

Phase retrieval in tomography with Kirkpatrick-Baez mirrors

Rajmund Mokso

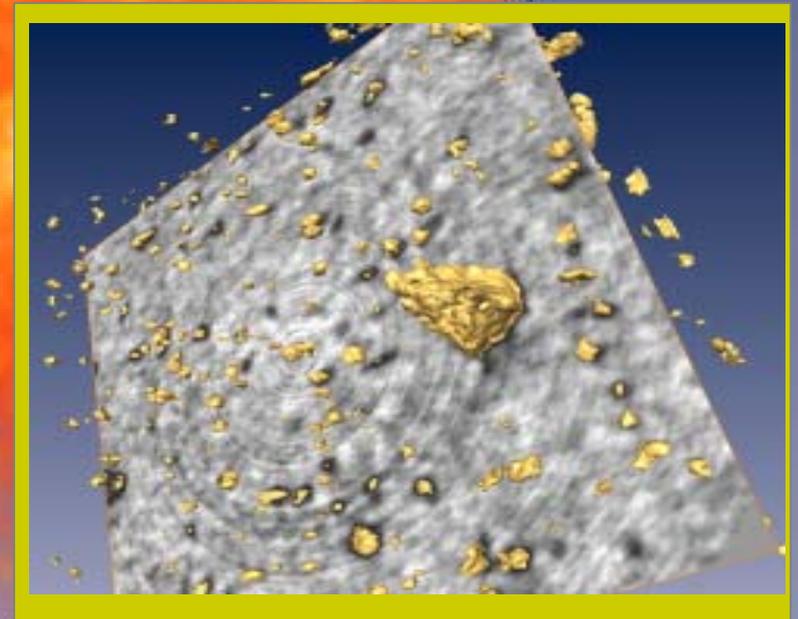
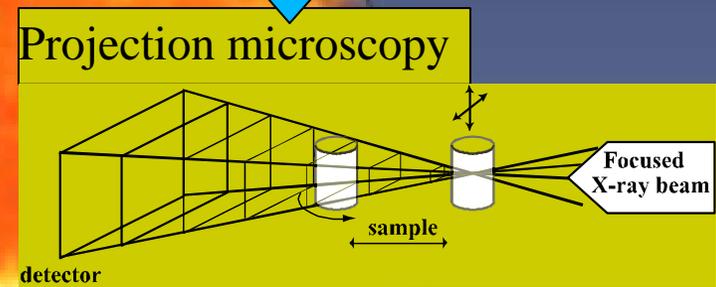
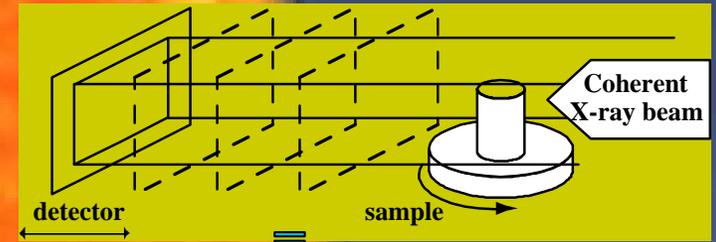
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Motivation

Improve the spatial resolution of tomographic scans

adapt phase retrieval algorithms to the focusing geometry



Outline

the geometry of
the imaging
setup

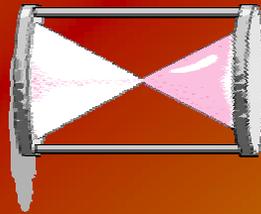
the nature of
artefacts

outlook

**HIGH SPATIAL RESOLUTION
(KB FOCUSING)**

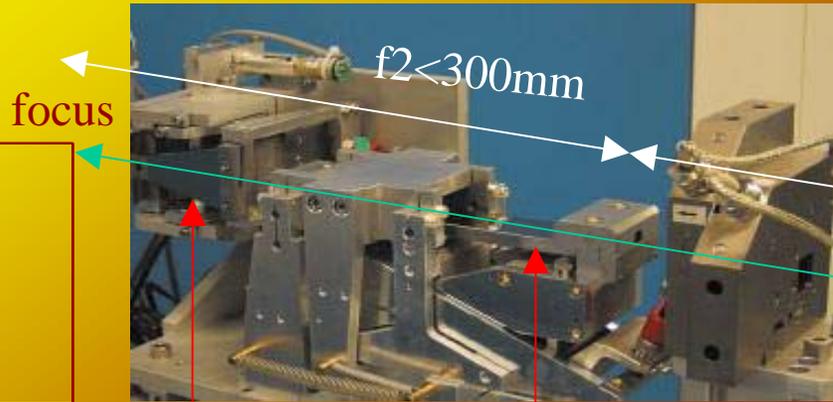
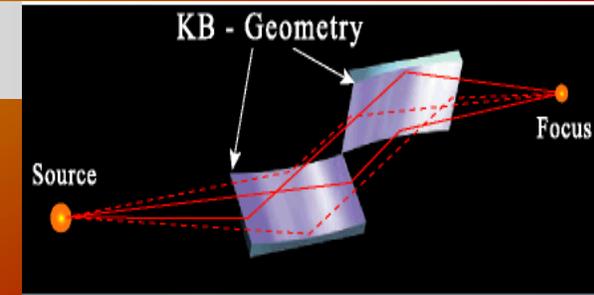
towards local
tomography

examples of
results (3D
rendering)



description of different phase retrieval
approaches and their comparison on
real data

THE GEOMETRY



demagnification:
 $f_1/f_2 \Rightarrow$
 theoretical best focus
 spot = $87 \times 45 \text{ nm}$

