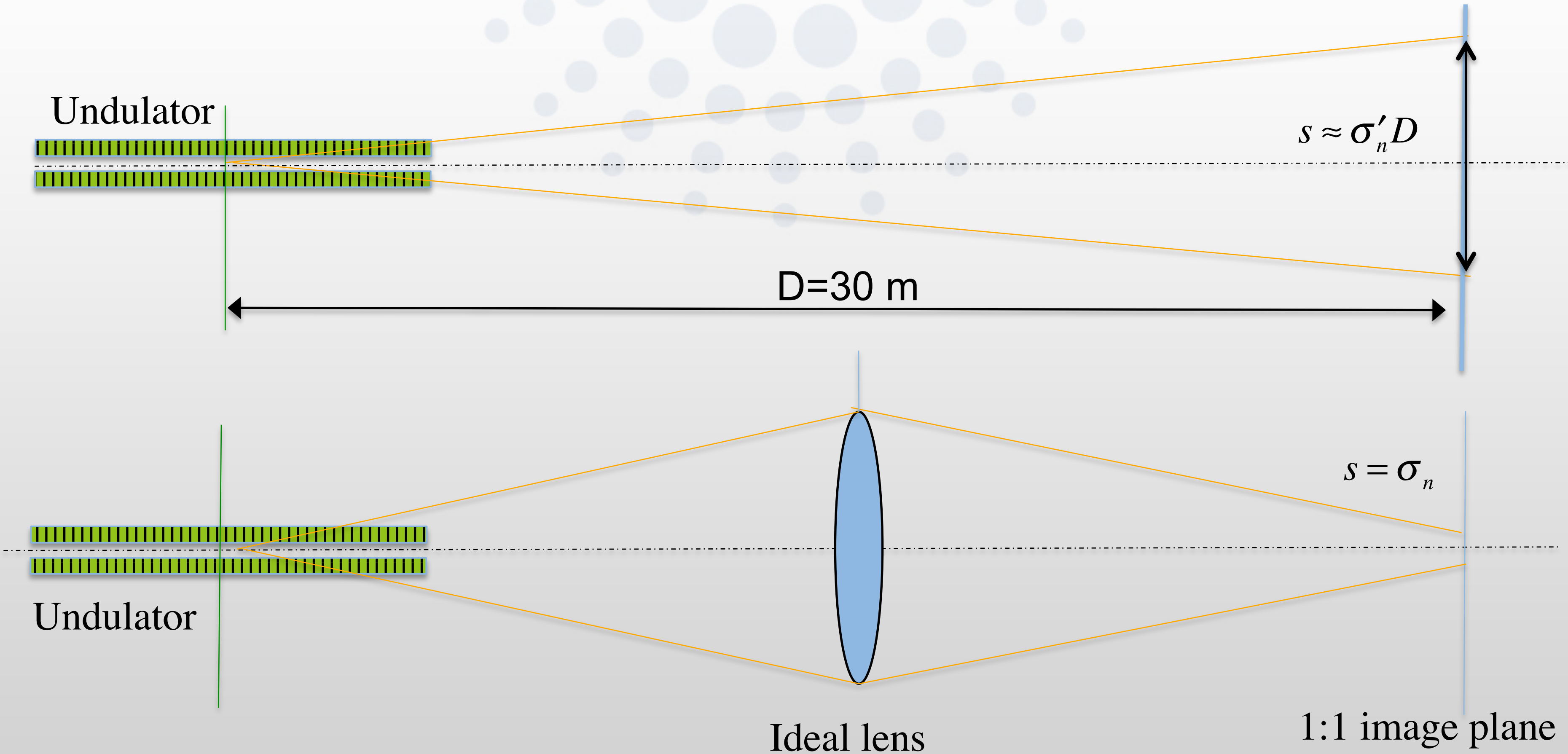
This do not impact on horizontal source size and divergence since dominated by electron beam

However in **vertical** plane the story is different:

At the middle of a straight section we have : σ_z=3 μm and σ'_z=1 μrad for ε_z=3 pm for the electron beam

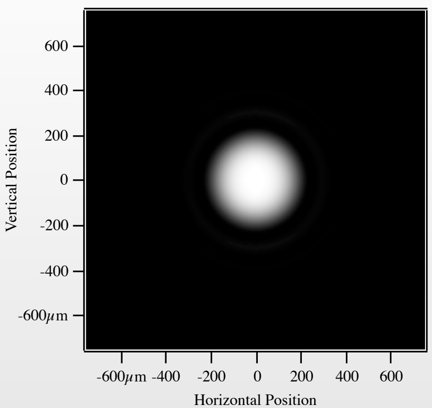
“natural” undulator size & divergence

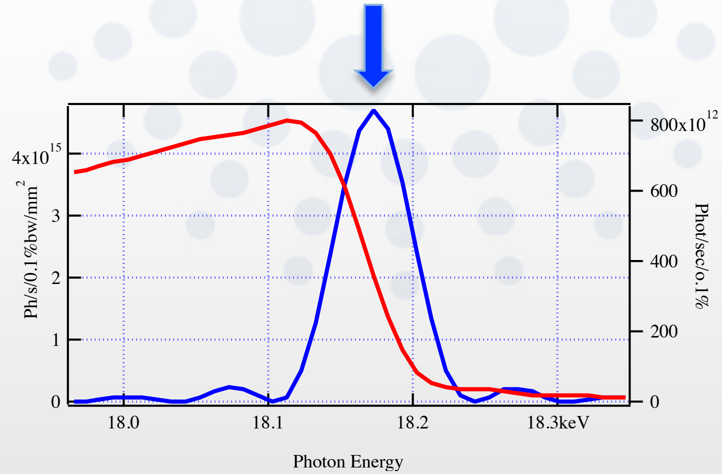
Evaluation of source size σ_n and divergence σ'_n (single electron)

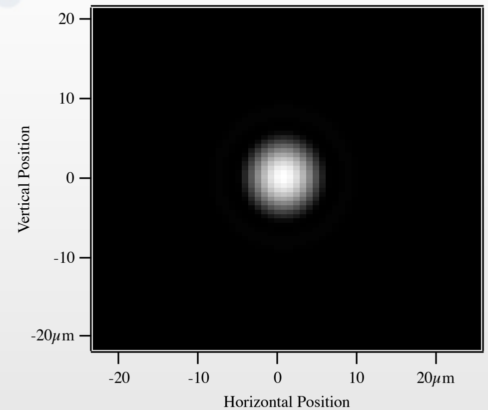


$\sigma_n \quad \sigma'_n$ rms values evaluated as second order moment: $\langle x^2 \rangle = \dfrac{\int_w x^2 f(x)dx}{\int_w f(x)dx}$

