X-ray Phase-Attenuation Duality and Phase Retrieval For Soft Tissue Imaging

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Phase-Attenuation Duality And Phase Retrieval

- Phase contrast imaging theory based on Wigner distributions

- Phase-Attenuation Duality

- Single-image phase retrieval with for inhomogeneous soft tissue

Modeling of Phase Imaging

- **An object is modeled by :**

$$T(\vec{r}) = \exp\left(i\phi(\vec{r}) - \frac{\mu_p(\vec{r})}{2} \right) = A_o(\vec{r}) e^{i\phi(\vec{r})}$$

X-ray Phase Change

$$\phi(\vec{r}) = -\frac{2\pi}{\lambda}\int \delta(\vec{r}, s)\,ds$$

Refractive Index Decrement

$$\delta = \left(\frac{r_e \lambda^2}{2\pi}\right)\sum_k N_k (Z_k + f_k^r)$$

Phase-Space Formulation for Phase Imaging

- **Wigner Distribution**

$$W(\vec{r},\vec{u};z)=\int J(\vec{r}+\vec{q}/2,\vec{r}-\vec{q}/2;z)\exp(-i2\pi\vec{q}\cdot\vec{u})d\vec{q}$$

$J(\vec{r}_1,\vec{r}_2;z)$ **is the Mutual Intensity of X-ray Wave**

- **Phase-space evolution: Solve for Louiville Equation**

$$W(\vec{r},\vec{u};R_1+R_2)=W(\vec{r}-\lambda R_2\vec{u},\vec{u};R_1)$$

Phase-Contrast Theory based on Wigner Distribution

$$I(\vec{r};R_1+R_2)=\int W(\vec{r},\vec{u};R_1+R_2)d\vec{u}=\int W(\vec{r}-\lambda R_2\vec{u},\vec{u};R_1)d\vec{u}$$

$$\Downarrow$$

$$\begin{aligned}\tilde{I}\left(\vec{u}/M;R_1+R_2\right)&=I_{in}\tilde{\mu}_{in}\left(\lambda R_2\vec{u}/M\right)\cdot OTF_{\det}\left(\vec{u}/M\right)\times\\&\times\left\{\begin{aligned}&\cos\left(\pi\lambda R_2\vec{u}^2/M\right)\cdot\left(FT(A_o^2)-i(\lambda R_2/M)\vec{u}\cdot FT(\phi\nabla A_o^2)\right)+\\&2\sin\left(\pi\lambda R_2\vec{u}^2/M\right)\cdot\left(FT(A_o^2\phi)+i(\lambda R_2/4M)\vec{u}\cdot FT(\nabla A_o^2)\right)\end{aligned}\right\}\end{aligned}$$

- X. Wu and H. Liu, *Med. Phys.* 31 (2004)

Formula For Intensity of Phase-Contrast Image

In Cases With $\pi \lambda R_2 \vec{u}^2 / M \ll 1$

$$\tilde{I}\left(\frac{\vec{u}}{M}; R_1 + R_2\right) = I_{in} \tilde{\mu}_{in}\left(\frac{\lambda R_2 \vec{u}}{M}\right) OTF_{\det}\left(\frac{\vec{u}}{M}\right) \times$$
$$\left\{\left(\hat{F}(A_o^2) - i\frac{\lambda R_2}{M}\vec{u}\cdot\hat{F}(\phi \nabla A_o^2)\right) + 2\frac{\pi \lambda R_2 \vec{u}^2}{M}\hat{F}(A_o^2 \phi)\right\}$$

When $\tilde{\mu}_{in}(\lambda R_2 \vec{u}/M) OTF_{\det}(\vec{u}/M) = 1$

this theory is reduced to the TIE-based theories:

Paganin&Nugent, *PRL*, 80 (1998)

