Phase Retrieval – An Overview

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Coherence

Start with Mutual Optical Intensity – the quasimonochromatic coherence function – which describes the correlations in the field between two points

$$\boldsymbol{J}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\left\langle \boldsymbol{E}(\boldsymbol{r}_1)\boldsymbol{E}^*(\boldsymbol{r}_2)\right\rangle$$

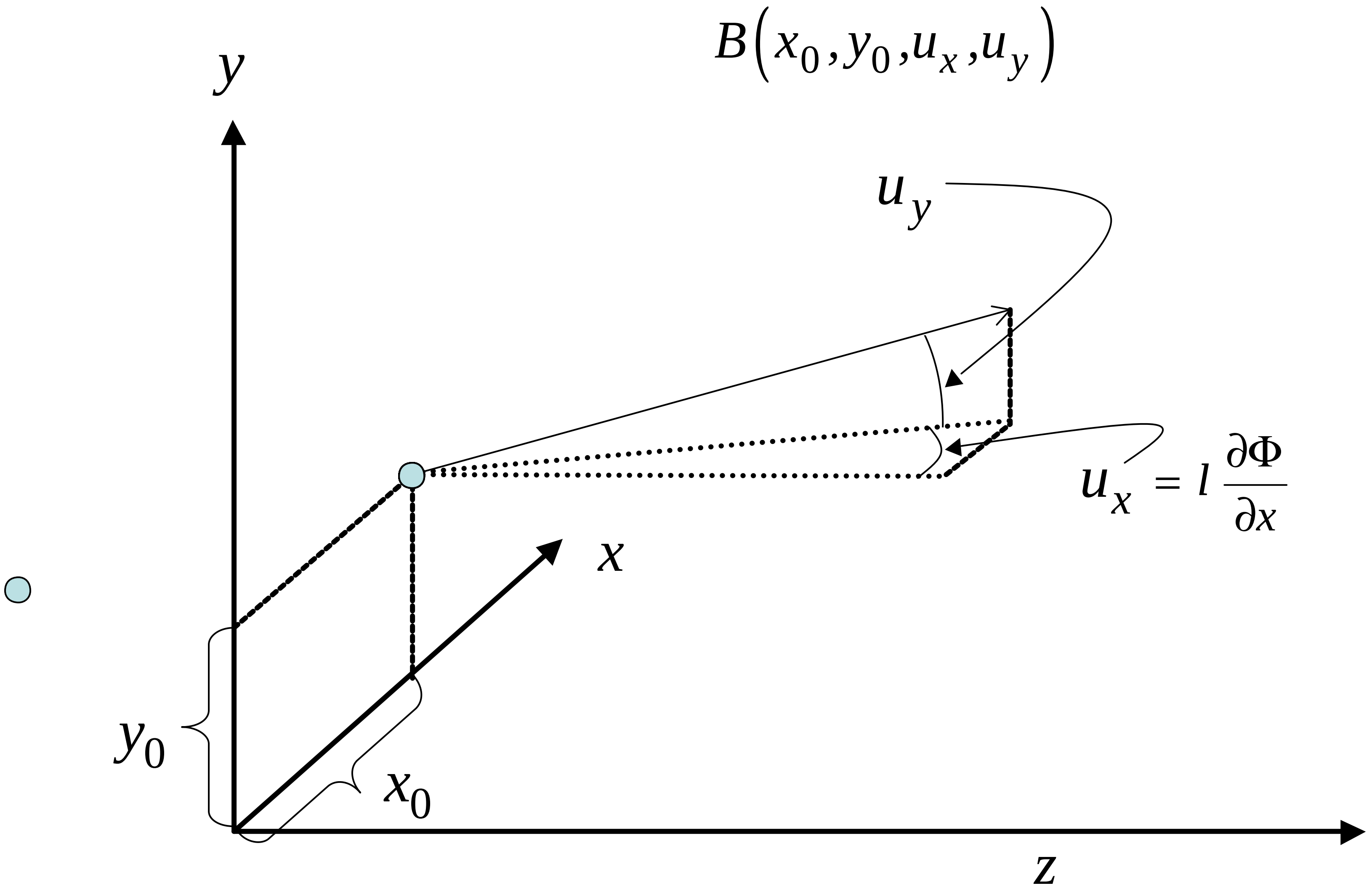
Coherence and Phase-Space

I propose to consider phase recovery from a very general perspective that uses the <u>Generalised Radiance</u> (Wigner function) of the field.

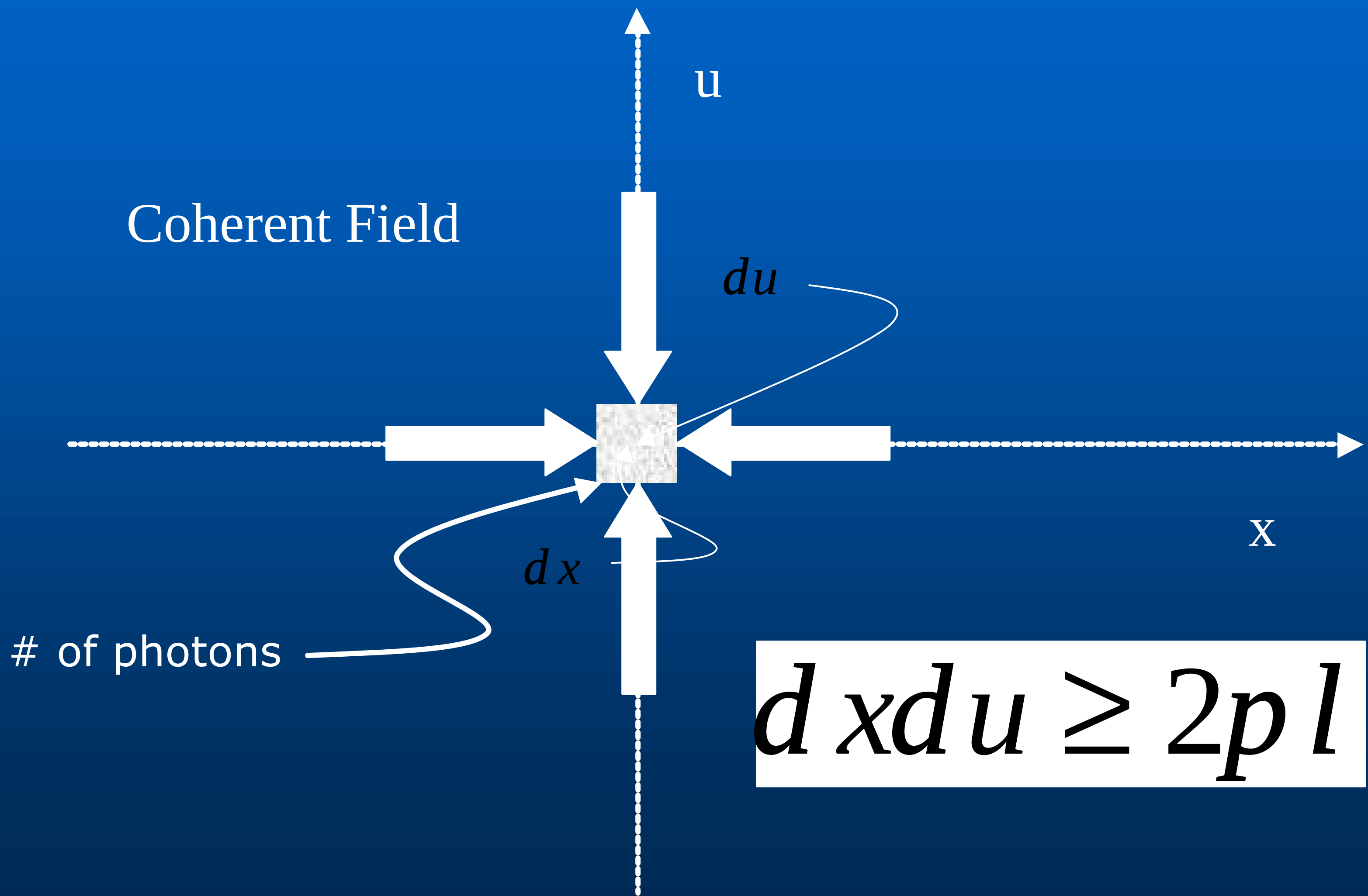
This describes the field in terms of the number of photons as a function of position and momentum. The quantity is *real* and so phase does not appear directly. It allows a very intuitive approach to partially coherent optics.

$$B(\mathbf{r},\mathbf{u}) = \left(k/2p \right)^2 \int J\left(\mathbf{r} + \frac{\mathbf{x}}{2}, \mathbf{r} - \frac{\mathbf{x}}{2} \right) exp\left[ik\mathbf{x} \bullet \mathbf{u} \right] d\mathbf{x}$$