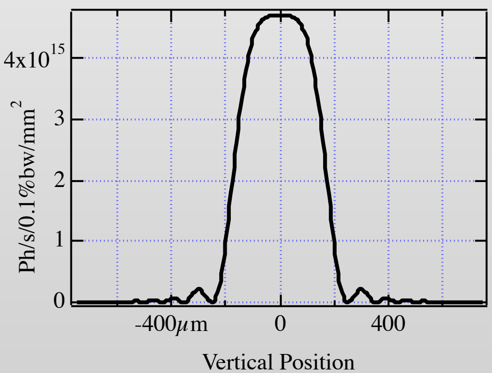
At on axis resonance







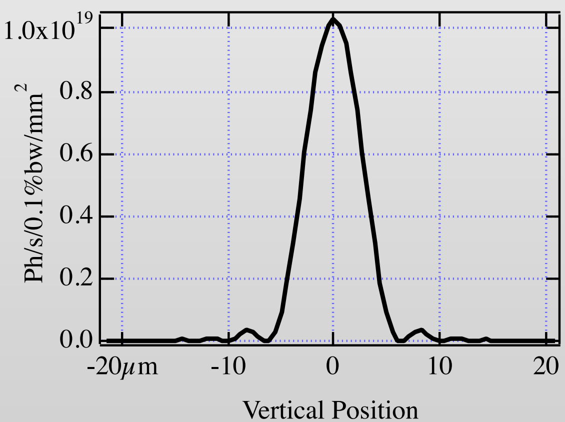
D=30 m



$$\sigma'_n \approx \sqrt{\lambda / 2L}$$

$$\sigma_n \approx \frac{\sqrt{2\lambda L}}{2\pi}$$

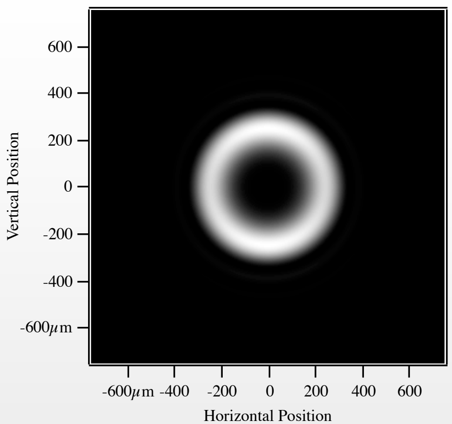
$$\varepsilon_n = \sigma_n \sigma'_n \approx 2\frac{\lambda}{4\pi}$$

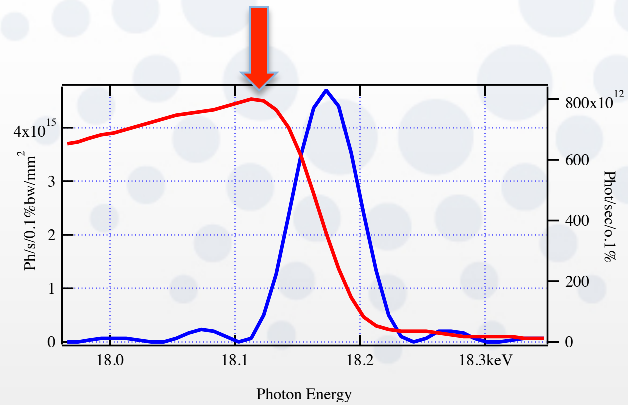


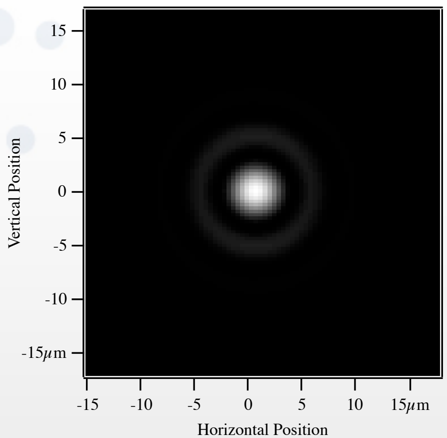
$\frac{\lambda}{4\pi}$ Diffraction limit for Gaussian beam

Undulator beam is not Gaussian but fully coherent transversally

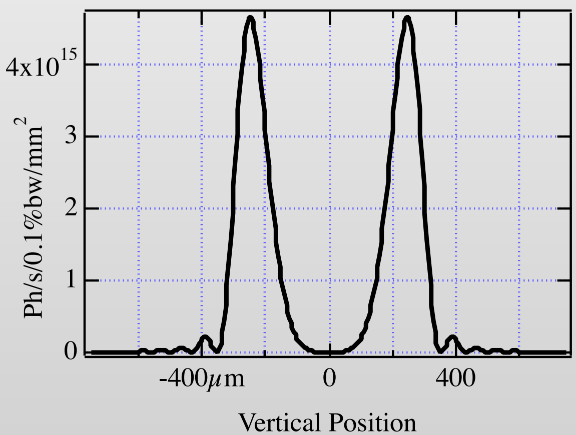
At peak flux







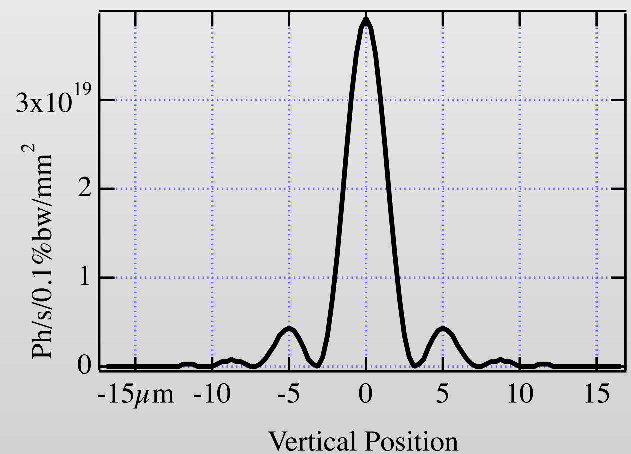
D=30 m



$$\sigma'_n \approx 2.1\sqrt{\lambda/2L}$$

$$\sigma_n \approx 0.9\frac{\sqrt{2\lambda L}}{2\pi}$$

$$\varepsilon_n = \sigma_n\sigma'_n \approx 3.8\frac{\lambda}{4\pi}$$



Phase space area ε_n is minimum at resonance

σ_n and σ'_n can depend strongly on detuning from on axis resonance

Energy spread

Electron beam energy spread impact also on source size & divergence:
pointed out at SPRING8 [1]
Had to be taken into account for NSLSII expected performances [2]