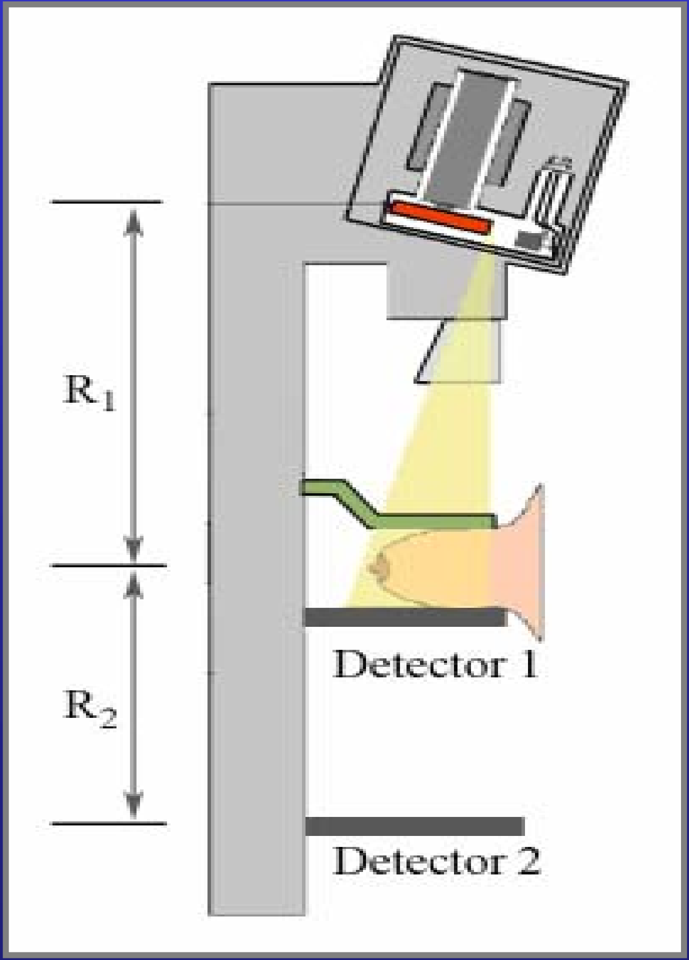
Phase Retrieval

In general <u>at least two</u> images are needed

The Attenuation Image $A_o^2(\vec{r})$

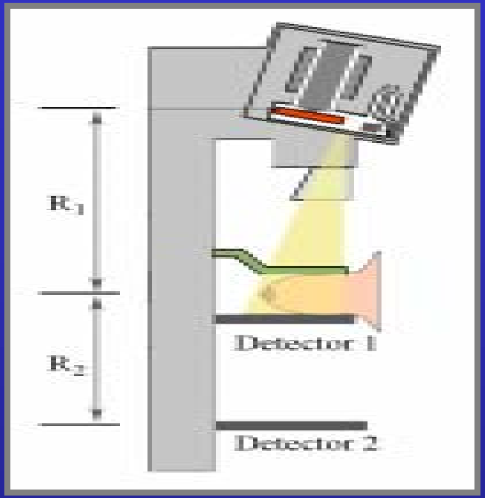
The Phase-Contrast Image $I\left(M\vec{r};R_1+R_2\right)$

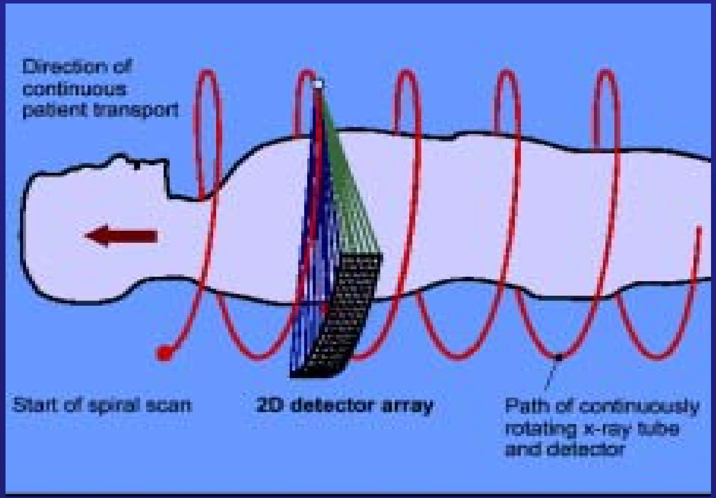
Multiple-image Approaches



- **Multiple exposures acquired at multiple distances**
- **One exposure with dual detectors** Wu&Liu, *JXST*, 2004.
- **Series short exposures of single photon-events to get energy-resolved images** Gureyev, Mayo, Wilkins, et al. *PRL*, 86 (2001).

Drawbacks of Multiple-image Approaches





- **Radiation doses**

  US FDA Limit on glandular dose to breast:

  3 mGy for a 4.2 cm breast of 50% adipose and 50% glandular tissue

- **Motion artifact**

- **Hard to implement for quantitative phase tomography**

  Bronnikov, *JOSA*, A19 (2002).

Single-Image Phase-Retrieval: Single-Material Objects

- **For a homogeneous object of <u>single material</u>**

$$\phi(\vec{r}) \propto \ln\left(A_o^2(\vec{r})\right) \propto T(\vec{r})$$

Paganin, Mayo, Gureyev, et al. *J. Micro.* <u>206</u> (2002).