Coherence and Phase-Space

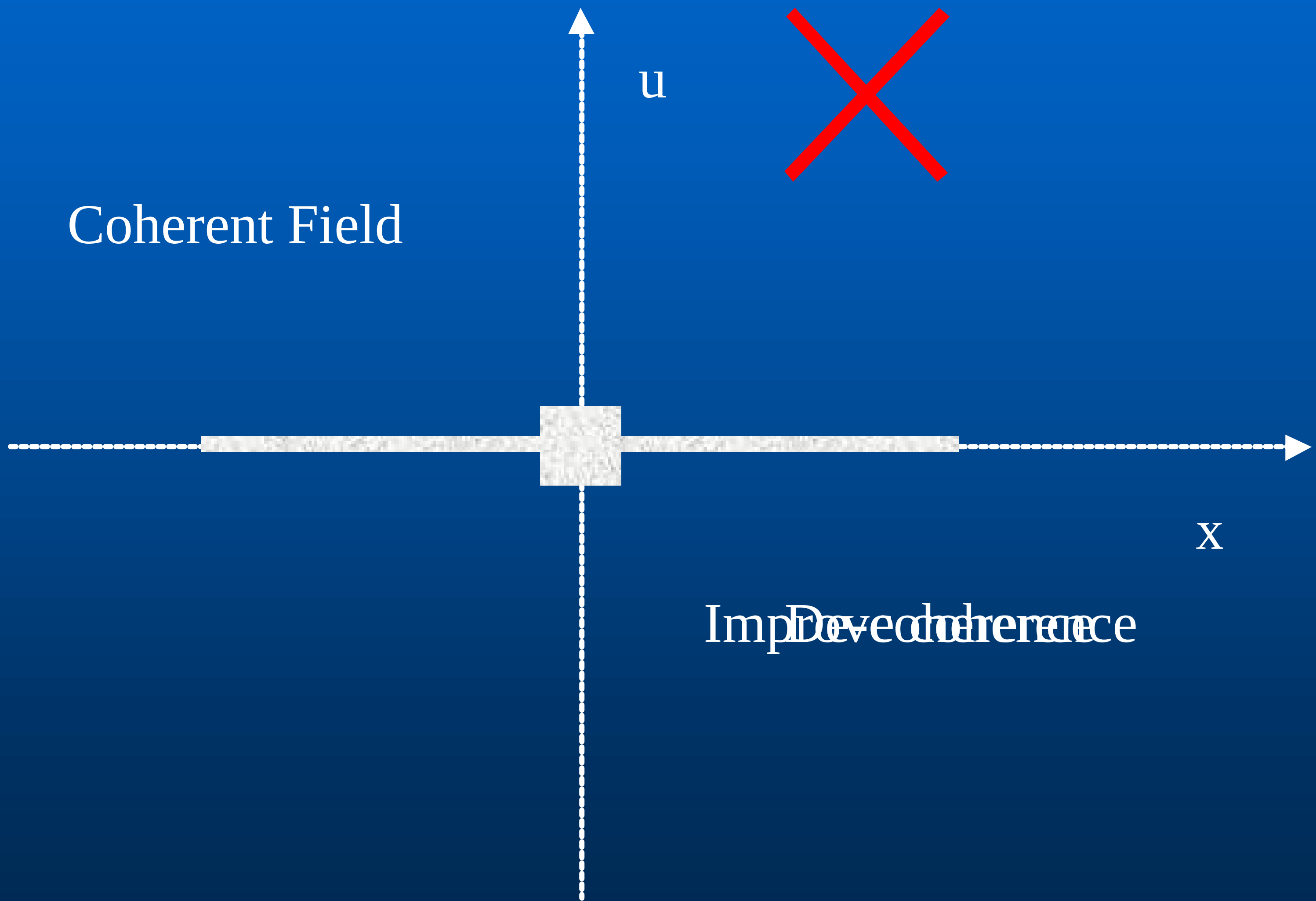


Phase Space



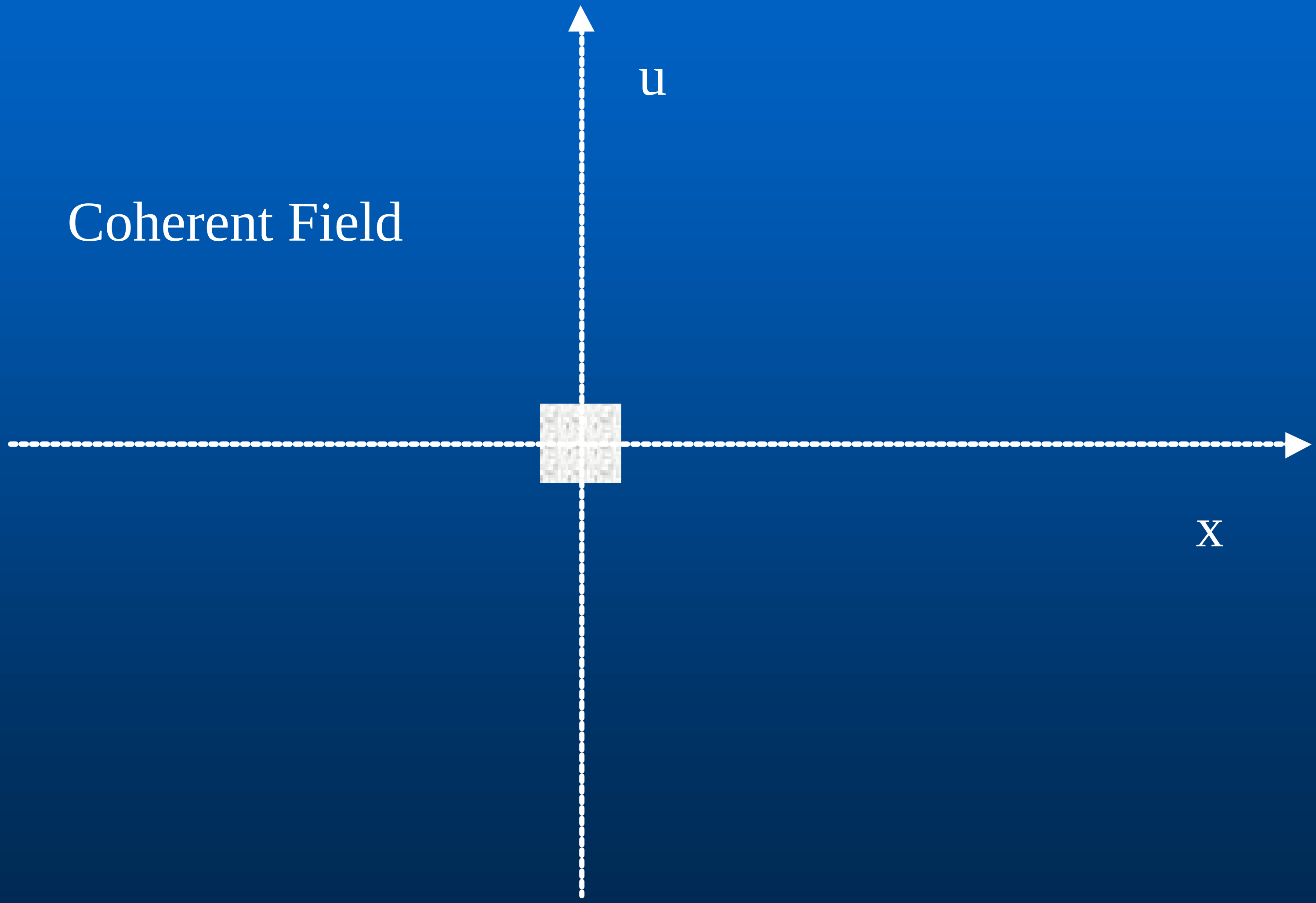
$$dxdu \geq 2pl$$

Phase Space



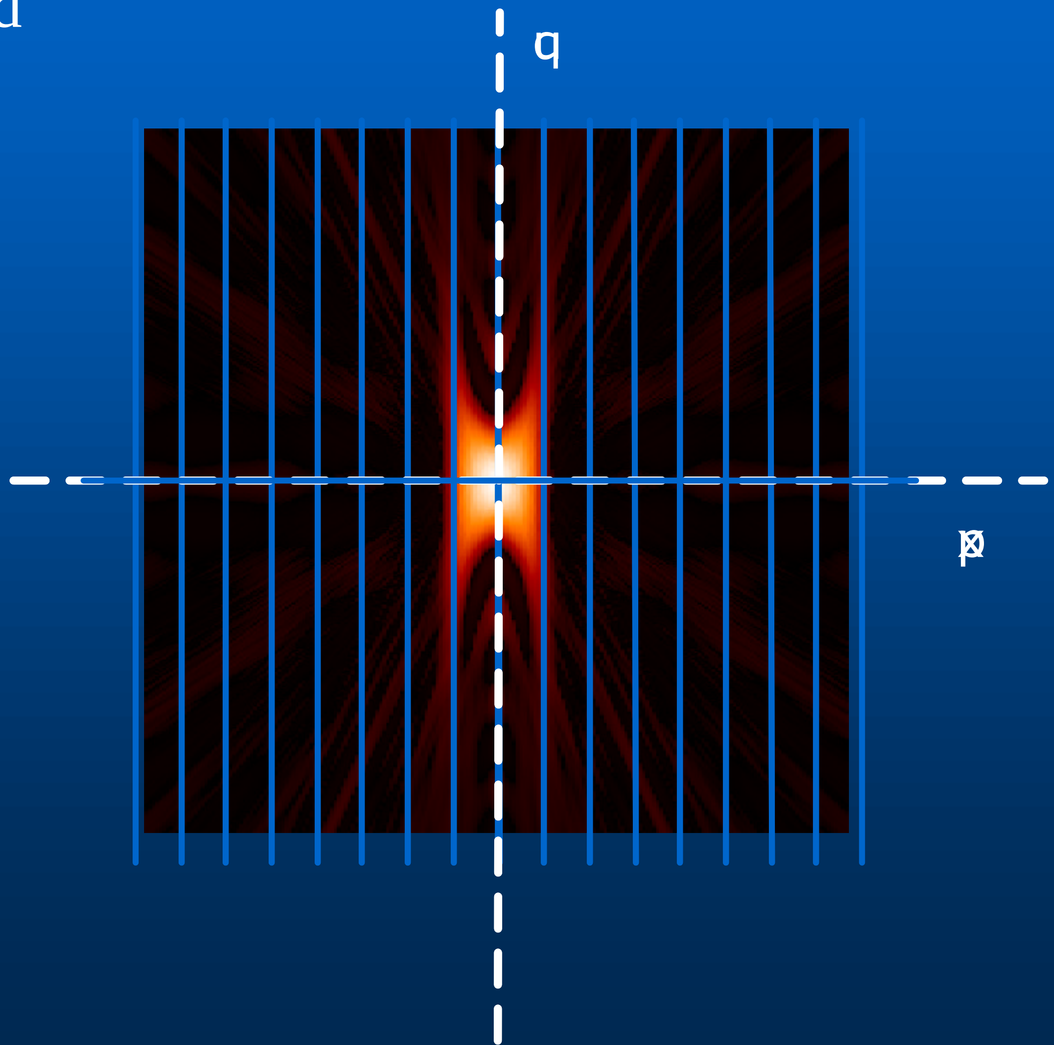
Phase Space

Coherent Field



Fourier transform of Phase Space

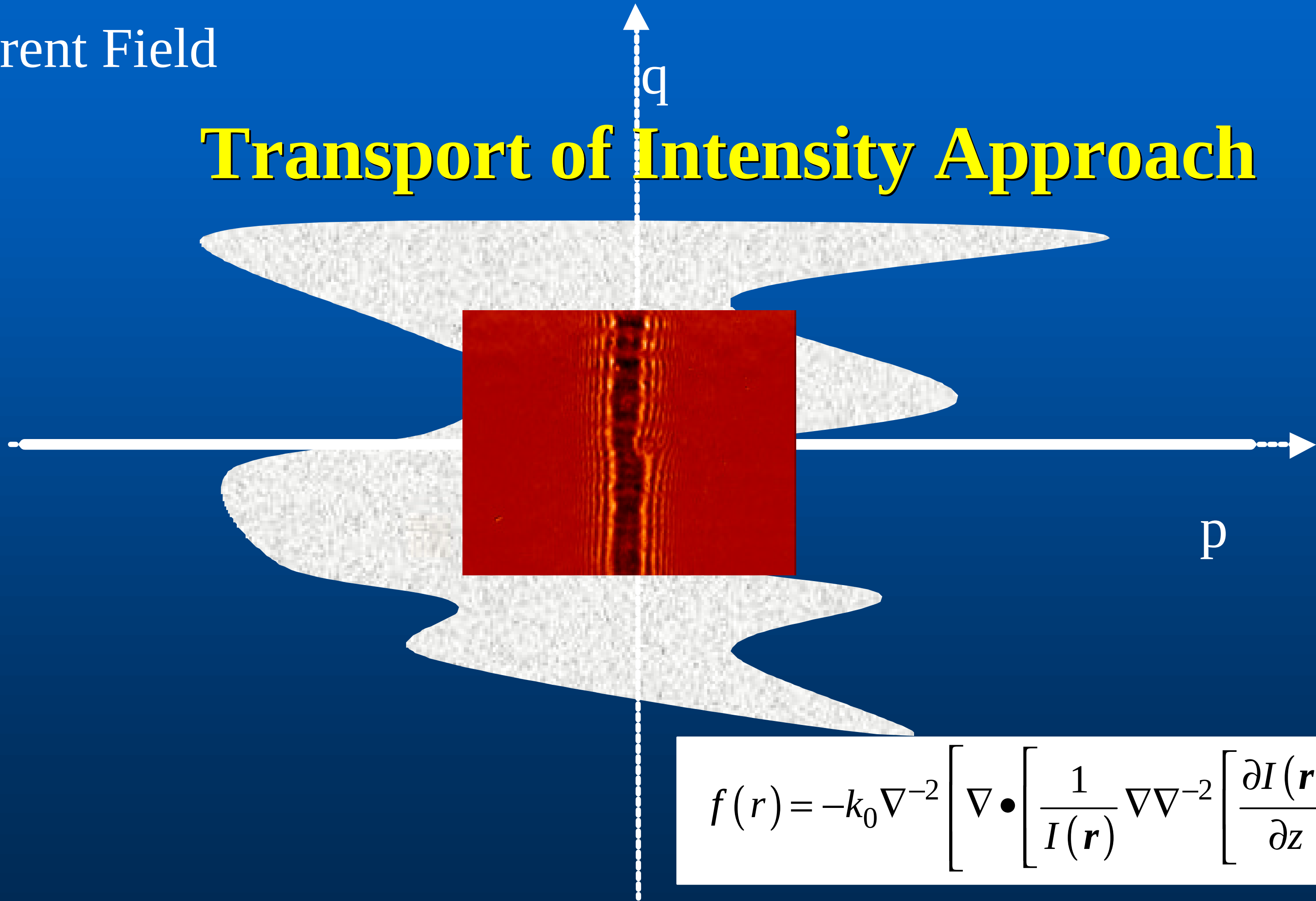
Coherent Field



Phase Recovery – Image Plane

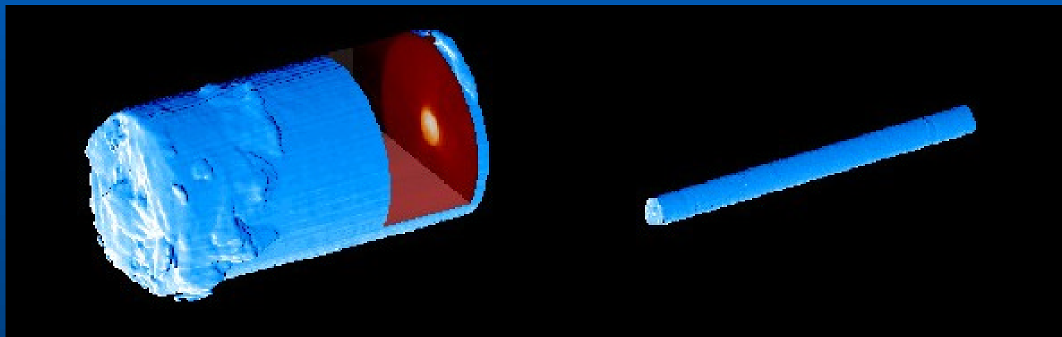
Coherent Field

Transport of Intensity Approach

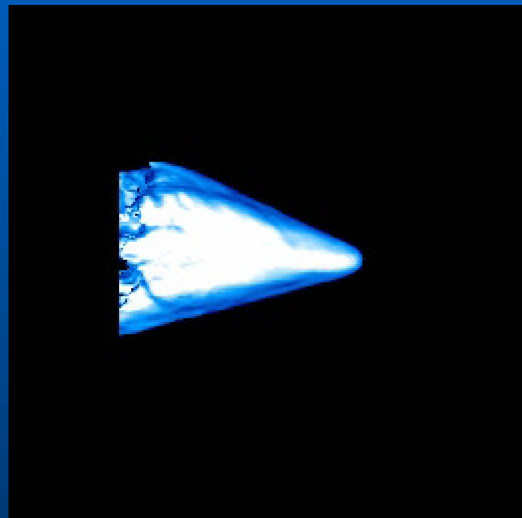


$$f(r) = -k_0 \nabla^{-2}\left[\nabla \bullet \left[\frac{1}{I(\boldsymbol{r})}\nabla\nabla^{-2}\left[\frac{\partial I(\boldsymbol{r})}{\partial z}\right]\right]\right]$$

Phase Recovery – Image Plane



McMahon PJ, Peele AG, Paterson D, Nugent KA, Snigirev A, Weitkamp T, Rau C, *X-ray tomographic imaging of the complex refractive index*, Applied Physics Letters, **83**, 1480-1482 (2003)