$f_1 = 145 \text{ m}$

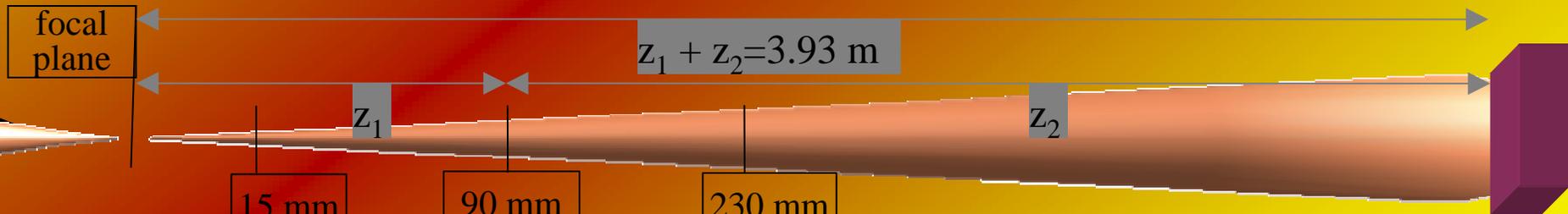
source



horizontally
focusing mirror

vertically
focusing mirror
with multilayer

dose efficient
 overcome resolution of detector
 compatible with fluorescence



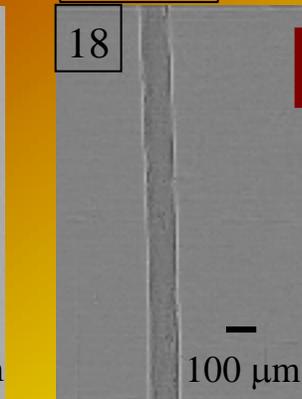
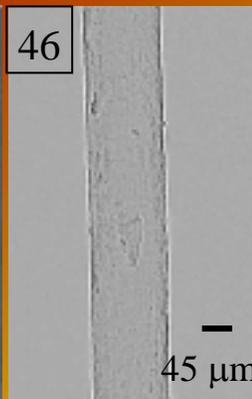
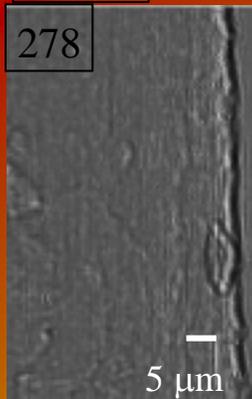
Magnification:

278

46

18

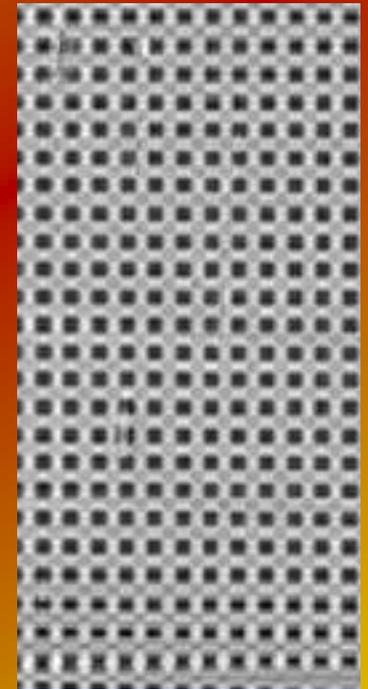
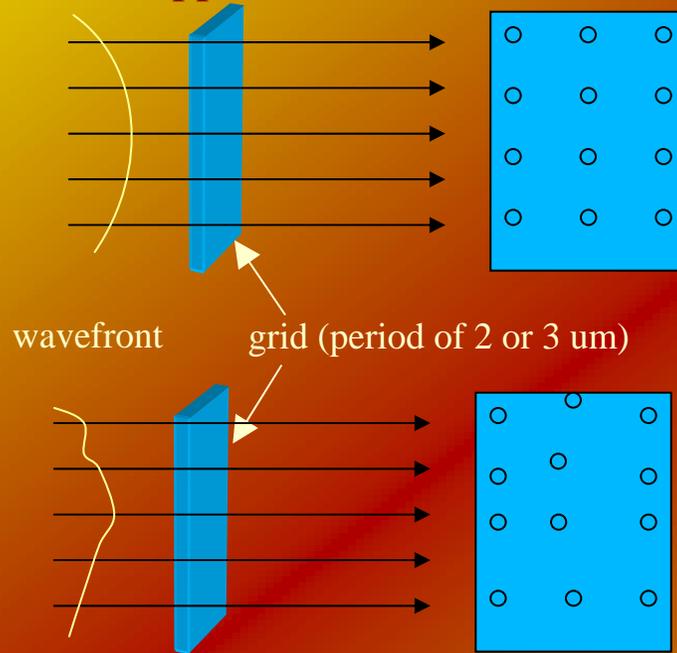
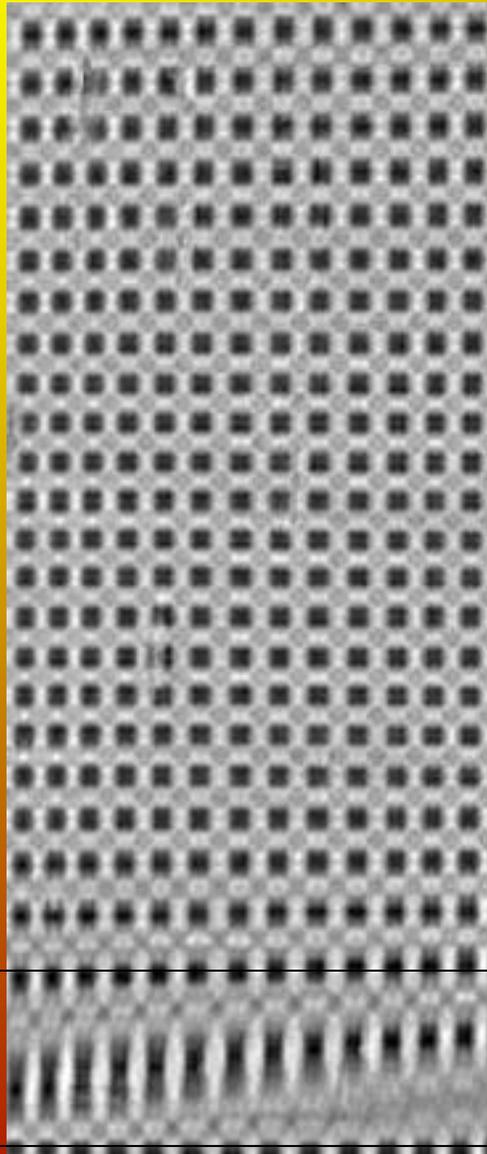
human hair