- **Phase map is reduced to the projected thickness map**

X-Ray Phase and Attenuation

- Phase and attenuation are closely related

- In clinical imaging 10 keV < E < 150 keV

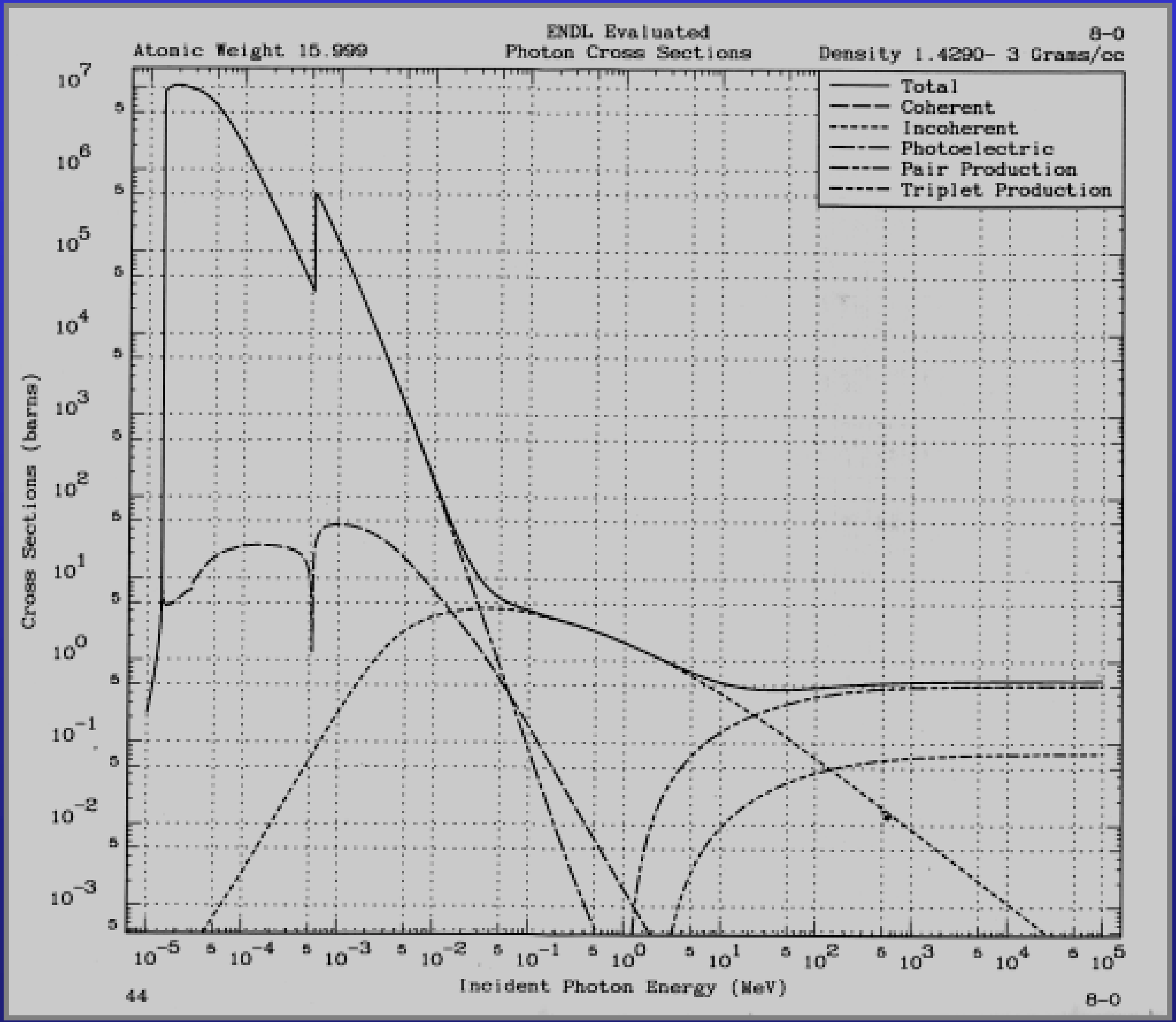
  Tissue attenuation results from:

  - Photoelectric absorption
  - Incoherent scattering
  - Coherent scattering

Soft Tissues Attenuation

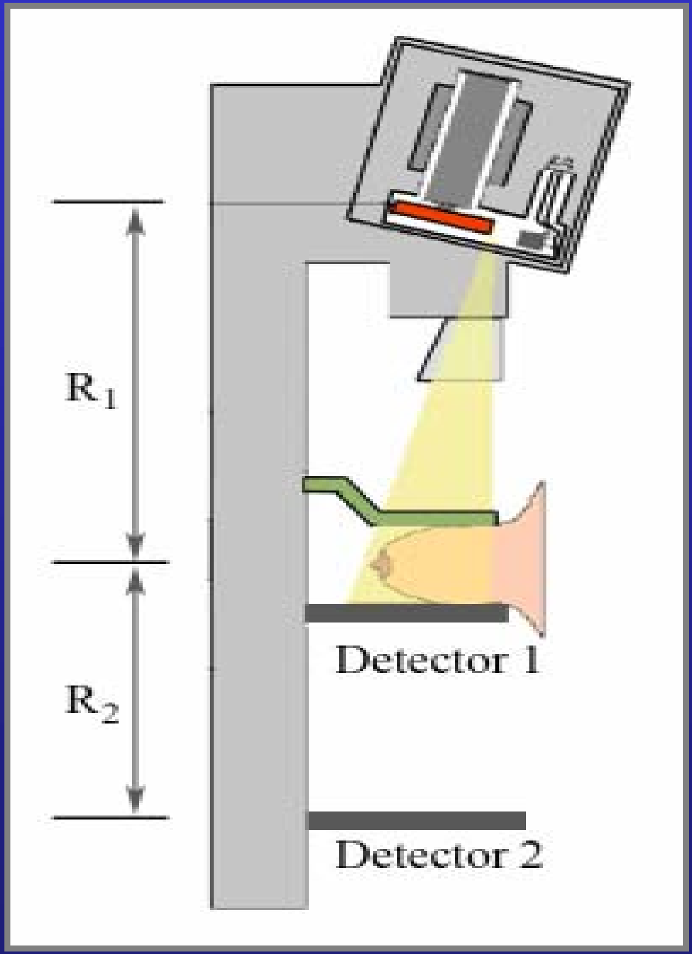
- Soft tissue attenuation decreases with x-ray energy for E > 10 keV