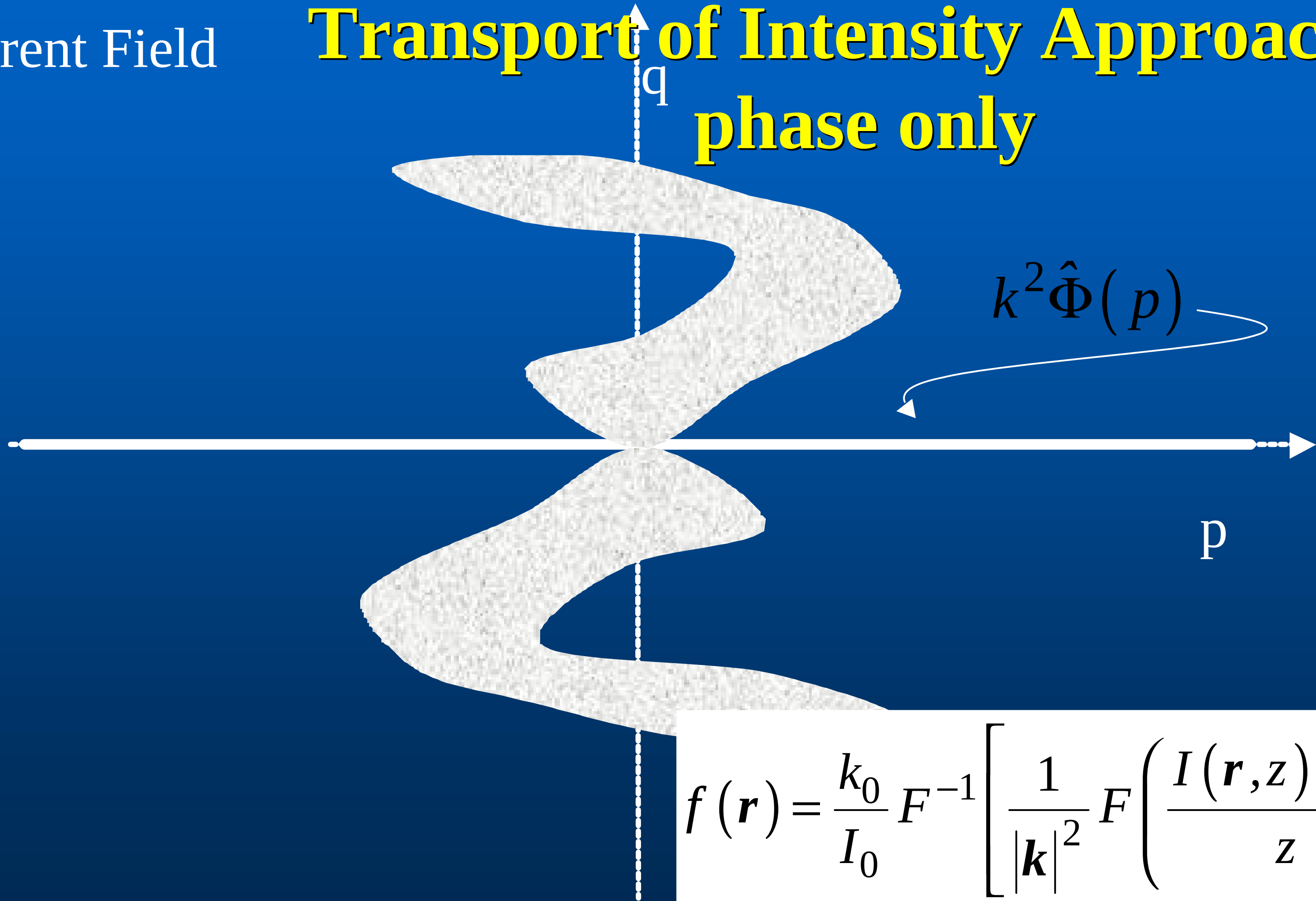


P. J. McMahon, A. G. Peele , D. Paterson, J. Lin, T. H. K Irving, I. McNulty and K. A. Nugent *Quantitative Sub-Micron Scale X-ray Phase Tomography*, Opt.Commun., **217**, 53-58 (2003)

Phase Recovery – Image Plane

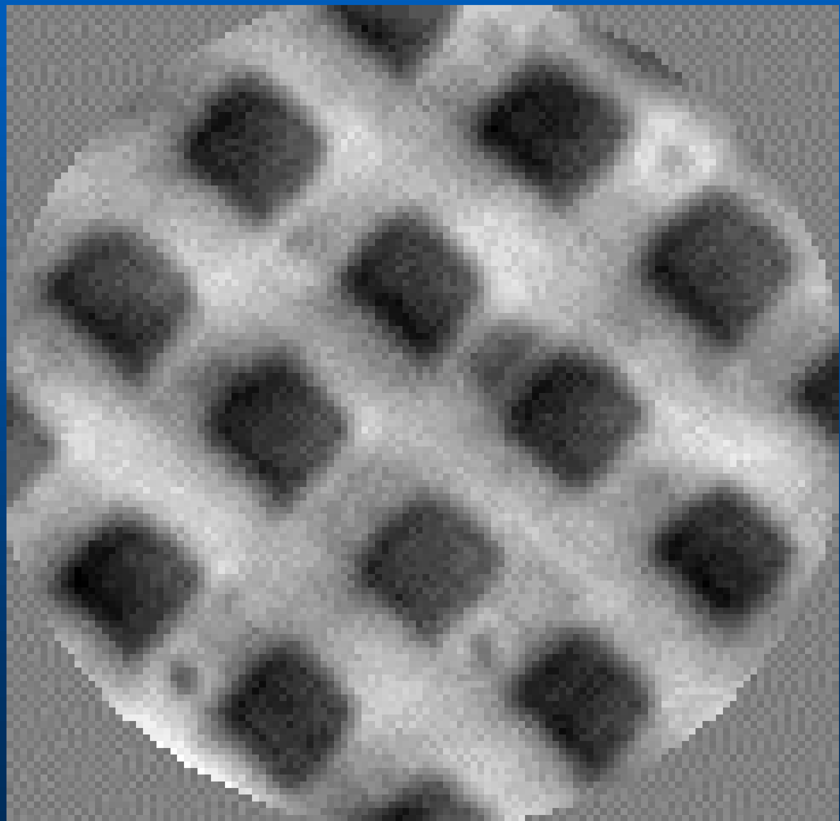
Coherent Field

Transport of Intensity Approach – phase only



$$f(\boldsymbol{r}) = \frac{k_0}{I_0} F^{-1}\left[\frac{1}{|\boldsymbol{k}|^2} F\left(\frac{I(\boldsymbol{r},z)-I_0}{z}\right)\right]$$

Hard X-ray Phase

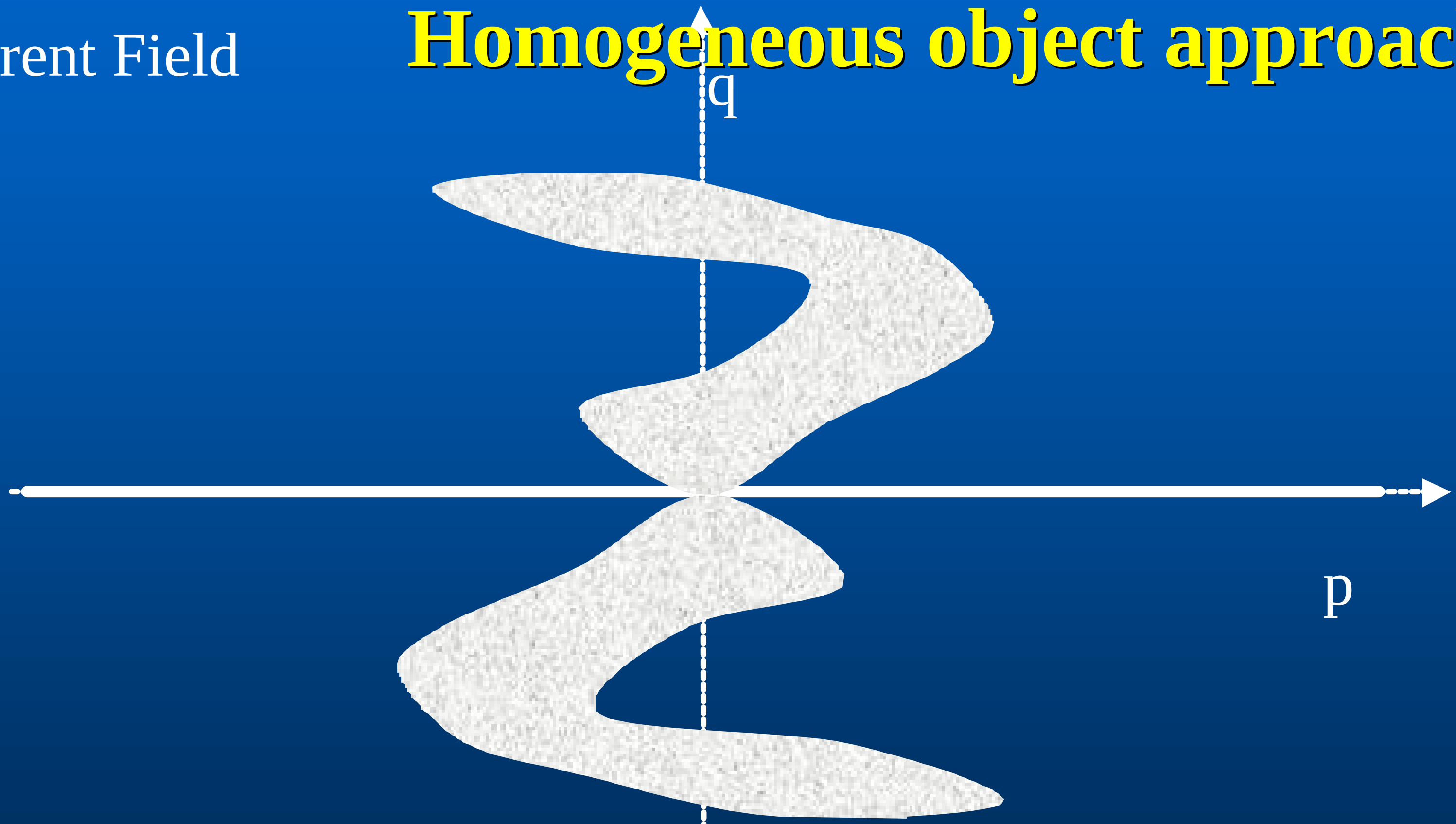


KA Nugent, T.E.Gureyev, D.F.Cookson, D.Paganin and Z.Barnea, *Quantitative Phase Imaging Using Hard X-Rays,* Phys.Rev.Letts, 77, 2961-2964 (1996)