For example

$$\sigma'_n \approx \sqrt{\lambda/2L}\,F(\sigma_\varepsilon)$$

$$\sigma_n \approx \frac{\sqrt{2\lambda L}}{2\pi}G(\sigma_\varepsilon)$$

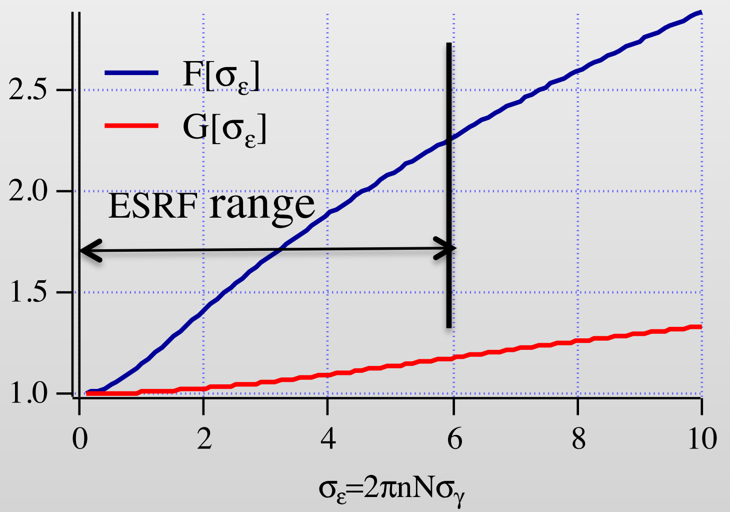
at resonance

$\sigma_\varepsilon = 2\pi n N \sigma_\gamma$ normalized energy spread

n undulator harmonic number
N number of periods

F, G universal functions of σ_ε

$\sigma_\gamma = 0.001 @ ESRF$



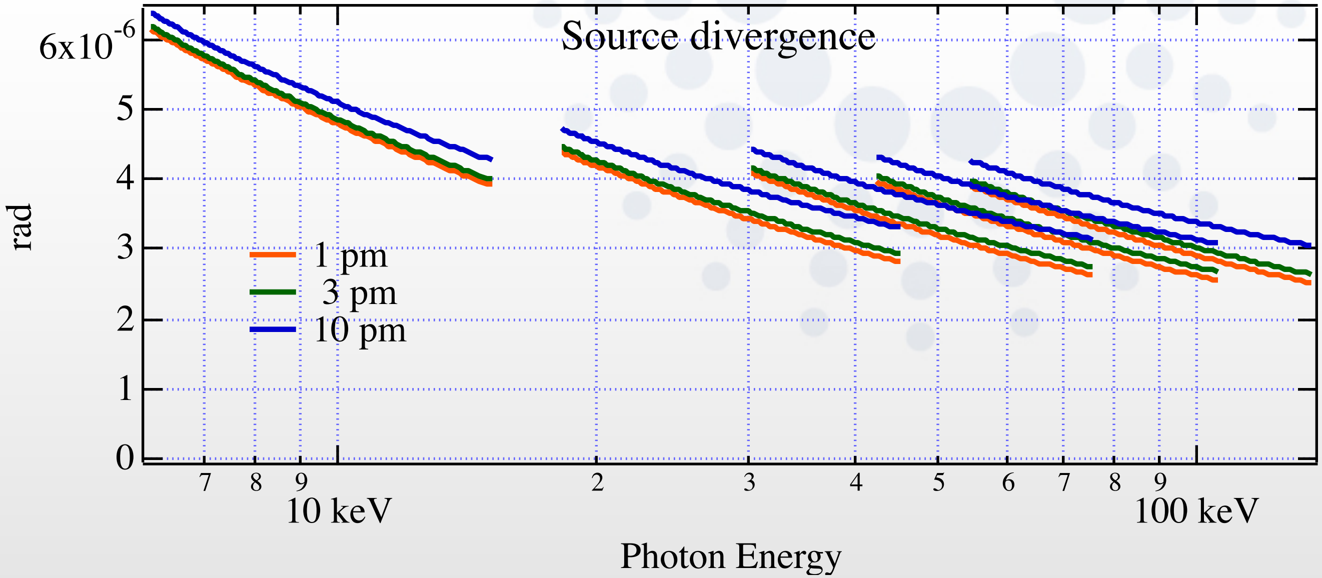
Behind this effect is $\lambda(\theta) \approx \frac{\lambda_0}{2\gamma^2}(1+\frac{K^2}{2}+\gamma^2\theta^2)$ again

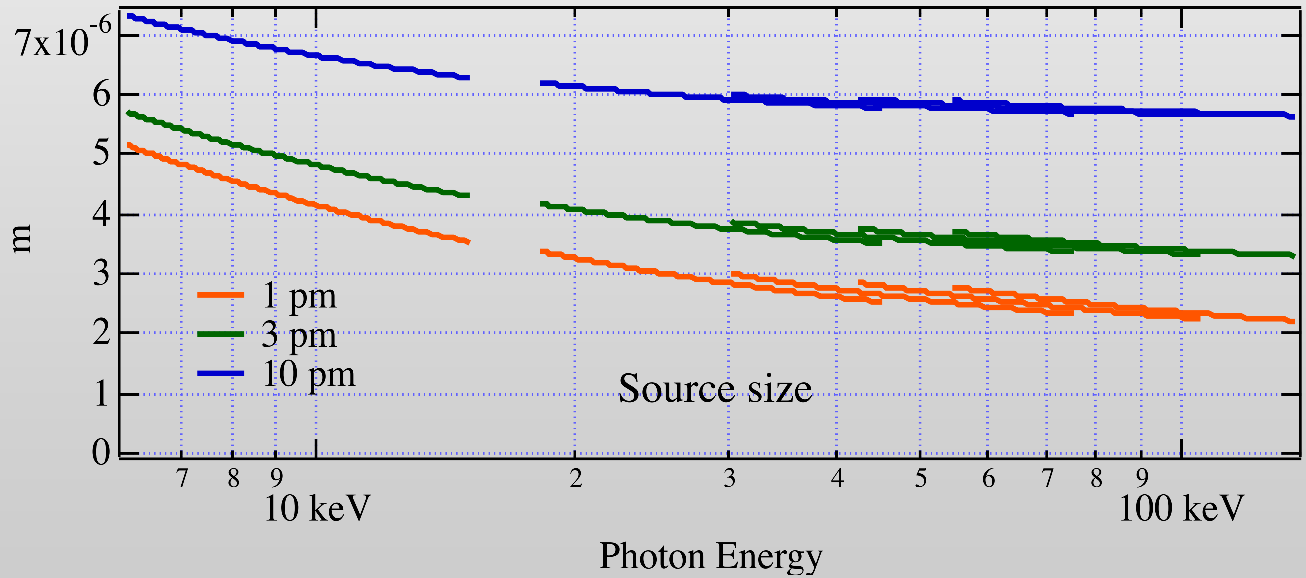
The impact is mostly on source divergence