- For X-rays of 60 keV < E < 500 keV
  - X-ray cross section is approximated by that of incoherent scattering
  - With Errors of 10%-0.1% depending on E and Z



ENDL Evaluated
Photon Cross Sections
Atomic Weight 15.999
8-0
Density 1.4290- 3 Grams/cc
Total
Coherent
Incoherent
Photoelectric
Pair Production
Triplet Production
Cross Sections (barns)
Incident Photon Energy (MeV)
44
8-0

Attenuation-Based Mammography



- Currently the most effective method for early detection of breast cancer
- Use “low energy” photons of 18-25 keV for tissue -lesion contrast
- Yet not sensitive enough

Phase-Attenuation Duality

$$
{}_a\sigma^{incoh}(Z,E) \approx Z\,{}_a\sigma_{KN}(E) - \frac{1}{\sqrt{Z}}\,{}_a\sigma^{coh}(Z,E)
$$
$$
\approx Z\,{}_a\sigma_{KN}(E)
$$

$$
\phi(\vec{r}) = -\lambda r_e \rho_{e,p}(\vec{r})
$$

$$
A_o^2(\vec{r}) = \exp\left(-\sigma_{KN}\rho_{e,p}(\vec{r})\right)
$$

- **Dual relationship : Incoherent Scattering Function S(q) vs. Coherent Scattering Form Factor F_0(q)**
- **Soft tissue's phase and attenuation are all determined by projected tissue-electron density**
  <u>for 60 keV < E < 500 keV</u>

Klein-Nishina Cross-Section

$$\sigma_{KN}(E) = 2\pi r_e^{\,2}\left\{\frac{1+\eta}{\eta^2}\left[\frac{2(1+\eta)}{1+2\eta} - \frac{1}{\eta}\log(1+2\eta)\right]\right\}$$

$$+2\pi r_e^{\,2}\left\{\frac{1}{2\eta}\log(1+2\eta) - \frac{(1+3\eta)}{(1+2\eta)^2}\right\}$$

$$\eta = \frac{E}{m_e C^2}$$

$\sigma_{KN}(60keV)$ = 5.4 x 10^{-29} m^2

Phase-Contrast Imaging

$$\tilde{I}\left(\frac{\vec{u}}{M};R_1+R_2\right)=I_{in}\tilde{\mu}_{in}\left(\frac{\lambda R_2\vec{u}}{M}\right)OTF_{\det}\left(\frac{\vec{u}}{M}\right)\times$$
$$\left\{\left(\hat{F}(A_o^2)-i\frac{\lambda R_2}{M}\vec{u}\cdot\hat{F}(\phi\nabla A_o^2)\right)+2\frac{\pi\lambda R_2\vec{u}^2}{M}\hat{F}(A_o^2\phi)\right\}$$

$$\Downarrow$$

$$\Leftarrow$$

$$\phi(\vec{r})=-\lambda r_e\rho_{e,p}(\vec{r})$$

$$A_o^2(\vec{r})=\exp\left(-\sigma_{KN}\rho_{e,p}(\vec{r})\right)$$

$$\tilde{I}\left(\frac{\vec{u}}{M};\vec{\theta}\right)=I_{in}\tilde{\mu}_{in}\left(\frac{\lambda R_2\vec{u}}{M}\right)OTF_{\det}\left(\frac{\vec{u}}{M}\right)$$
$$\times\left\{1+\frac{2\pi r_e\lambda^2R_2\vec{u}^2}{M\sigma_{KN}}\right\}\hat{F}(A_o^2)$$

Phase Retrieval Based on Phase-Attenuation Duality

$$\rho_{e,p}(\vec{r}) = -\frac{1}{\sigma_{KN}} \log_e \left(\hat{F}^{-1} \left\{ \frac{\hat{F}\{M^2 I(M\vec{r}, R_1 + R_2)\}}{I_{in}\tilde{\mu}_{in}\left(\frac{\lambda R_2 \vec{u}}{M}\right) OTF_{\det}\left(\frac{\vec{u}}{M}\right)\left(1 + 2\pi\left(\frac{r_e \lambda^2 R_2}{M\sigma_{KN}}\right)\vec{u}^2\right)} \right\} \right)$$