2 degrees of freedom:
 - distance from focal plane
 - sample rotation

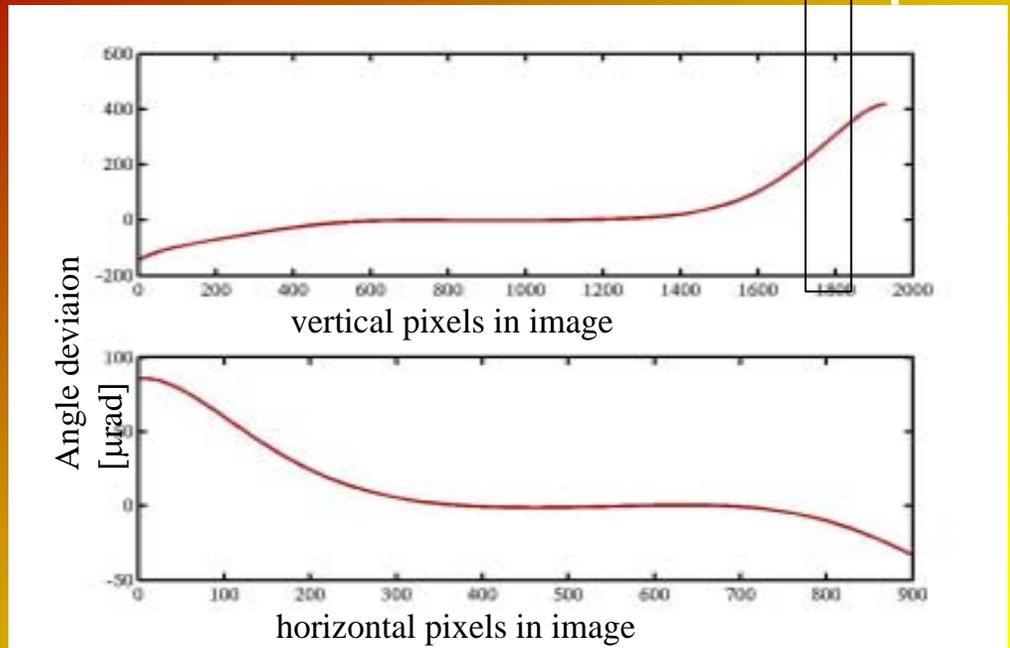
THE DISTORTION

determination of wavefront aberrations caused by mirror bending defects (geometrical approximation)

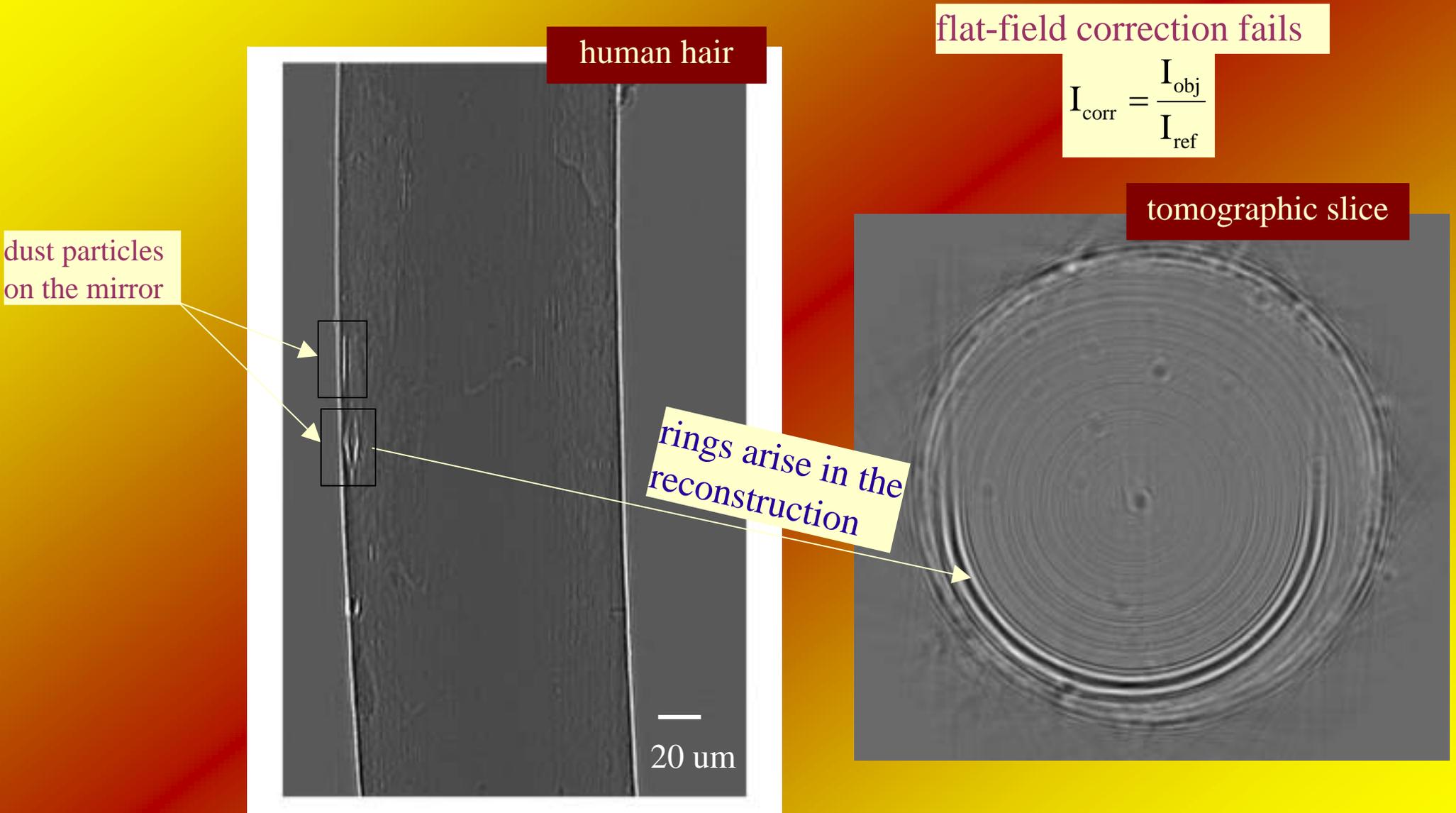


relation between the phase and the angle:

$$\alpha = (\lambda/2) \cdot \delta\phi / \delta z$$



refraction by the sample

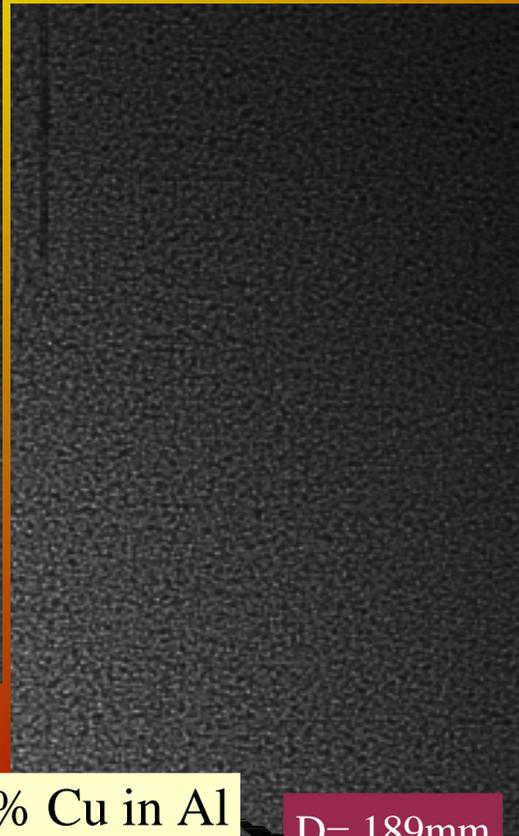


solution: **LOCAL TOMOGRAPHY**

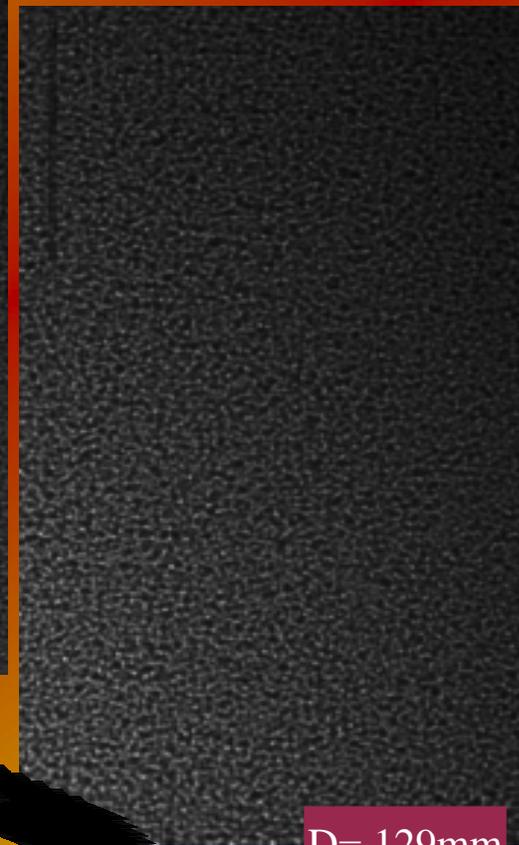
Local tomography



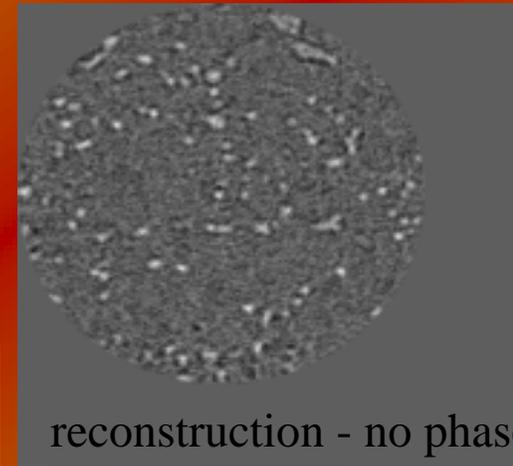
D= 269mm



D= 189mm



D= 129mm



reconstruction - no phase
retrieval



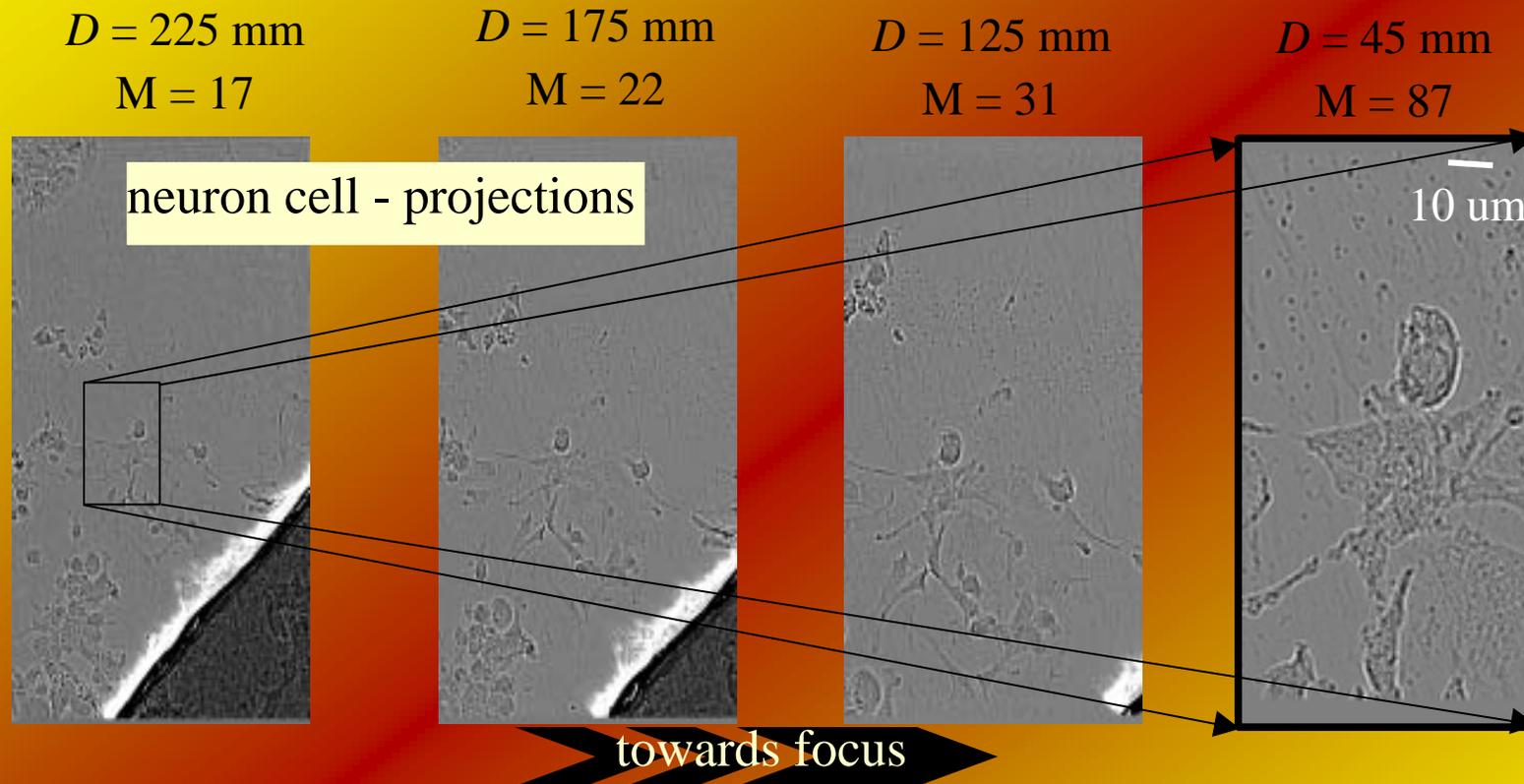
20 um

D= 71mm

cylindrical sample of 7% Cu in Al
much larger than the field of view

towards focus

the projections - preprocessing of data