[1] Takashi Tanaka* and Hideo Kitamura, J. Synchrotron Rad. (2009). 16, 380–386

[2] see NSLS II conceptual design report, radiation sources

Example



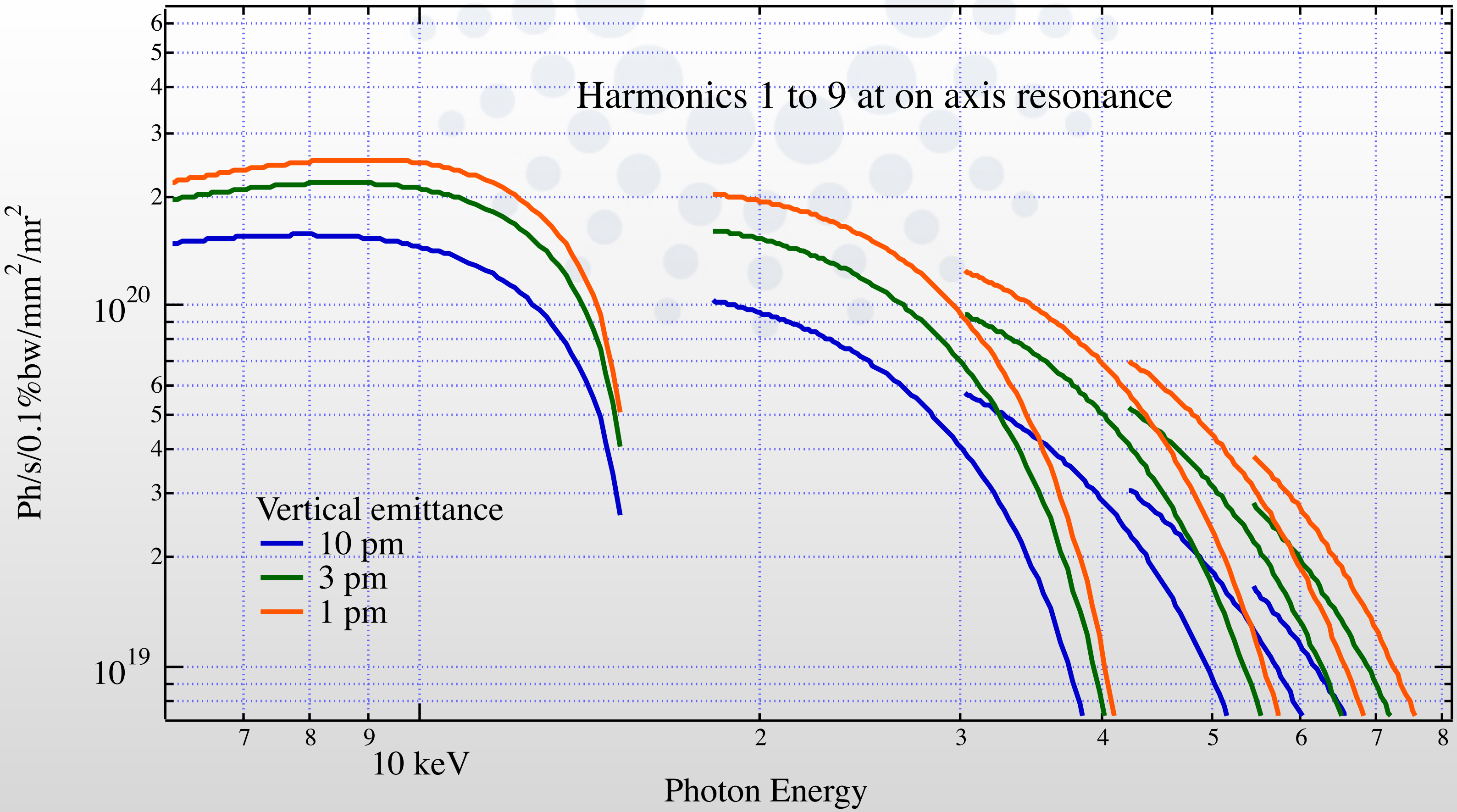


Undulator:
Period λ_0= 22 mm
Number of period
N=90
K max =1.79

Electron beam
E=6.04 Gev
I=0.2 A
ESRF low beta

Evaluation
At on axis resonance

Resulting brilliance



Undulator:
Period λ_0= 22 mm
Number of period
N=90
K max =1.79

Electron beam
E=6.04 Gev
I=0.2 A
Horizontal emittance: 4 nm
ESRF low beta

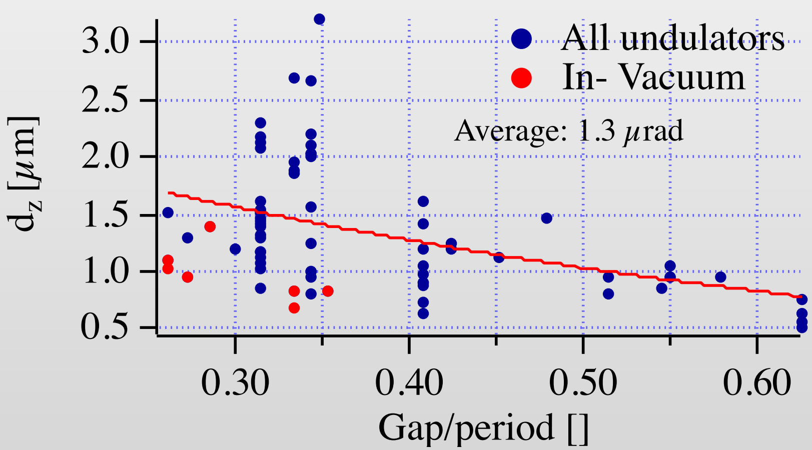
Residual field errors in undulators

Undulators have residual small horizontal along all magnetic structure
-> small vertical random motion of electron along undulator