- X. Wu H. Liu and Aimin Yan, *Optics Letters* 30 (2005)

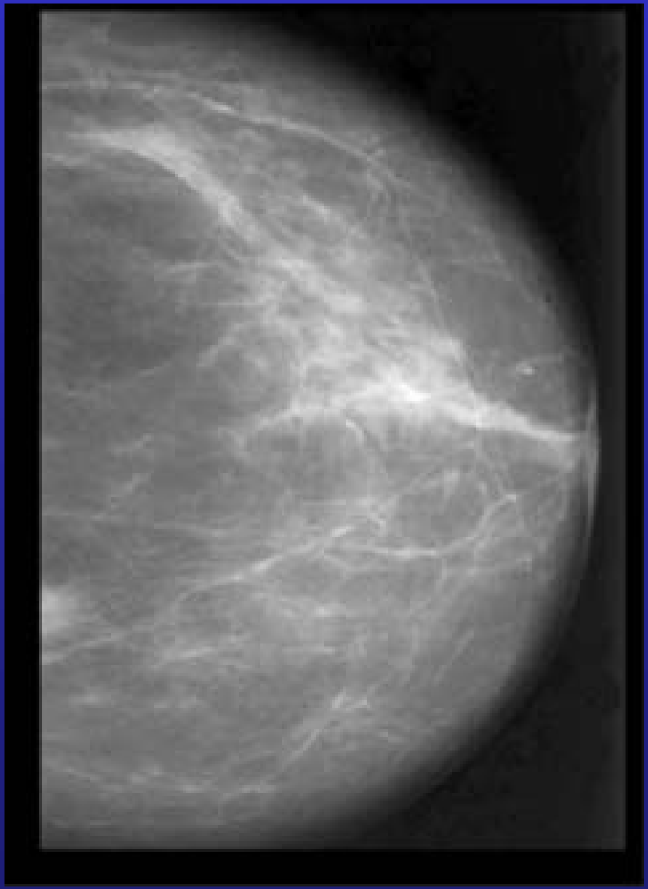
Partial Coherence Effects on Phase Retrieval

- Reduced Complex Degree of Coherence
  - For anode sources : OTF for Geometric unsharpness
  - For undulator sources:

$$\tilde{\mu}_{in}\left(\frac{\lambda R_2 \vec{u}}{M}\right) = \exp\left[-\frac{1}{2}\left(\left(\frac{(M-1)u_x\sigma_x}{M}\right)^2 + \left(\frac{(M-1)u_y\sigma_y}{M}\right)^2\right)\right]$$

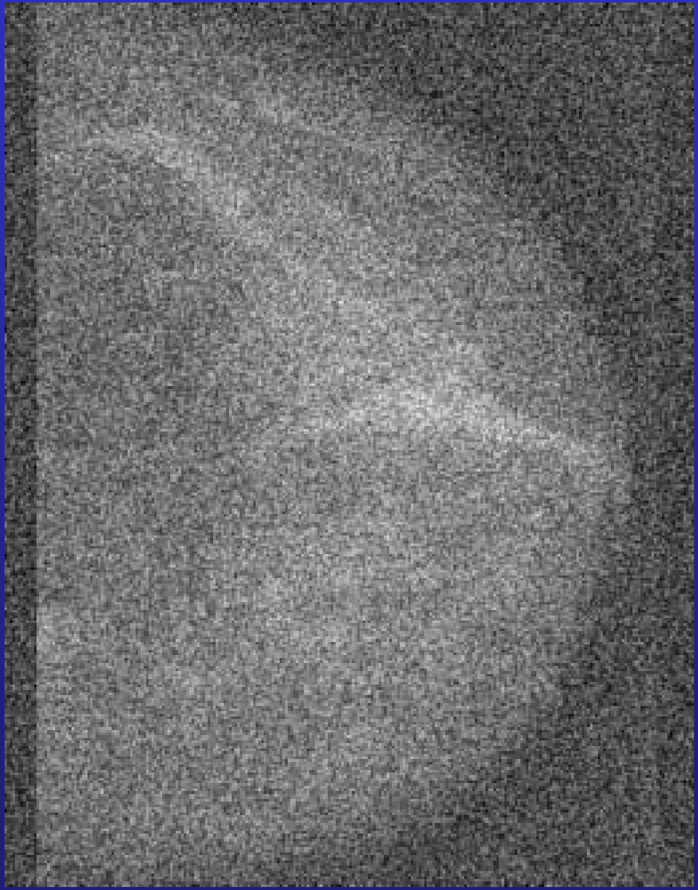
$$\sigma_x = 2\pi\sqrt{\left(\sigma_{ex}^2 + \frac{1}{4u_{1x}^2}\right) - \frac{1}{4(2\pi\sigma_{ex}'/\lambda)^2 + 4u_{1x}^2}}$$