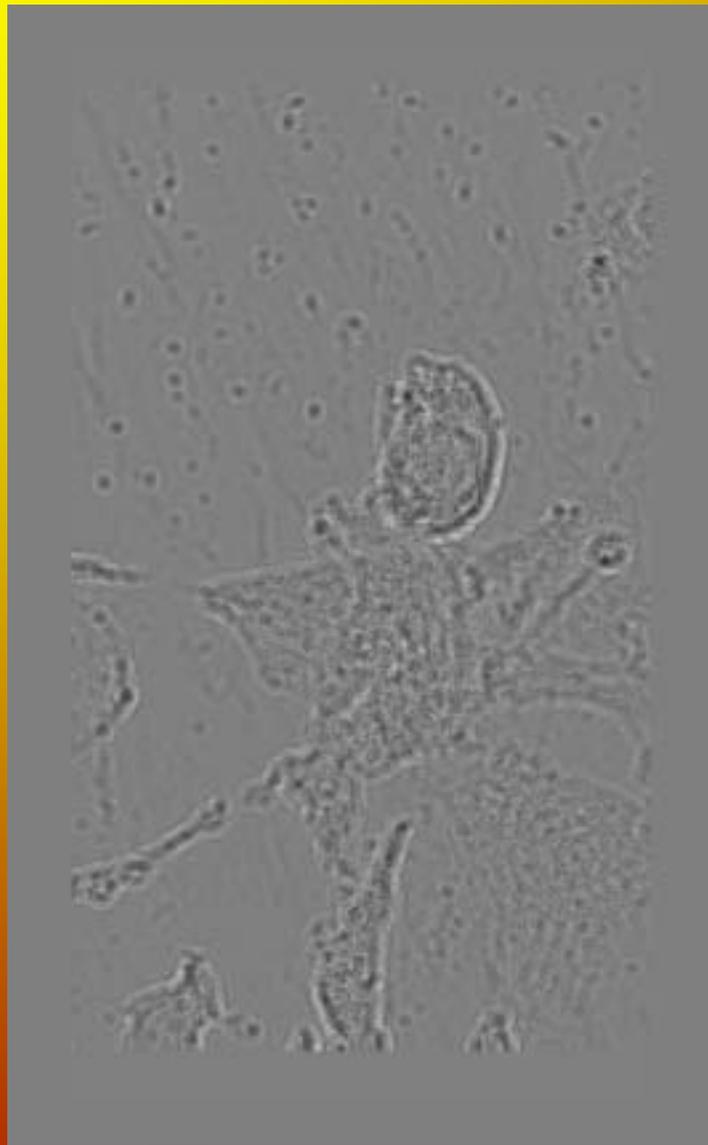


The equivalences between
parallel and cone beam

$$D = \frac{z_1 \cdot z_2}{z_1 + z_2}$$

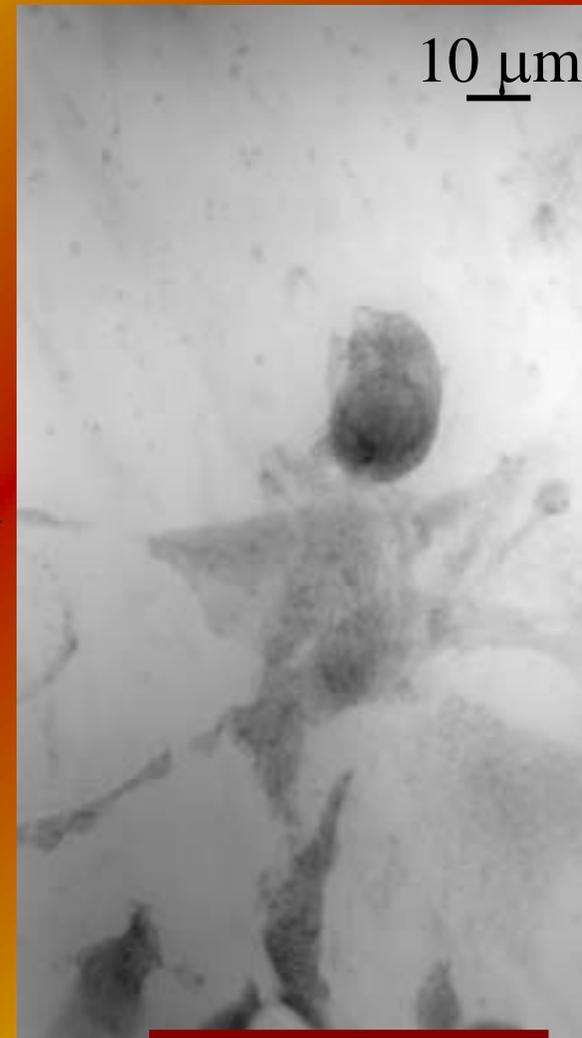
$$M = \frac{z_1 + z_2}{z_1}$$

Rel. phase map from magnified fresnel diffraction patterns



$D = 115\text{mm}$

5 dist. →



relative phase map

Rel. phase map from magnified fresnel diffraction patterns brought to same magnification

phase retrieval

The cost functional

$$C_s = \sum_d \left| \tilde{I}_d^{\text{exp}}(f) - \tilde{I}_d^{\text{calc}}(\tilde{T}(f)) \right|^2$$