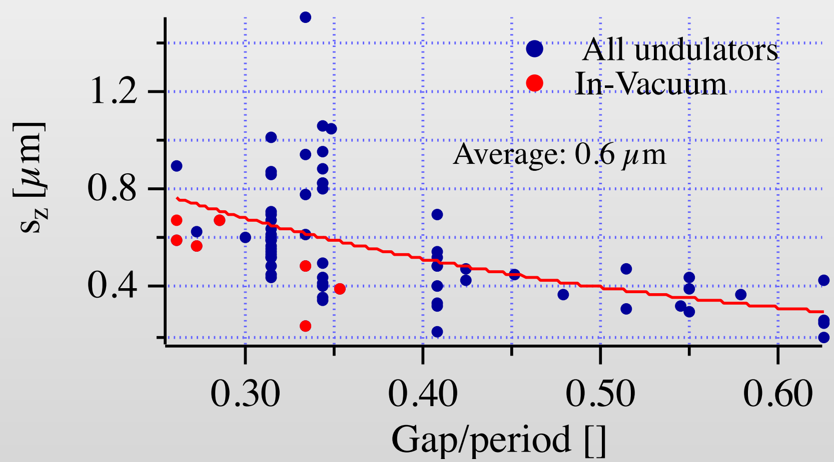
This generate an additional contribution to vertical source size and divergence

Has no impact on electron beam closed orbit and vertical emittance

Divergence



Size

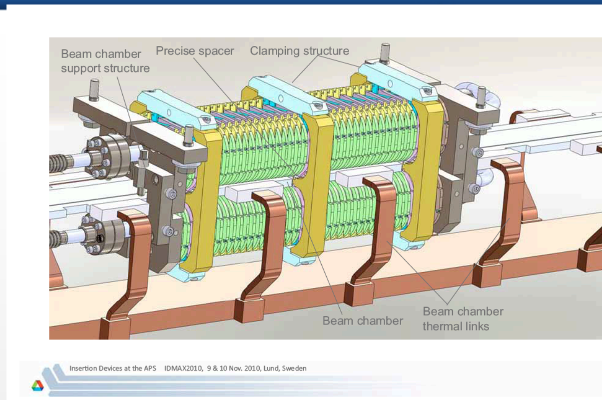


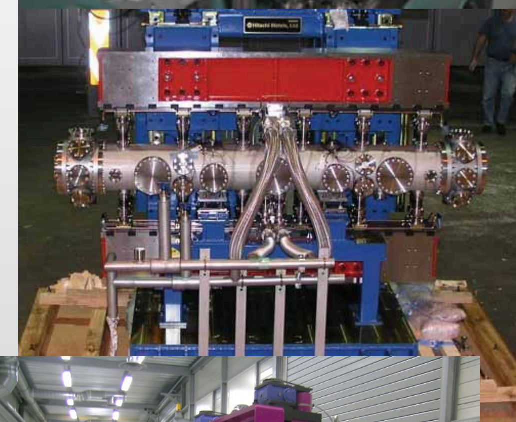
All Undulators @ minimum gap:

Gives an equivalent extra vertical phase space area of ~ 0.8 pm

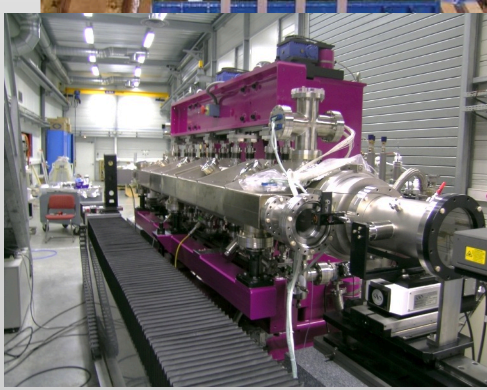
Ambient field along straight section also contribute ->to be investigated

Technological trends in Insertion Devices





Overview of Insertion Devices at SR facilities
Small gap & conventional undulators
Cryogenic devices



Undulator demand

Driven by new constructions & upgrades

Many Medium energy rings :2.7-3.5 GeV

SOLEIL, DIAMOND, CLS, ALBA, SSRF, TPS ,Australian Synchrotron, NSLS II …

High energy rings (≥ 6.GeV)

SPRING 8

ESRF Upgrade

APS Upgrade

Petra III

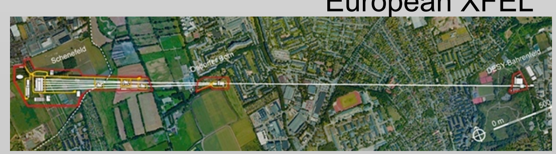
X FELs

- LCLS (Stanford)
- SACLA (SPRING8)
- Flash, European XFEL (Hamburg)
- Fermi@ elettra
- …..

LCLS

SACLA

European XFEL



Fermi

Types of Insertion Devices

Medium Energy Rings

1- In-Vacuum undulators
2- Superconducting wigglers
2- Elliptically polarized Undulators

Access to photon energy above 10 KeV rely only on ID performance

High energy rings

1- Conventional (In-air planar undulators) (ESRF,APS, PETRA III)
2- IVUs (SPRING 8 ,ESRF, planned at PETRA III)
3- Elliptically polarized Undulators
4- Superconducting undulator development (APS)

X-FELS

1- Conventional in-air planar undulator: LCLS (fixed gap), European X-FEL
2- IVUS (SACLA-SPRING8)
3- EPU (Fermi)