Phase Recovery – Image Plane

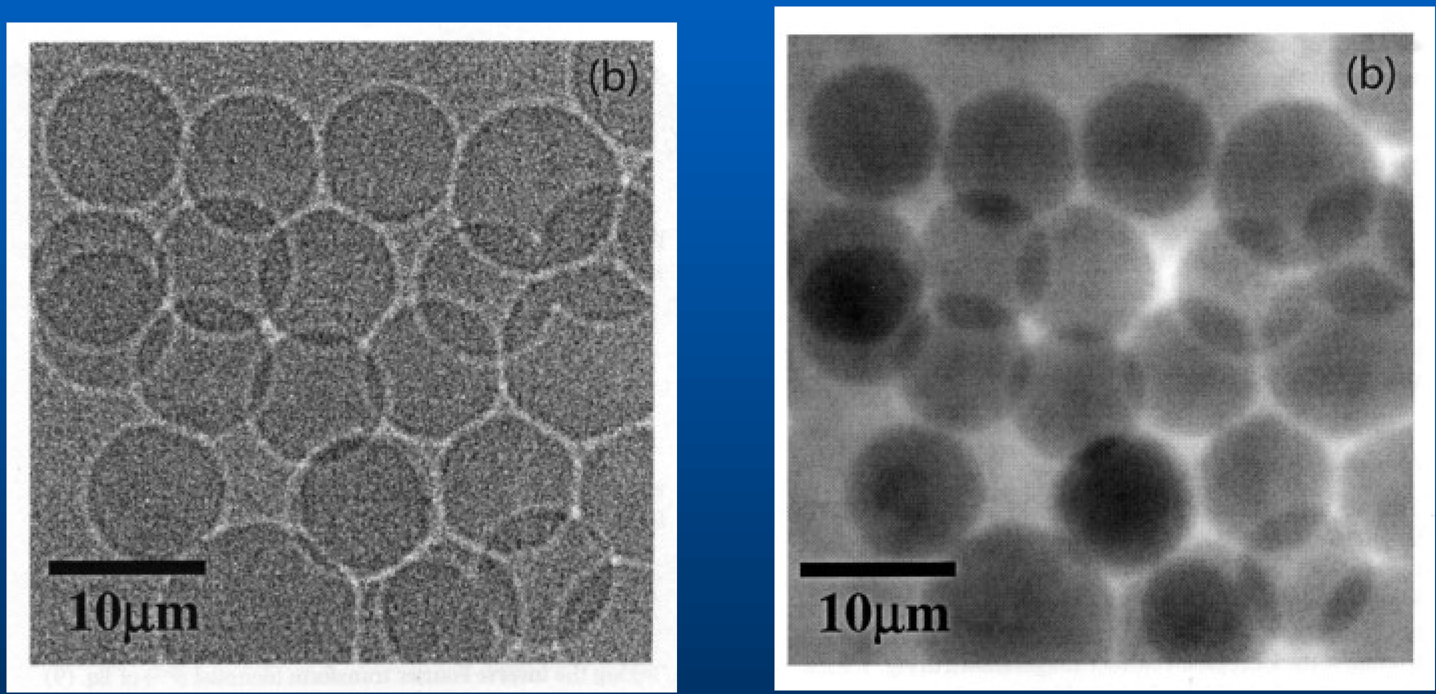
Coherent Field

Homogeneous object approach



$$t(r) = -\frac{1}{2k_0 \boldsymbol{b}} \ln F^{-1}\left[\frac{\boldsymbol{b}}{\boldsymbol{b} + dp\, l\, z\,|\boldsymbol{k}|^2} F\left[\frac{I(\boldsymbol{r})}{I_0}\right]\right]$$

Experimental Results



D. Paganin, S. C. Mayo, T. E. Gureyev, P. R. Miller and S. W. Wilkins, "Simultaneous phase and amplitude extraction from a single defocused image of a homogeneous object," J. Microscopy **206,** 33 – 40 (2001)

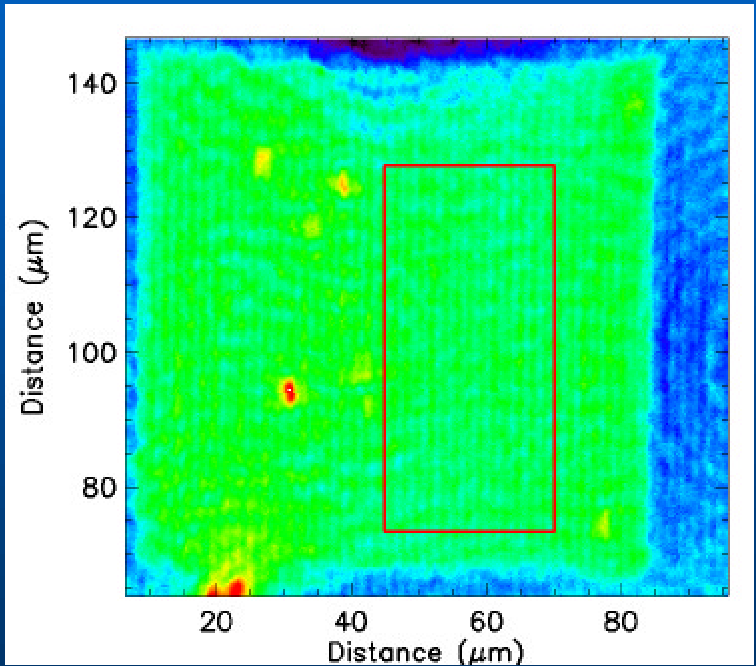
Phase Recovery – Image Plane

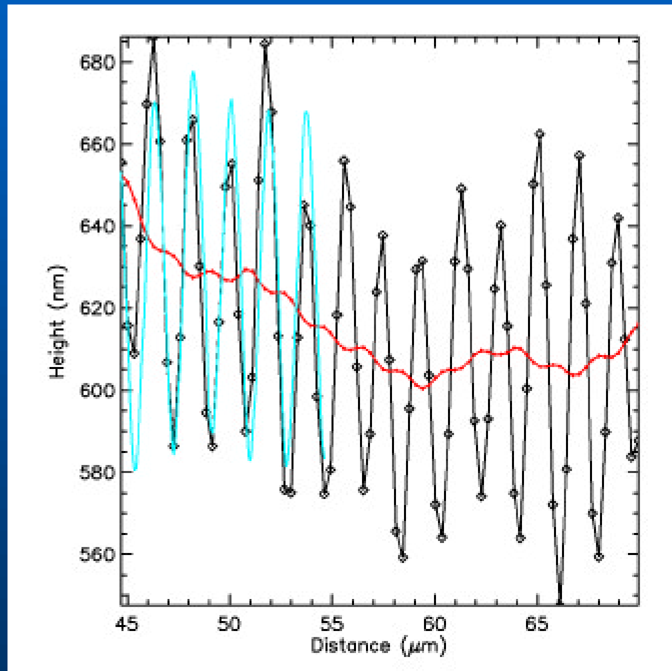
Coherent Field

Contrast Transfer Function Approach – weak phase/absorption

$$\tilde{I}(\mathbf{k},z) = |f_0|^2 \left\{ d(\mathbf{k}) - 2\cos\left(\pi \lambda z |\mathbf{k}|^2\right) \tilde{m}(\mathbf{k}) + 2\sin\left(\pi \lambda z |\mathbf{k}|^2\right) \tilde{f}(\mathbf{k}) \right\}$$

Experimental Results



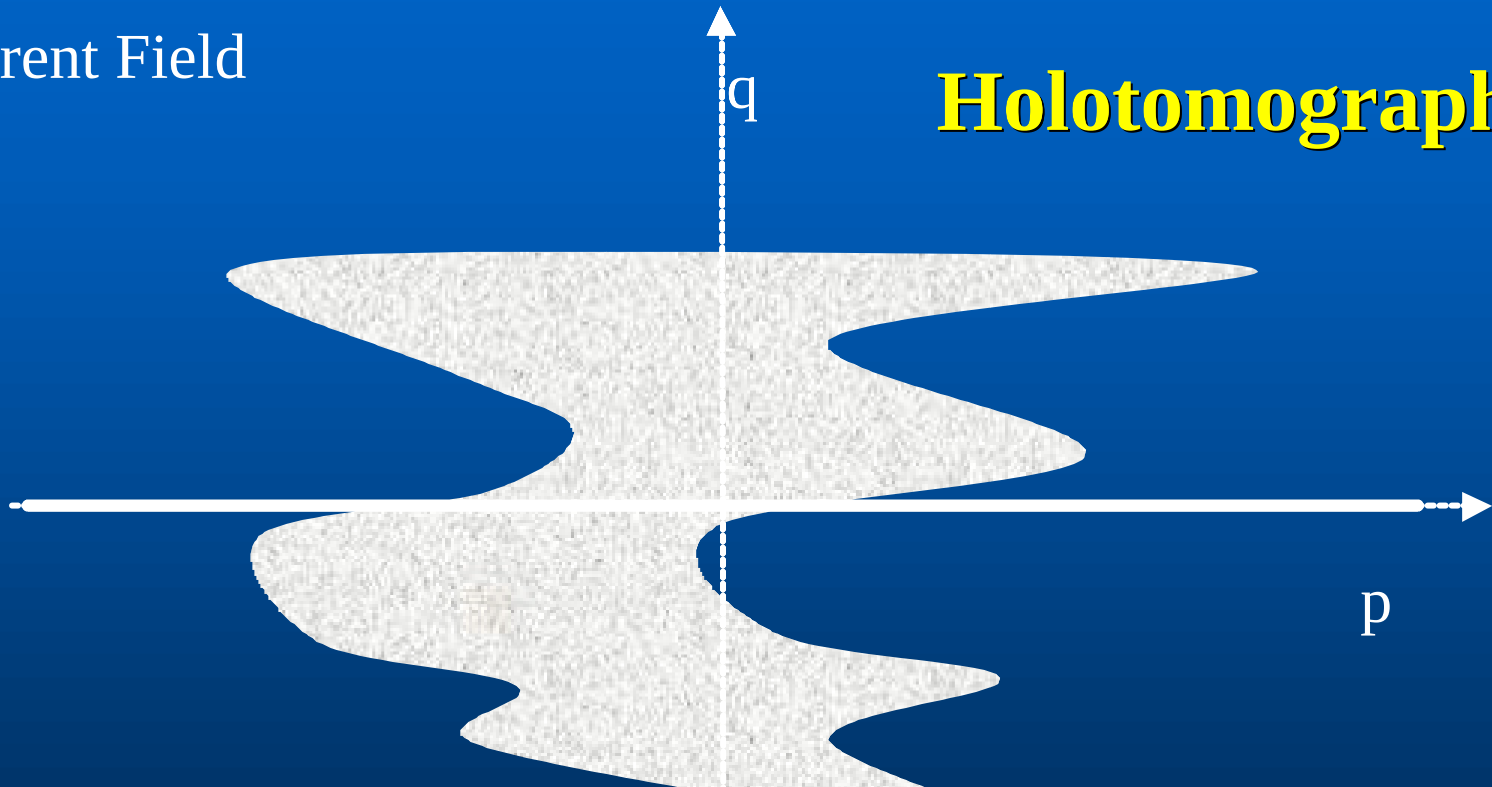


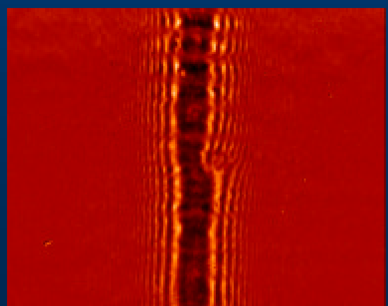
L. D. Turner, A. G. Peele, B. Dhal, A. P. Mancuso, R. E. Scholten, C. Q. Tran, K. A. Nugent, J. P. Hayes, D. Paterson, *X-ray phase imaging: Demonstration of extended conditions for homogeneous objects*, Optics Express, **12**, 2960-2965 (2004).