Direct linear methods
(filtering)

slowly-varying
phase

weak object

paraboloid method

linearization with
respect to:

$\tilde{\phi}$

$\tilde{\phi}, \tilde{B}$

\tilde{T}

Iterative method:

no restrictions on
object

Steepest descent to minimize
the cost functional

Slowly-varying phase

the approximations

$$T(\mathbf{x}) = e^{i\varphi(\mathbf{x})} \quad \& \quad \underbrace{|\varphi(\eta) - \varphi(\eta - \lambda Df)|}_{\lambda Df} \ll 1 \quad \forall \eta$$

the basic formula

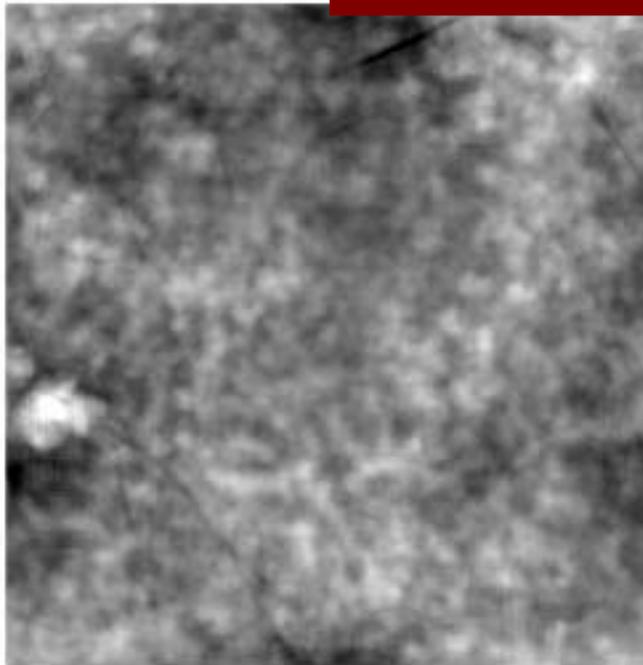
$$\tilde{I}_D(f) \approx \underbrace{\delta(f)}_{\text{FT of intensity}} + \underbrace{2\sin(\pi\lambda Df^2)}_{\text{contrast factor}} \cdot \underbrace{\tilde{\varphi}(f)}_{\text{FT phase}}$$