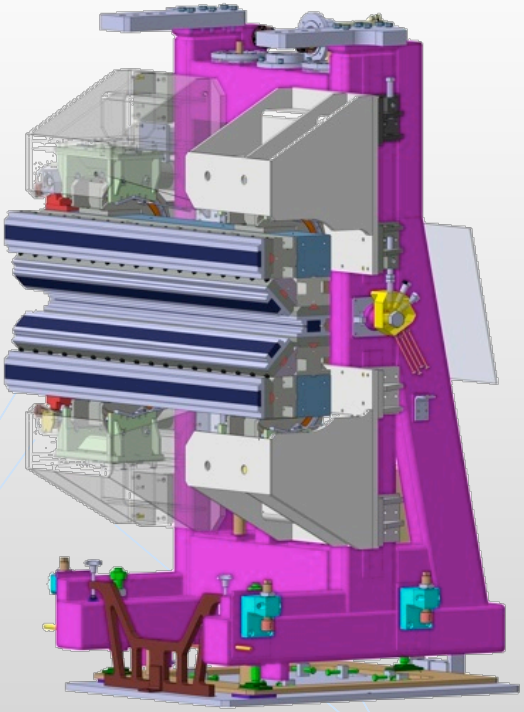
For the time being, X-FELs and SR facilities rely on same ID technology

" Conventional " undulators

Significant part of IDs in high energy rings ESRF, APS, PETRA III

Evolution toward revolver structure:
Connected to specialization of beamlines

Flexibility
Combines:
Tunable undulator for 2.5 - 30 keV (period 35 mm, Kmax>2.2)
+ Shorter period undulators for higher brilliance in limited energy range (period 18 ~ 27 mm, Kmax <1.5)
Interchangeable with other standard undulator segments

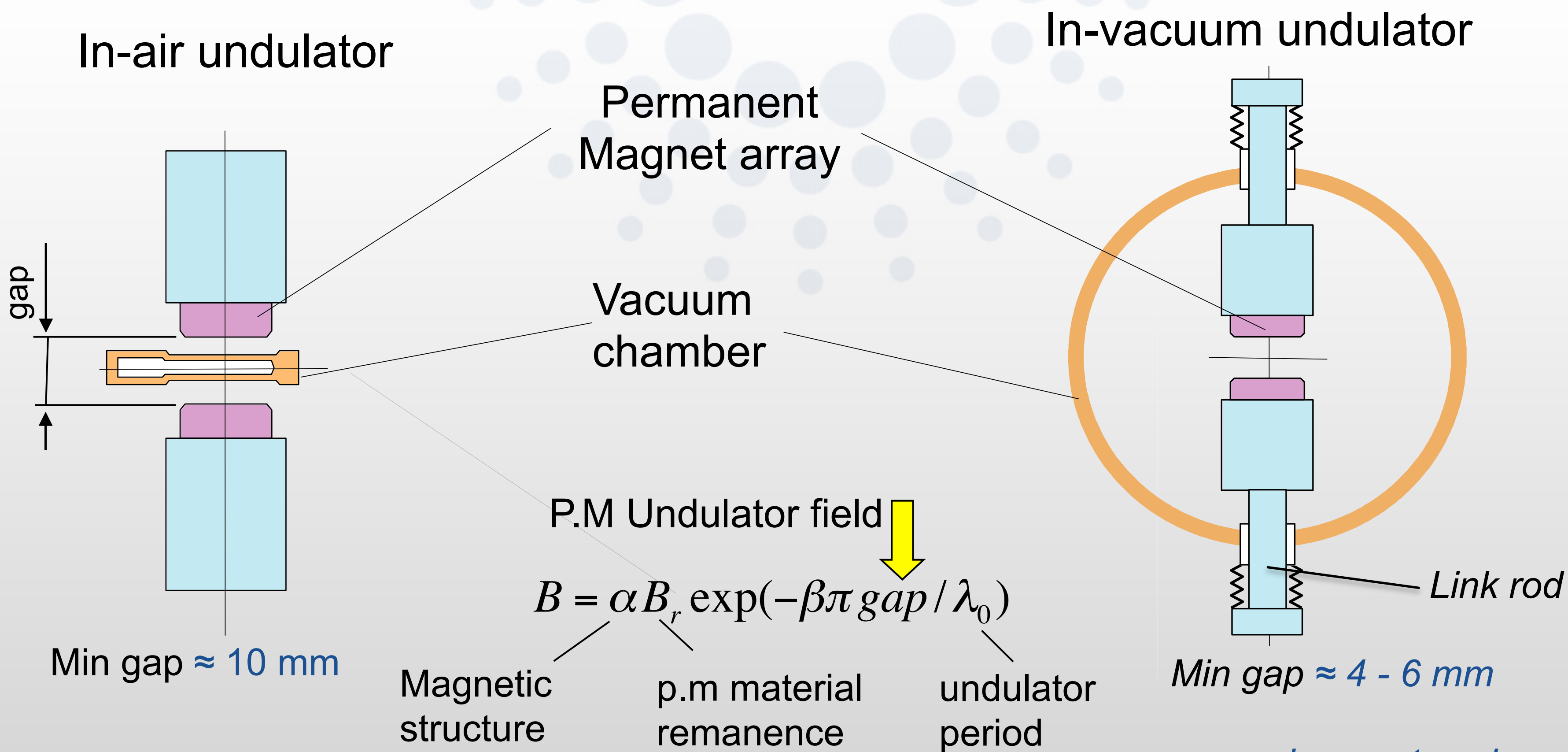


Noticeable demand for revolver devices at ESRF

Foreseen in the upgrade of APS

ESRF revolver undulator
3 different undulators

In-Vacuum undulators



$$B = \alpha B_r \exp(-\beta\pi\, gap / \lambda_0)$$

Min gap ≈ 10 mm

Min gap ≈ 4 - 6 mm

≈ beam stay clear

Large international development of IVUs
Minimum gap limited by effect on beam (beam losses, lifetime reduction ..)