Phase Recovery – Image Plane

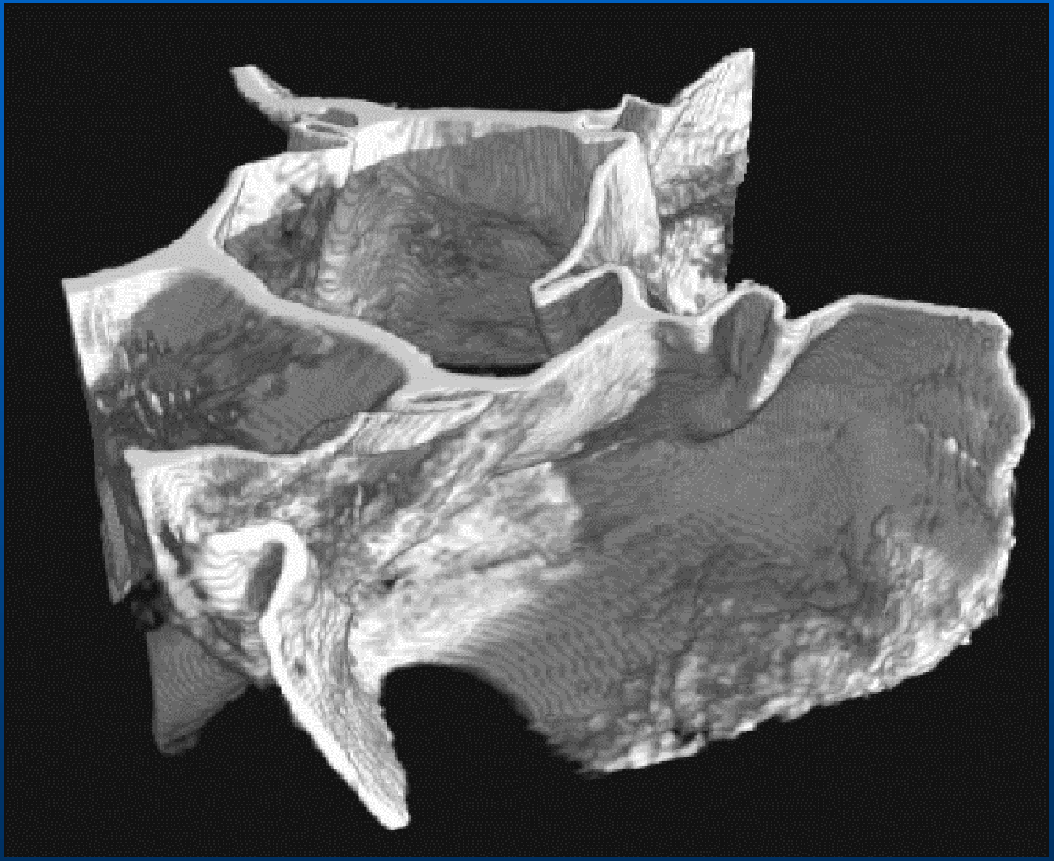
Coherent Field

Holotomography





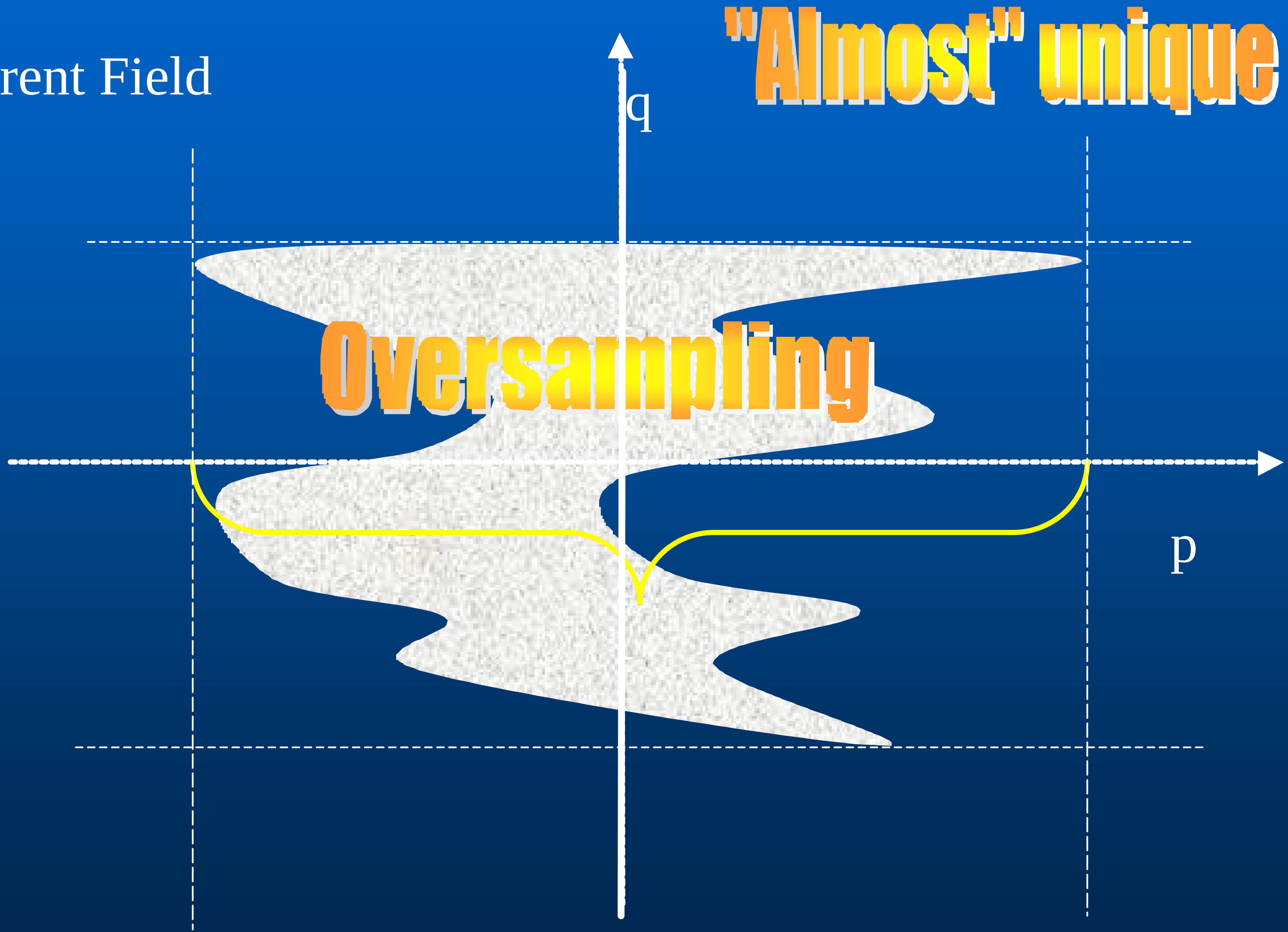
Holotomography - experimental



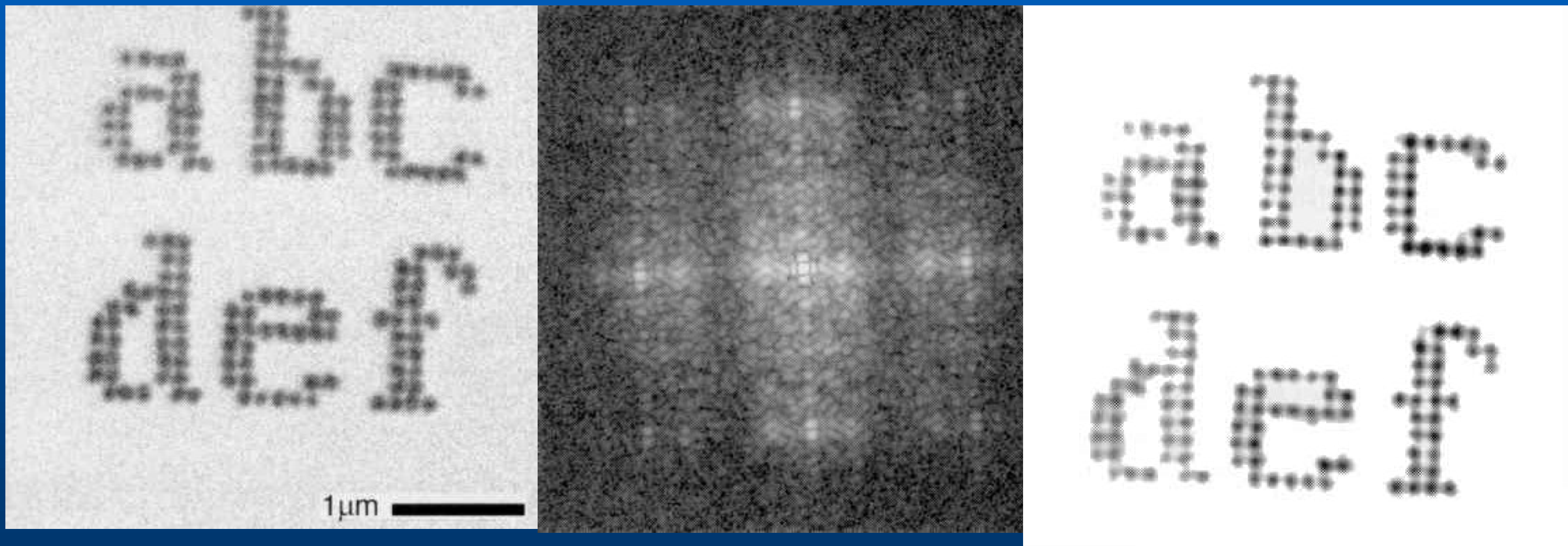
P.Cloetens, W.Ludwig, J.Baruchel, D.Van Dyck, J.Van Landuyt, J.P.Guigay and M.Schlenker, *Holotomography: Quantitative phase tomography with micrometer resolution using hard synchrotron radiation X-rays*, Appl.Phys.Lett. **75** 2912-2914 (1999).

Coherent Diffractive Imaging

Coherent Field



Experimental results

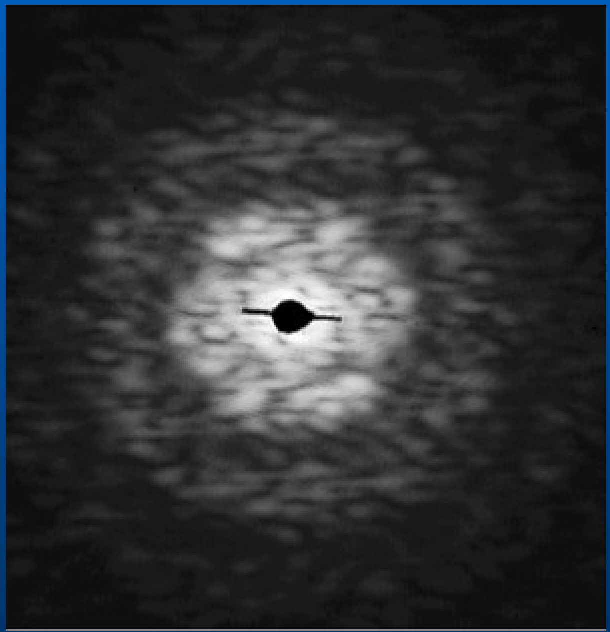


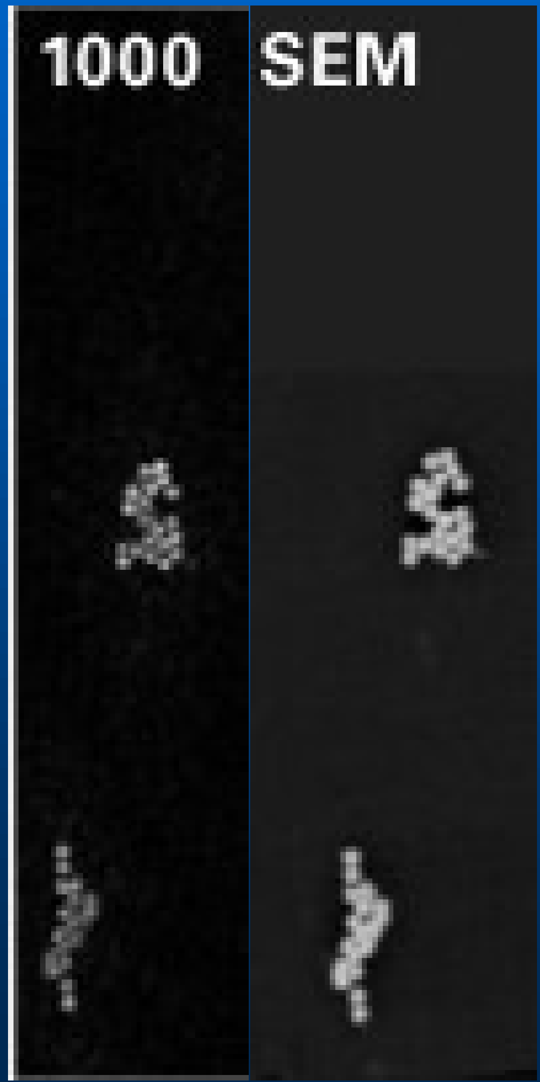
J.Miao, P.Charalambous, J.Kirz and D.Sayre, *Extending the methodology of X-ray crystallography to allow imaging of micrometer-sized non-crystalline specimens*, Nature, 342-344 (1999).