Simulated Image of Projected Electron Density for a 4cm-Thick Breast



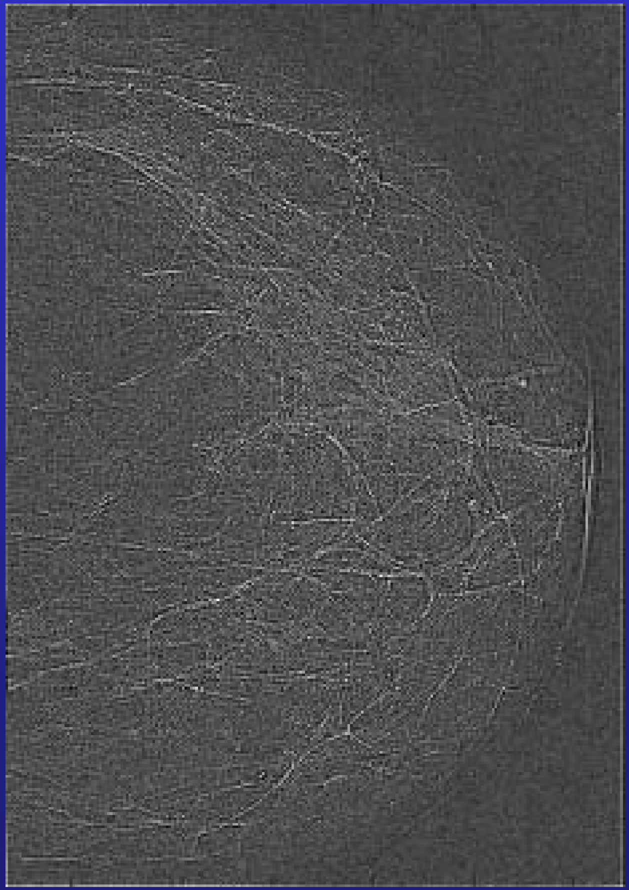
- Hypothetical Breast
- With very low tissue radiographic subject contrasts 0.72% for x-rays of 60 keV.

Simulated Image : $A_o^2(\vec{r})$ With Added Noise



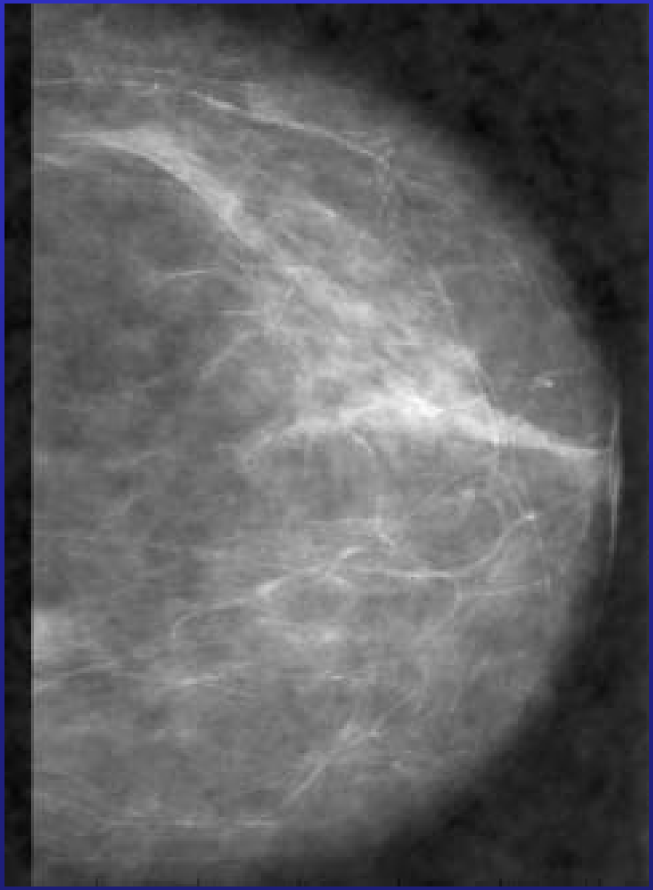
- Simulated attenuation image with added random noise in 0.5%
- The noise masked anatomical details

Simulated Phase Contrast Image: $\left(\vec{r}, R_1 + R_2\right)$



- Simulated phase-contrast image with added random noise in 0.5%
- 60-keV X-ray
- $R_1 = R_2 = 1$m
  9.67 μm pixel size