Minimum gap < 6 mm needs to be investigated at ESRF in near future

ESRV IVUs

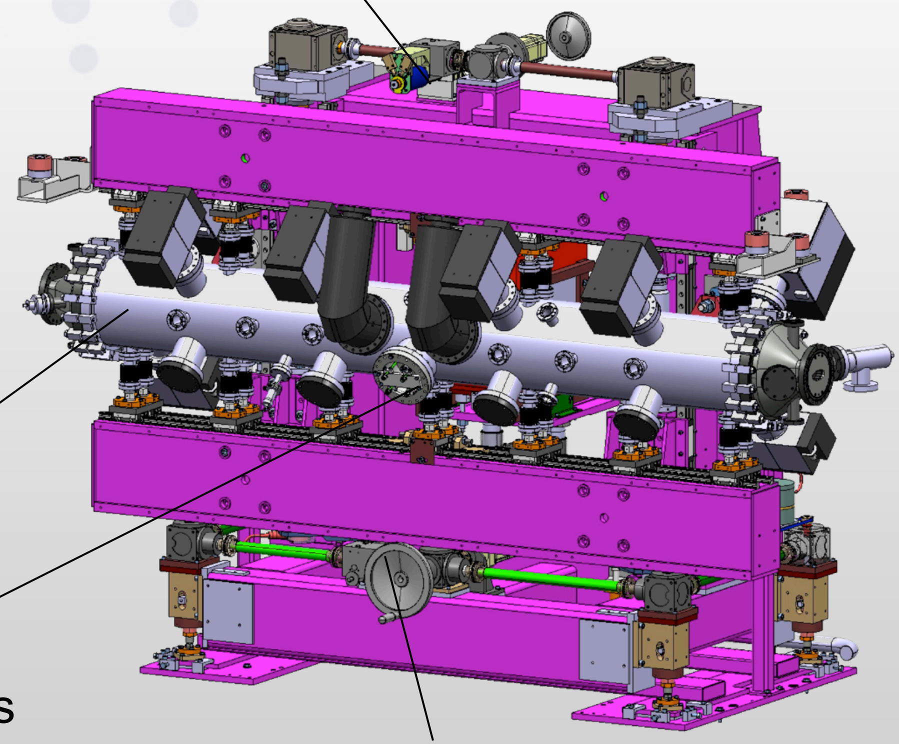
Nominal magnetic length 2m

New version with 2.5 m
Under construction (UPBL4)

Mature technology

Essential for High Photon energy

above 50 keV



Support structure compatible with room temperature IVU or CPMU

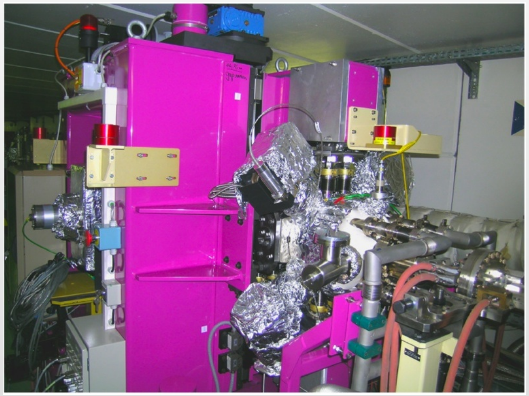
CPMUs

CPMU: Cryogenic Permanent Magnet Undulator

Affordable evolution of IVUs:

Cryogenic cooling of permanent magnet arrays:

- possible use of high performance magnets
- high resistance to demagnetization
- ~ 35 % gain in peak field vs standard IVUs



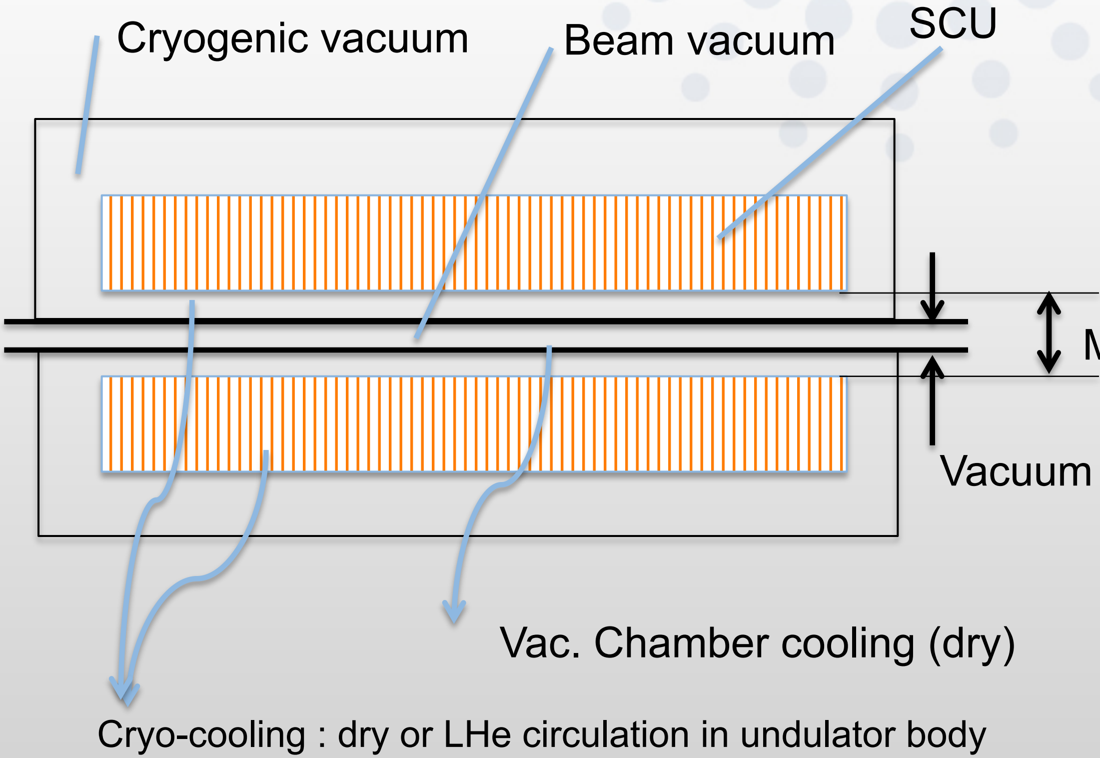
First device installed and operated at ESRF

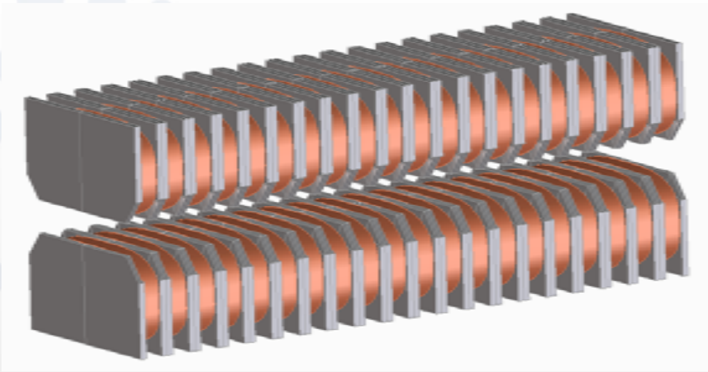
Second device completed: installation in January 2012 in ID11