Experimental results

Williams GJ, Pfeifer MA, Vartanyants IA, Robinson IK *Three-dimensional imaging of microstructure in Au nanocrystals*, PHYSICAL REVIEW LETTERS 90, Art. No. 175501 (2003)

Soft x-ray demonstration



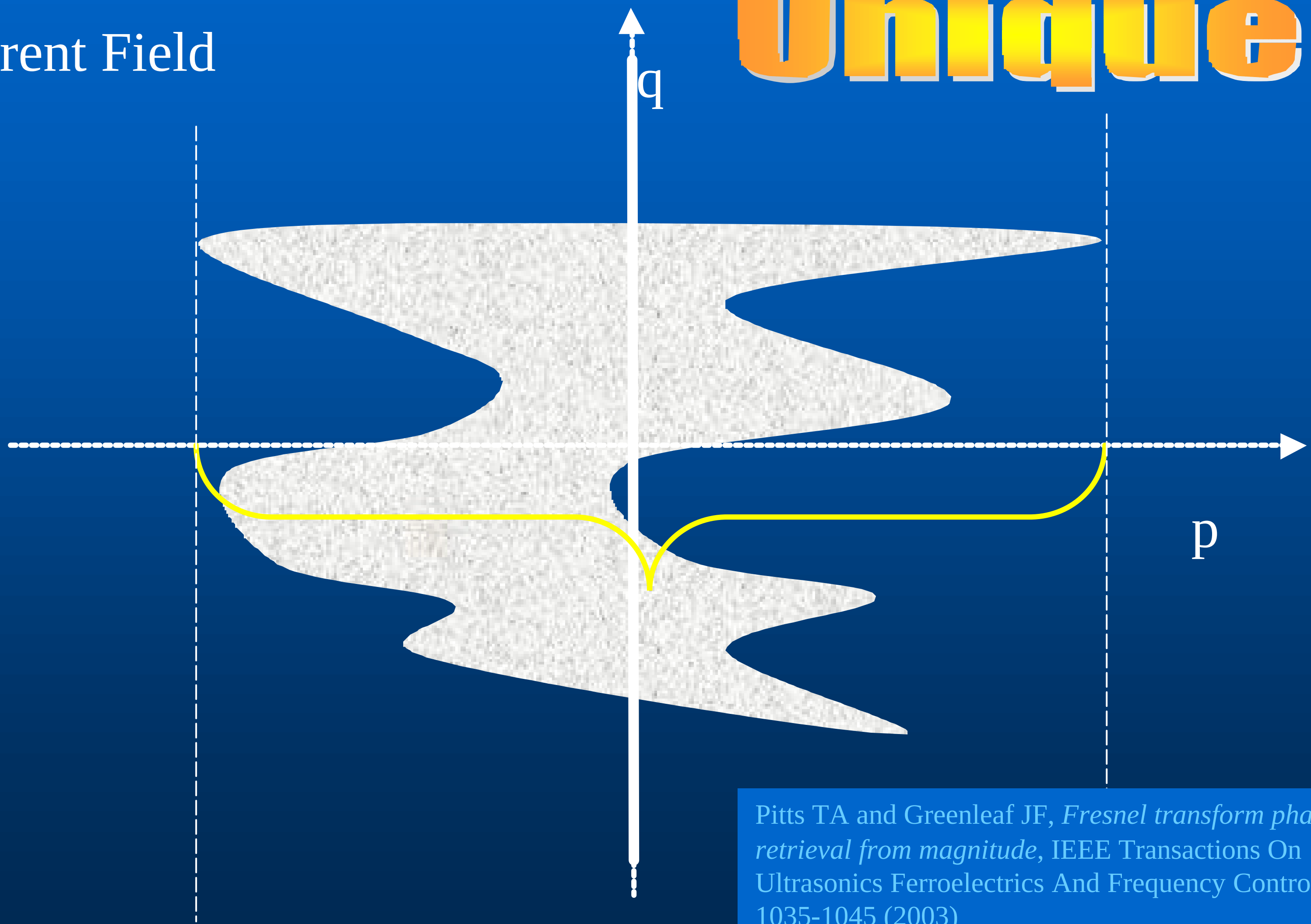


Marchesini S, He H, Chapman HN, Hau-Riege SP, Noy A, Howells MR, Weierstall U, Spence JCH *X-ray image reconstruction from a diffraction pattern alone*, PHYSICAL REVIEW B 68, Art. No. 140101 (2003)

Coherent Diffractive Imaging

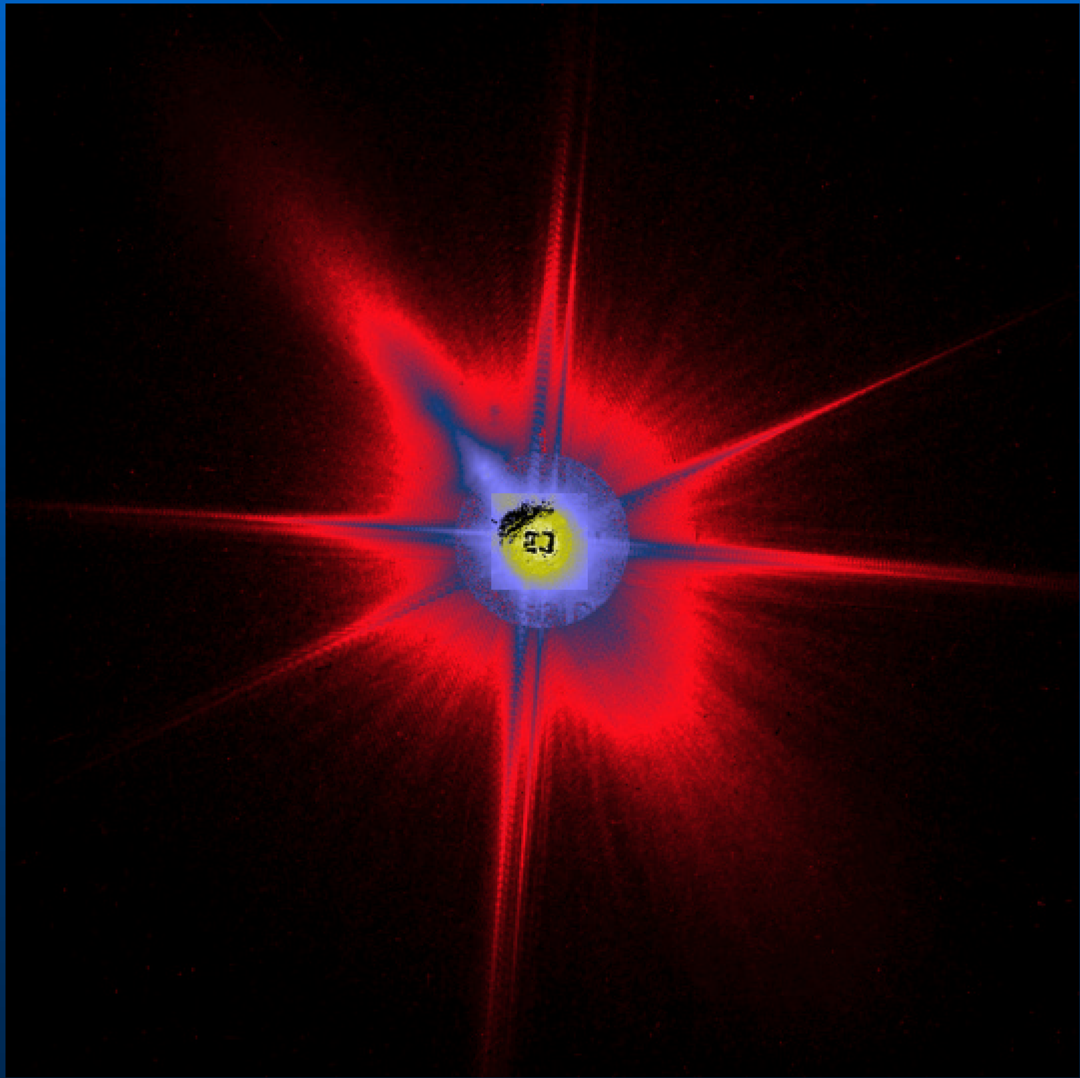
Coherent Field

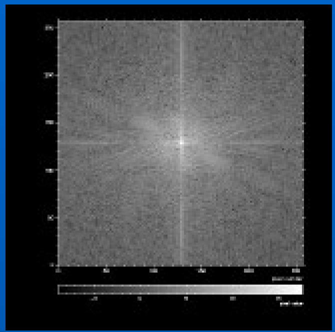
Unique

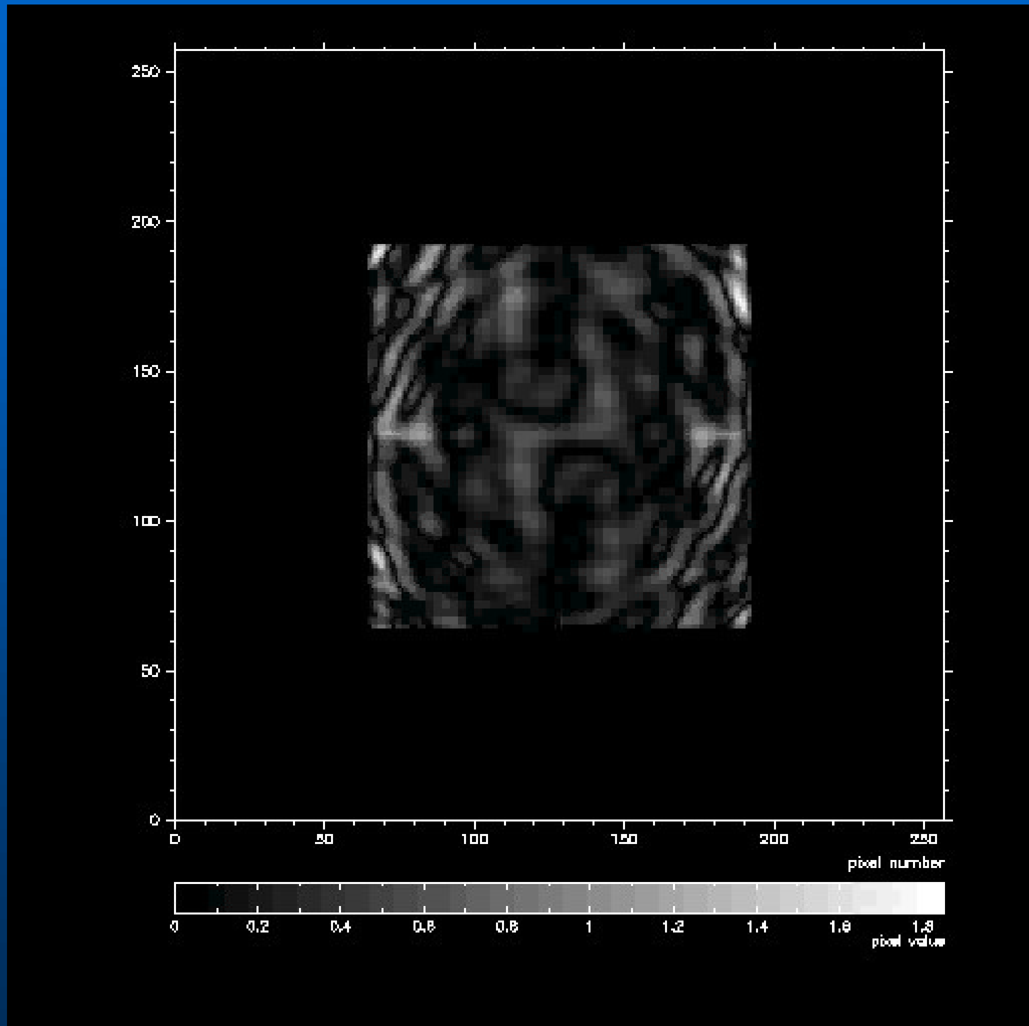


Pitts TA and Greenleaf JF, *Fresnel transform phase retrieval from magnitude*, IEEE Transactions On Ultrasonics Ferroelectrics And Frequency Control **50**, 1035-1045 (2003)