Reconstructed Phase Image: $(\vec{r})$

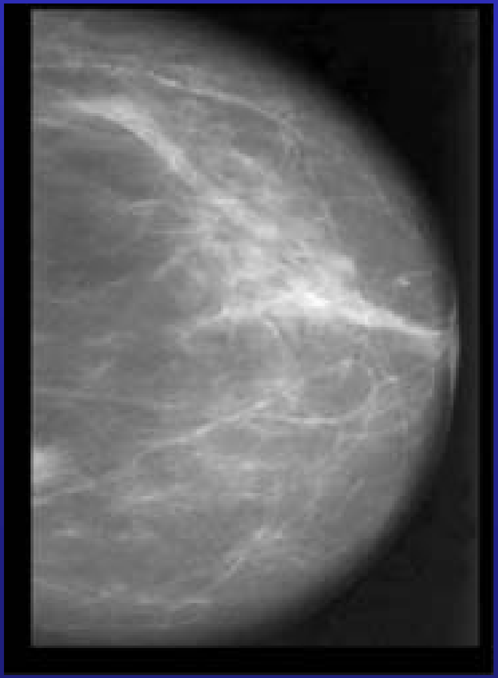
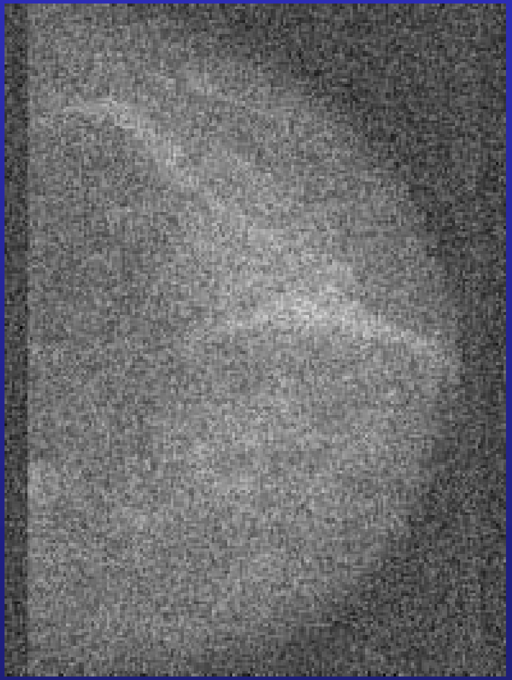
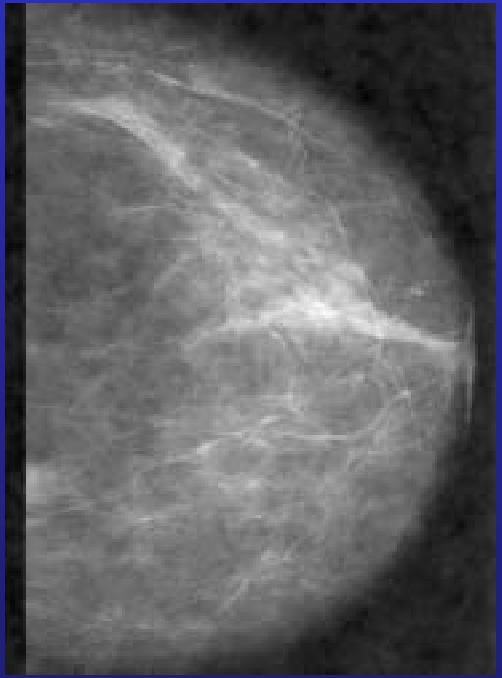


- Reconstructed <u>quantitative</u> phase-map for the breast
- Stable Retrieval
- Recover errors