FT of intensity

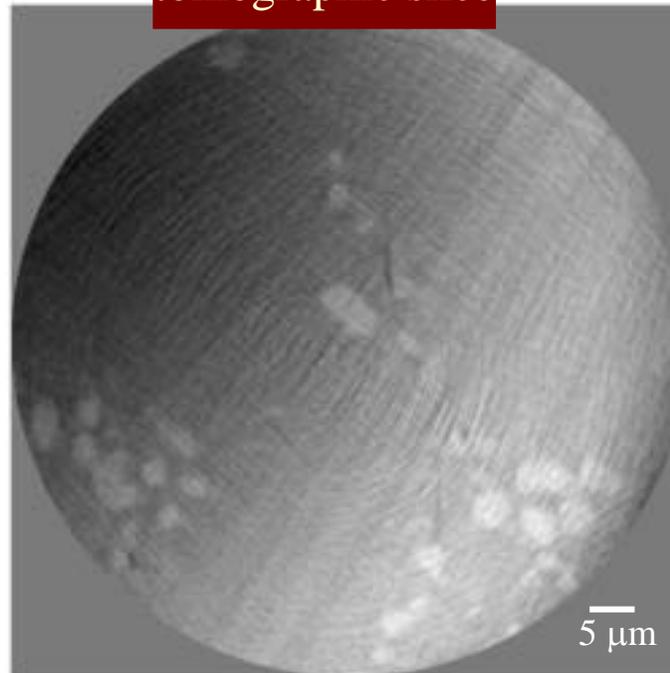
contrast factor

FT phase

phase map



tomographic slice



5 distances
900 projections

Weak object approximation

the approximations

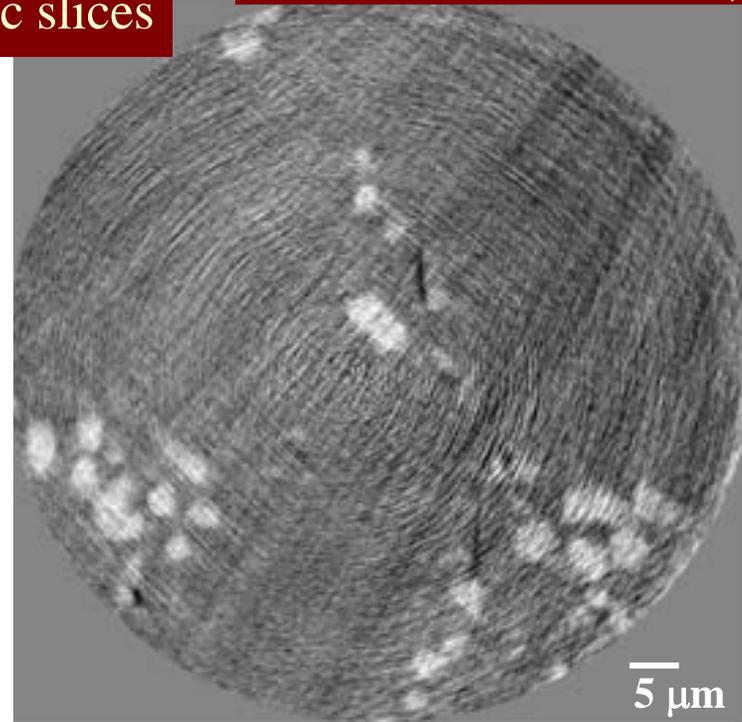
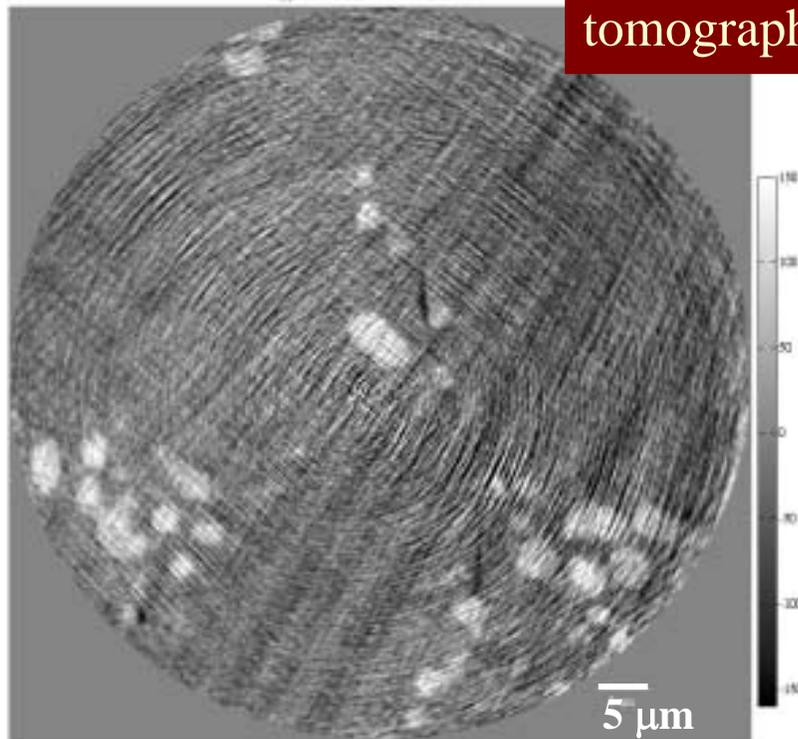
$$T = e^{-B(x)} \cdot e^{i\varphi(x)}, B(x) \ll 1$$

the basic formula

$$\tilde{I}_D(f) \approx \delta(f) + \underbrace{2 \sin(\pi\lambda Df^2)}_{\text{phase contrast factor}} \cdot \tilde{\varphi}(f) - \underbrace{2 \cos(\pi\lambda Df^2)}_{\text{absorption contrast factor}} \cdot \tilde{B}(f)$$

tomographic slices

5 iterations with calculated φ as initial guess



paraboloid method

the approximations

neglect the non-linear contributions to the intensity

the basic formulas

decomposition of the object: $T(x) = \alpha + \psi(x)$

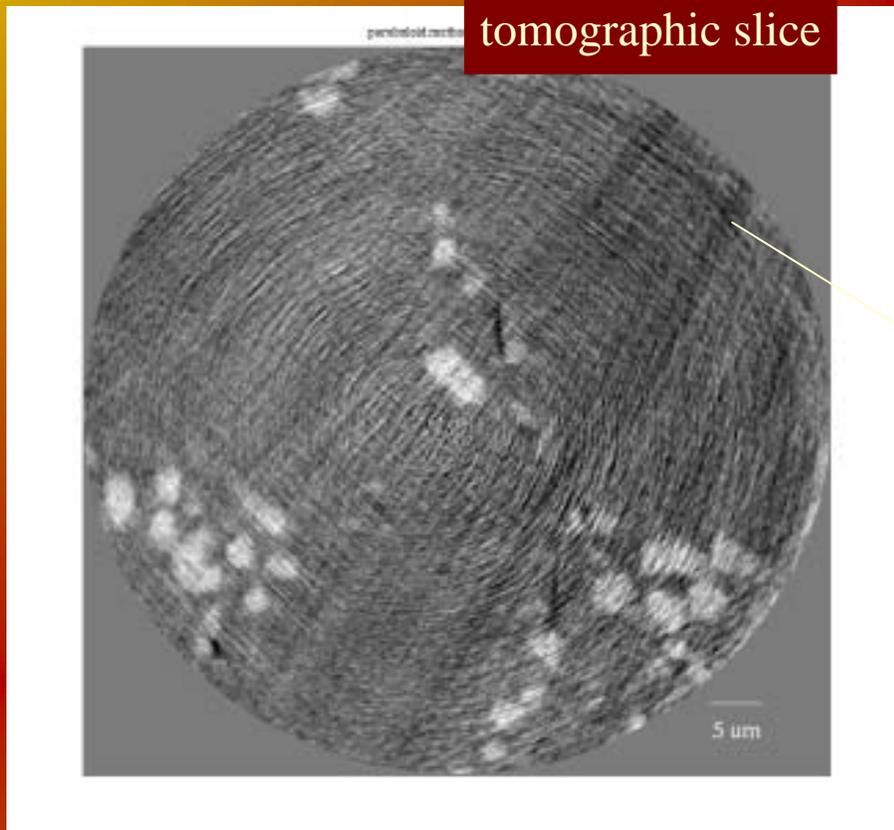
$$= \alpha^2 \cdot \delta(f) + \exp(-i\pi\lambda Df^2) \cdot \alpha + \exp(i\pi\lambda Df^2) \cdot \alpha + N.L.$$