Fresnel diffraction imaging






250
200
150
100
50
0
0
50
100
150
200
250
pixel number
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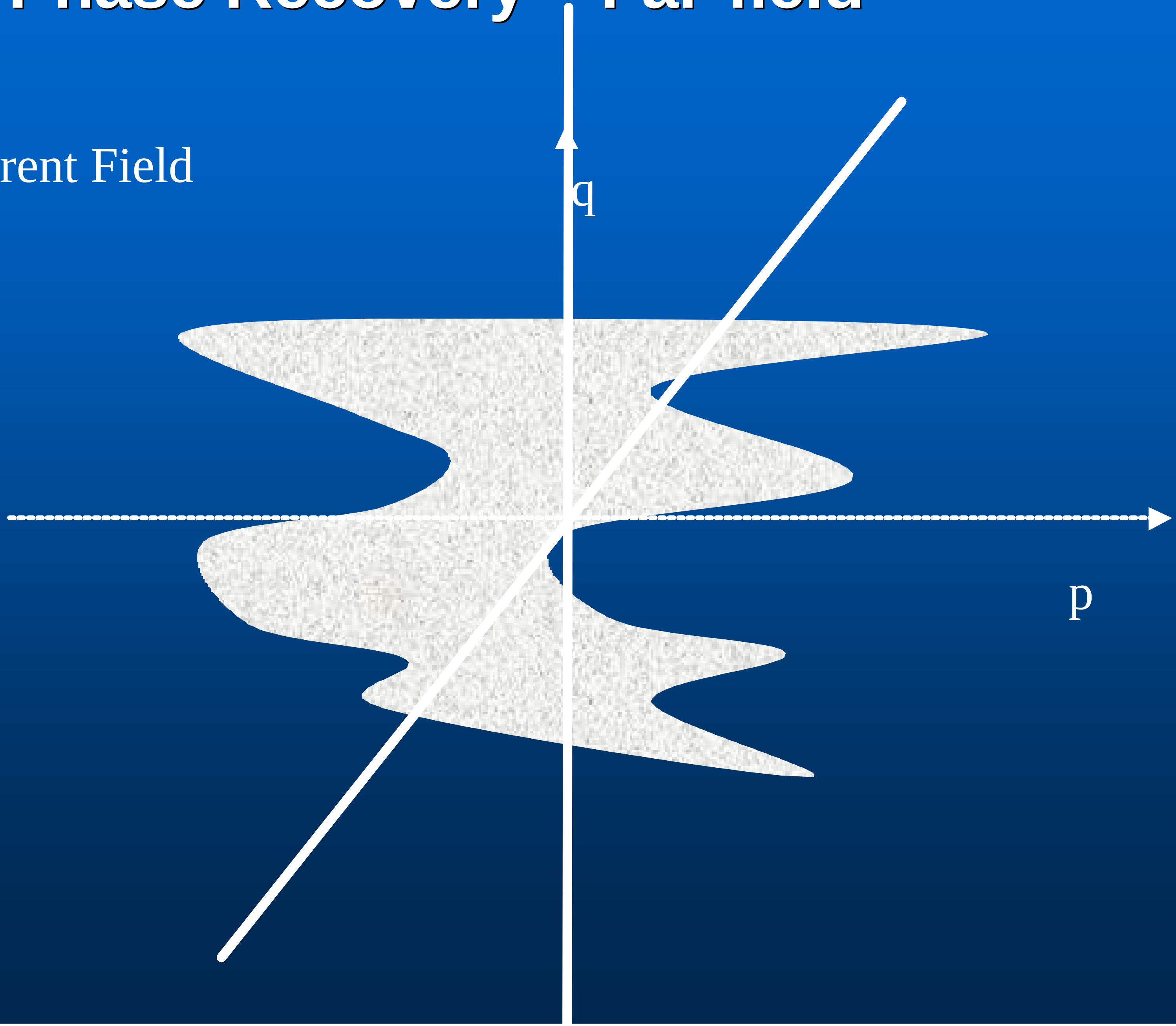
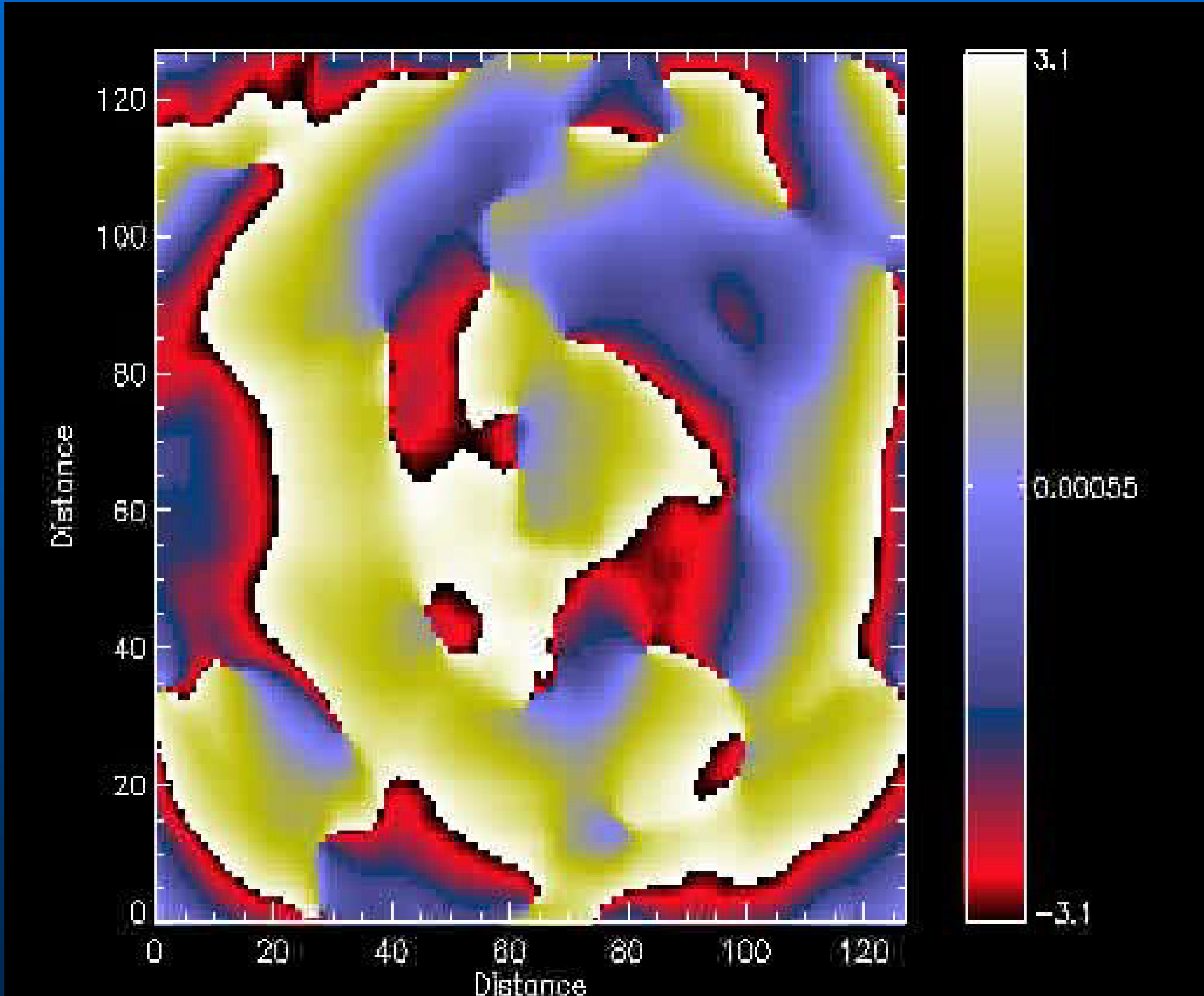
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
pixel value