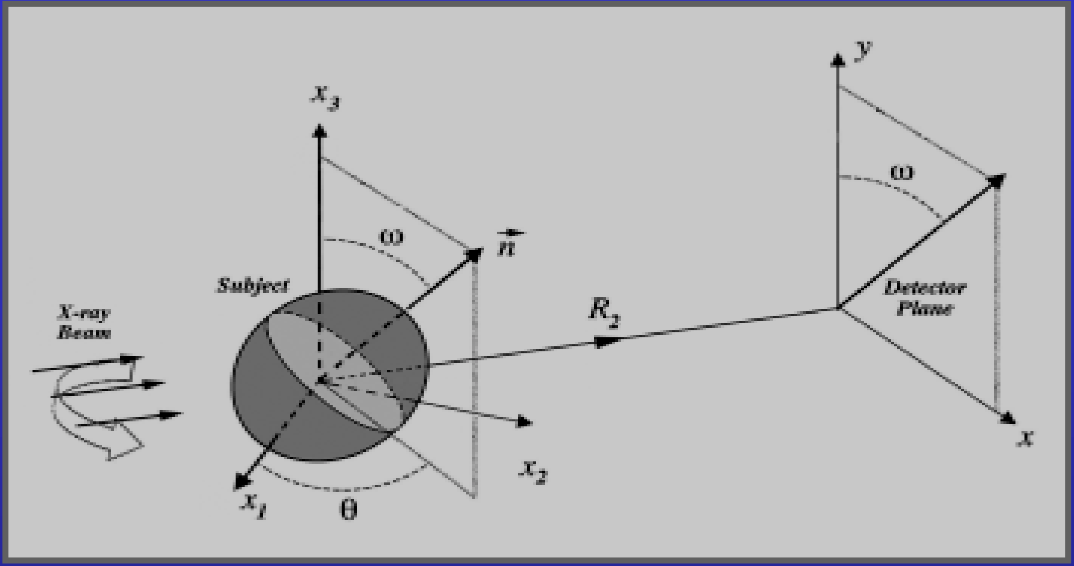
  Max 0.46%

  Min 2.9 $\times 10^{-8}$

Phase Retrieval
Based on Phase-Attenuation Duality

3-D Phase Tomography: Parallel Beam

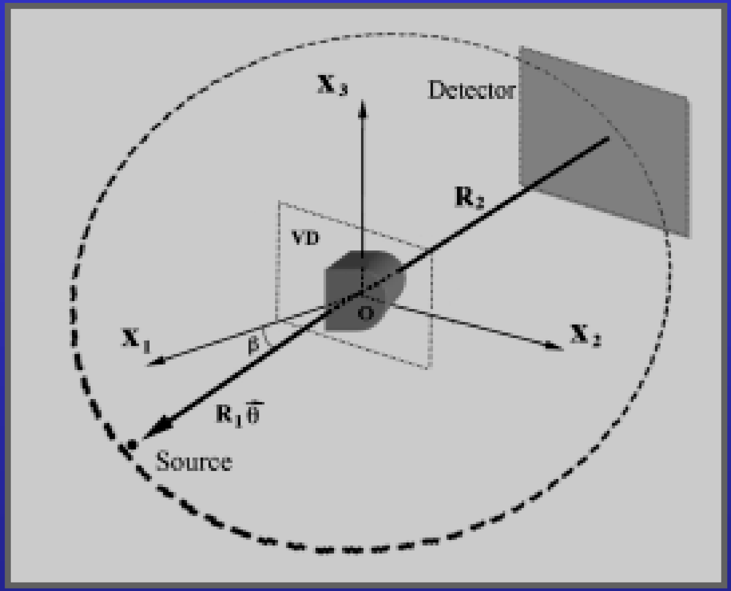


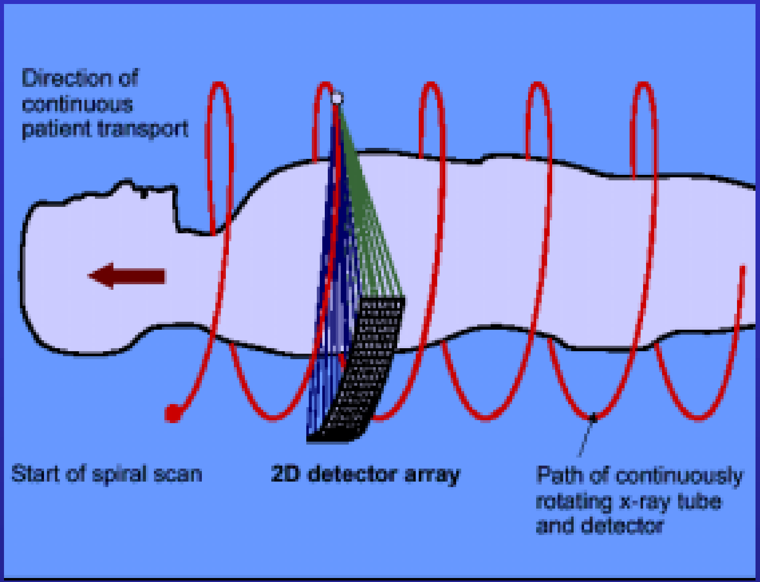
$$\delta(\vec{r}_o) = \frac{\lambda^2 r_e}{8\pi^3 \sigma_{KN}} \int_0^{\pi} \sin\omega \left\{ \int_0^{\pi} \int \Psi(\vec{r}_D;\theta)\, \delta^{(D)}(\vec{r}_D \cdot \vec{n}_D - s)\, d\vec{r}_D\, d\theta \right\} d\omega$$

$$\Psi(\vec{r}_D;\theta) = \frac{A^2(\vec{r}_D;\theta)\nabla^2 A^2(\vec{r}_D;\theta) - \left(\nabla A^2(\vec{r}_D;\theta)\right)^2}{\left(A^2(\vec{r}_D;\theta)\right)^2}$$

$$\vec{n}_D = (\sin\omega, \cos\omega)$$

Cone Beam Phase Tomography





Conclusion

- Phase-Attenuation Duality is an important notion for soft tissue phase imaging

- Single-Image phase retrieval based on Phase-Attenuation Duality is advantageous for applications such as clinical soft tissue imaging

Acknowledgment

- **This work is supported in part by NIH grants.**
- **We thank Dr. Aimin Yan and Hong-gang Liu for their help in preparation of this talk.**