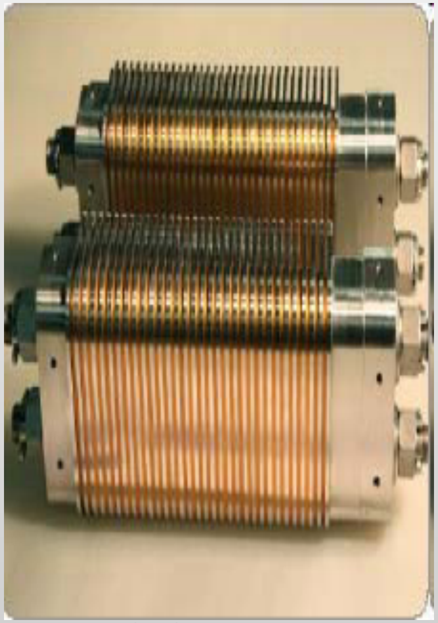
- period 18 mm
- peak field 1 T @ 148 K, gap 6 mm

A light for Science

2001 New concept of SCU (ANKA-ACCEL)





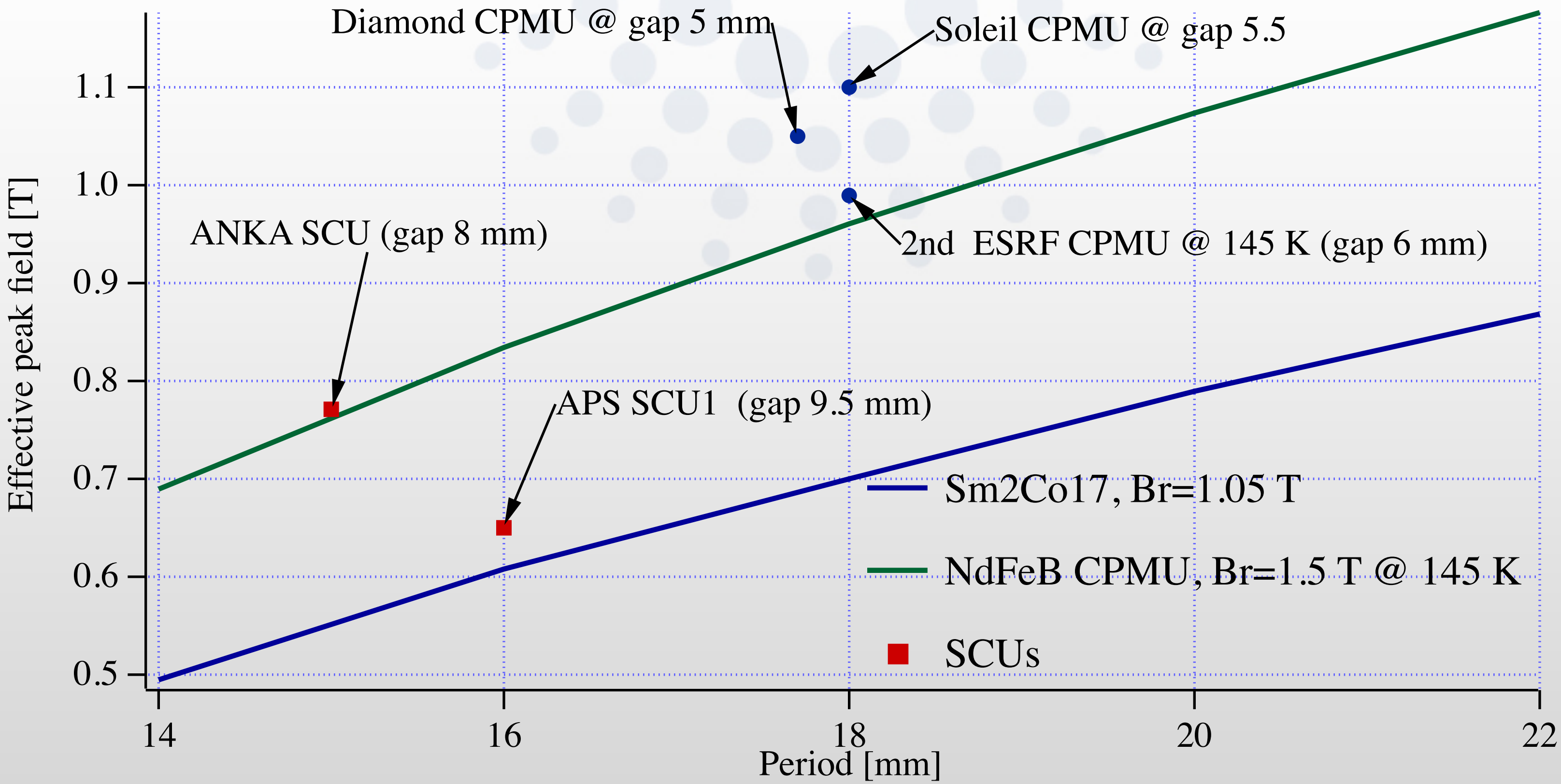


S. Casalbuoni

Magnetic gap= vacuum gap + D

D = 2 ~ 2.5 mm

SCUs Vs CPMUs



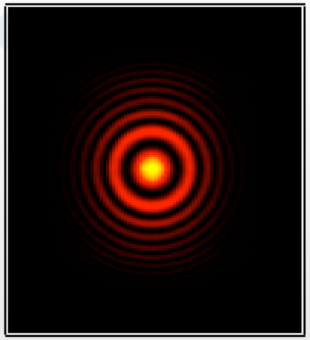
Plans to Use Nb3Sn instead of NbTi superconducting materials for SCUs

Present Limitation for SCUs: Magnetic gap vs vertical beam stay clear (heat budget)

Summary

- Basic principles of undulator radiation have been visited

- Undulator radiations have longitudinal and transverse “interference” patterns



- Limiting factors on undulator performances
  - horizontal emittance
  - energy spread on high undulator harmonics

- Beneficial improvement achieved through the reduction of vertical emittance
  - vertical source divergence close to saturation

- The technology of undulator evolves toward
  - higher flexibility
  - cryogenic devices