Phase Recovery – Far-field

Coherent Field



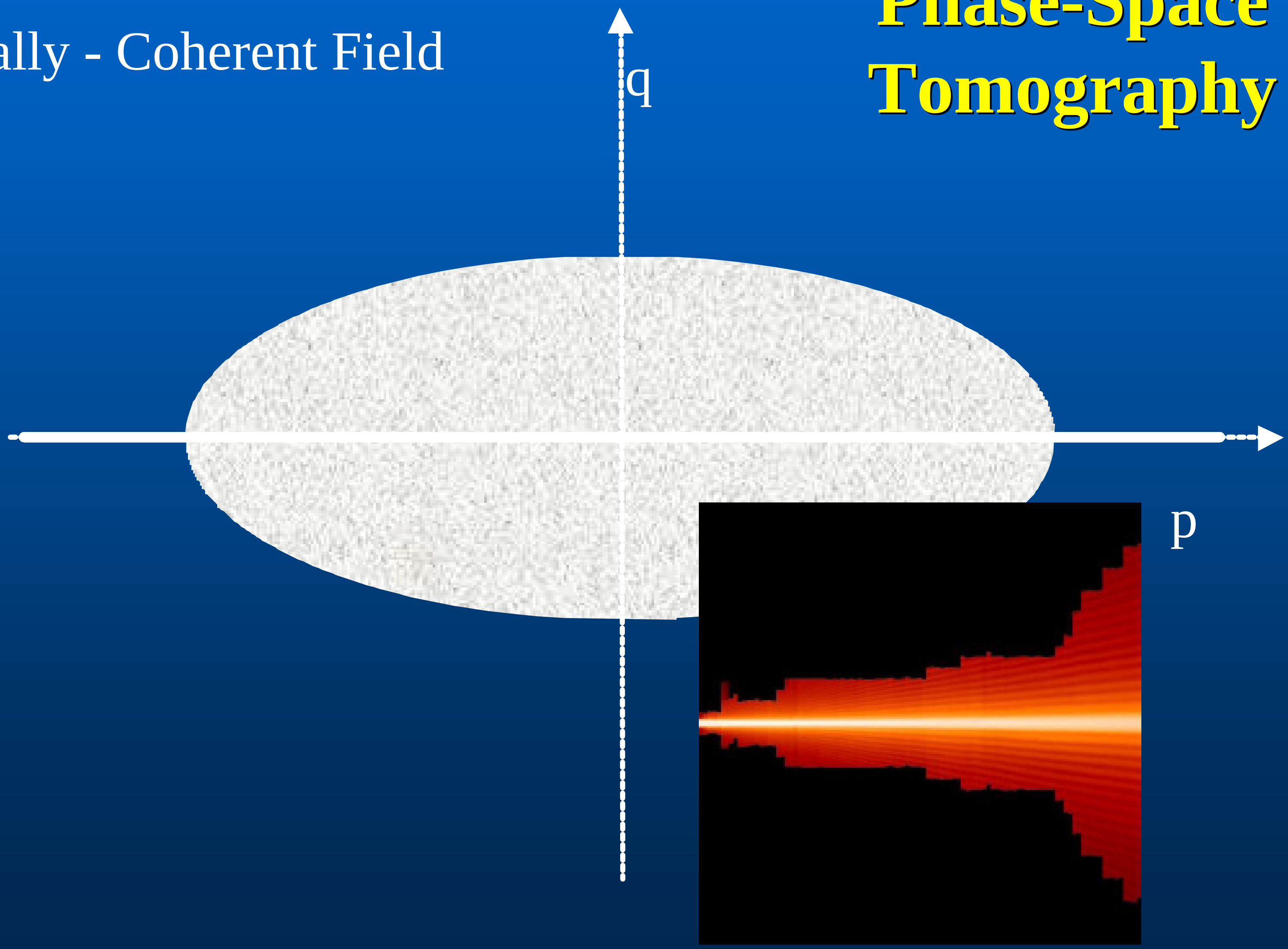


Distance
120
100
80
60
40
20
0
0
20
40
60
80
100
120
Distance
3.1
0.00055
-3.1

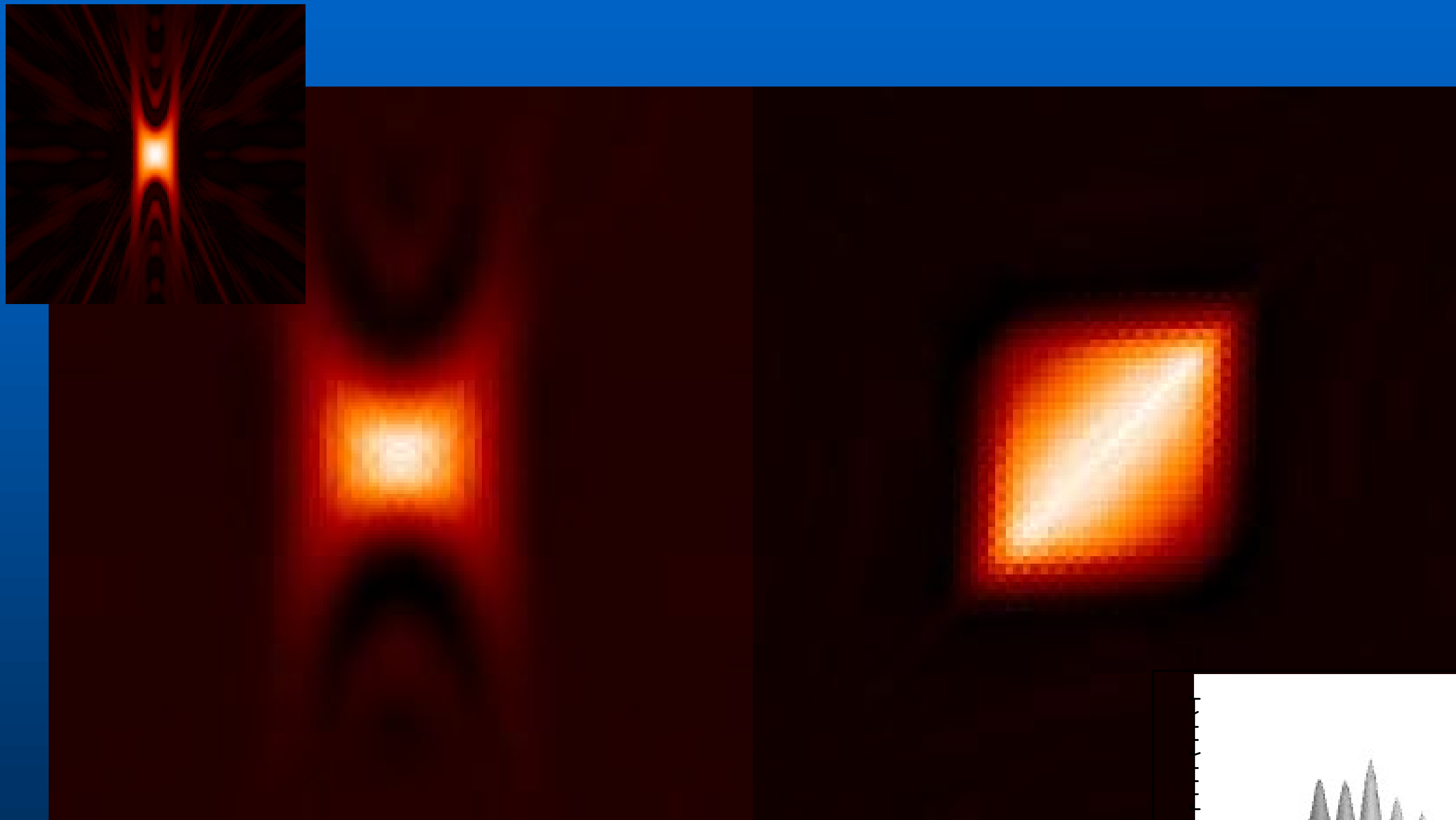
Partially coherent imaging

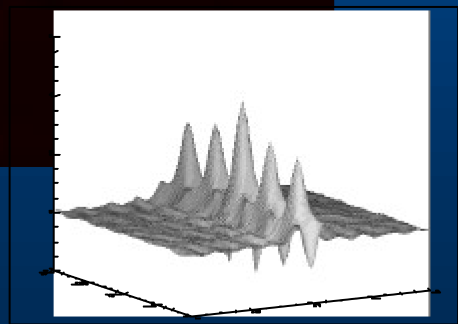
Partially - Coherent Field

Phase-Space Tomography



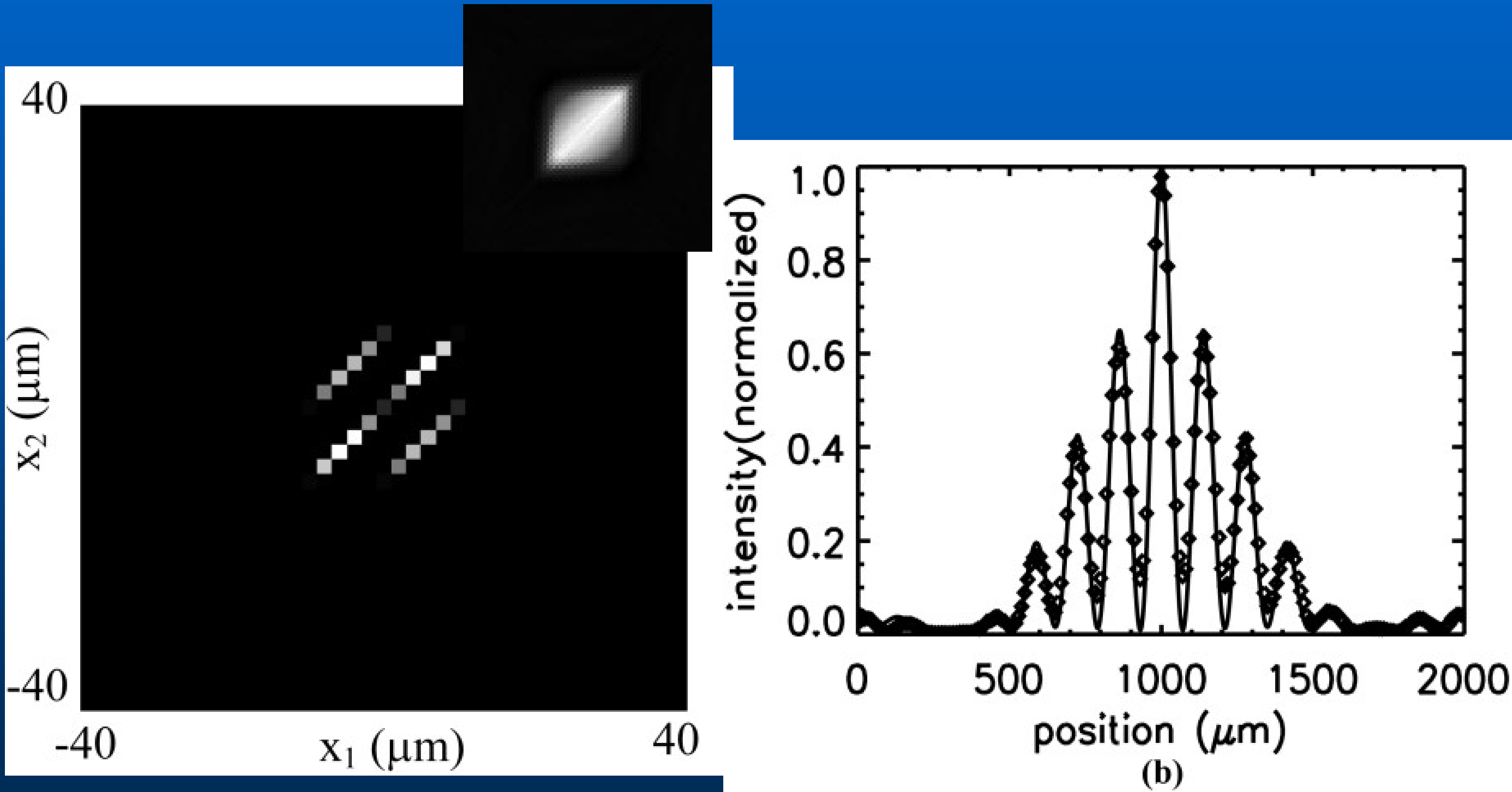
Recovered Coherence Function





C.Q. Tran, A.G. Peele, D. Paterson, A. Roberts, I. McNulty and K.A. Nugent, *Synchrotron Beam Coherence Measured using Phase-Space Tomography*, Optics Letters, **30**, 204-206 (2005).

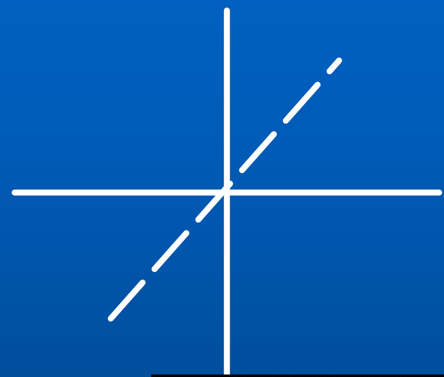
Extraction of coherent information

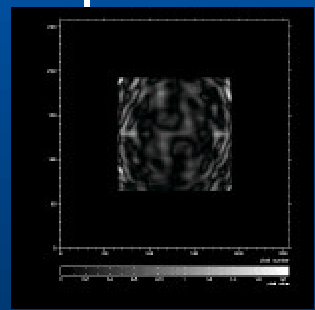


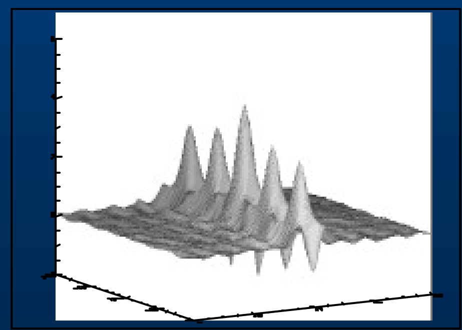
Chanh Q.Tran, Andrew G.Peele, Ann Roberts, Keith A.Nugent David Paterson and Ian McNulty *X-Ray Imaging: A Generalised Approach Using Phase Space Tomography*, J.Opt.Soc.Am. A., in press

Summary

- Phase recovery may be consistently understood from a phase-space perspective.
- Methods may be classified in terms of which projections are measured.
- Far-field measurements need not also be Fraunhofer measurements, and this allows a greater amount of information to be acquired
- This perspective meshes neatly with partially coherent measurements







Collaborators

- **Harry Quiney (UM)**
- **Andrew Peele (La Trobe)**
- **Adrian Mancuso (UM)**
- **Chanh Tran (UM)**
- **Bipin Dhal (UM)**
- **Ann Roberts (UM)**
- **David Paganin (now @ Monash U)**

- **David Paterson (now @ APS)**
- **Ian McNulty (APS)**
- **Barry Lai (APS)**
- **Zhonghou Cai (APS)**
- **Henry Chapman (LLNL)**