Fourier transform
intensity

FT object

FT c.c. object

non-linear
terms



large objects enter
the field of view

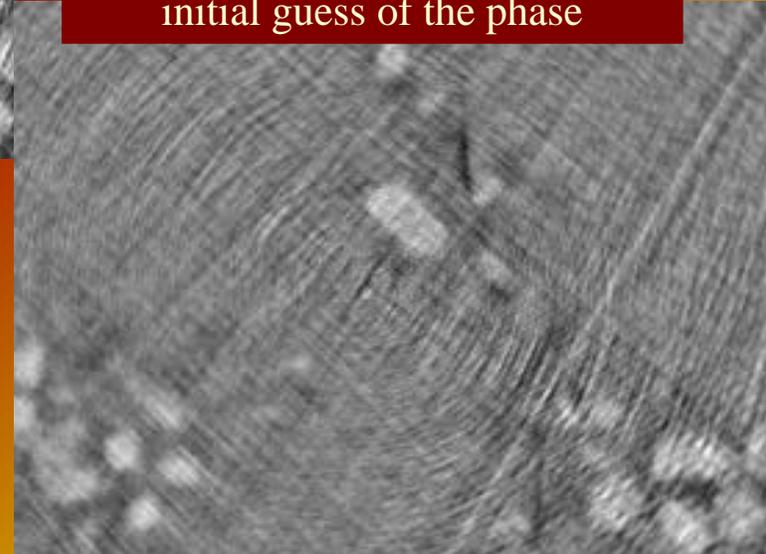
•phase unwrapping needed

comparing the different approaches

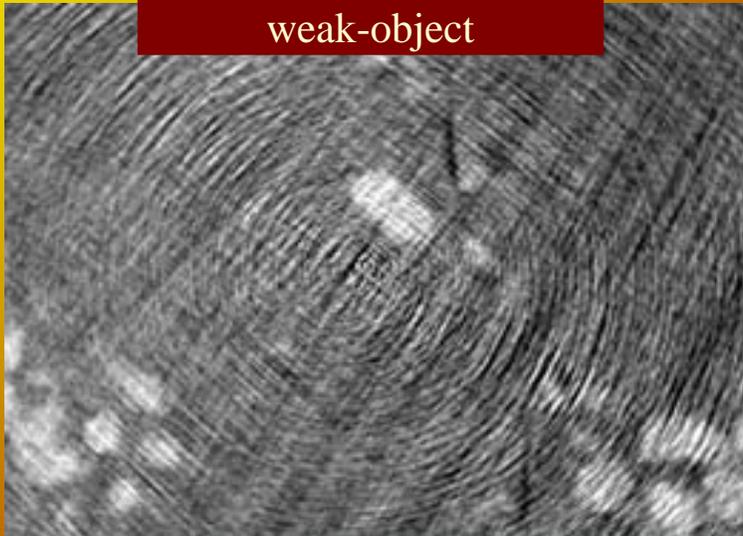
weak-object followed by 5 iterations



5 iteration without restrictions on initial guess of the phase

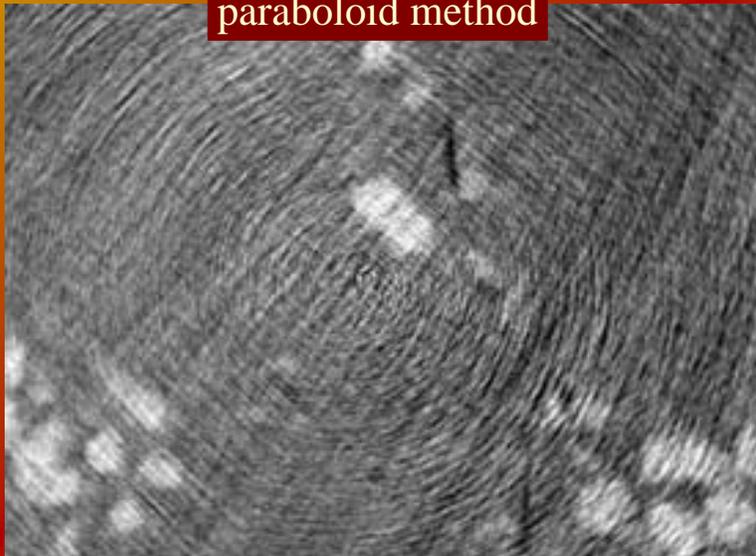


weak-object



5 μm

paraboloid method

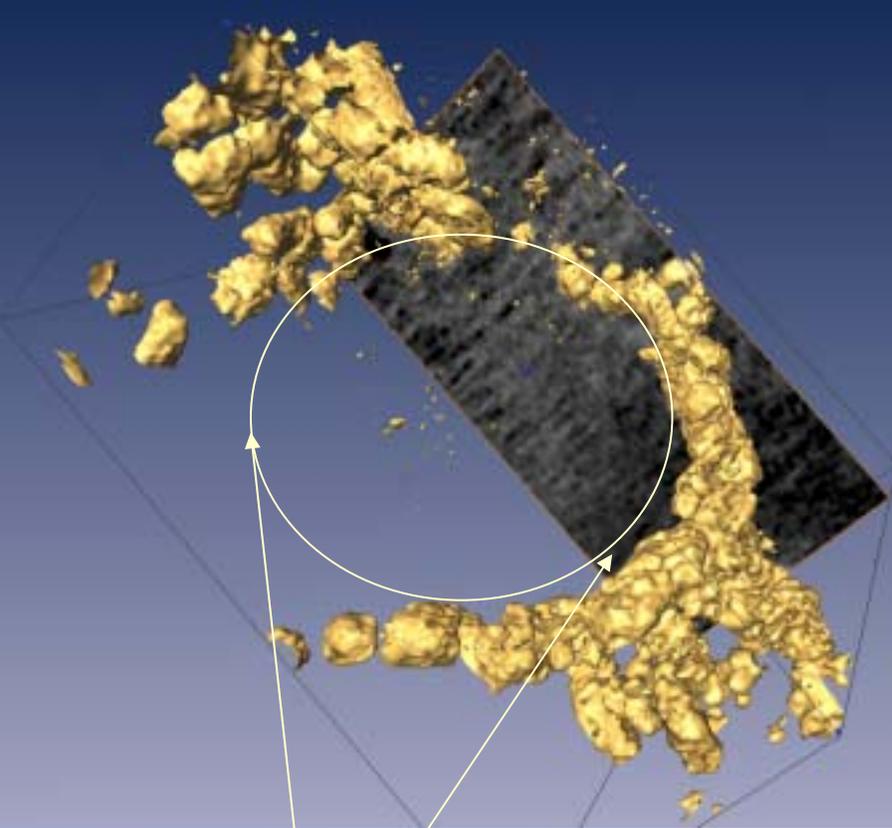


•the design of the filter amplifies ring artefacts

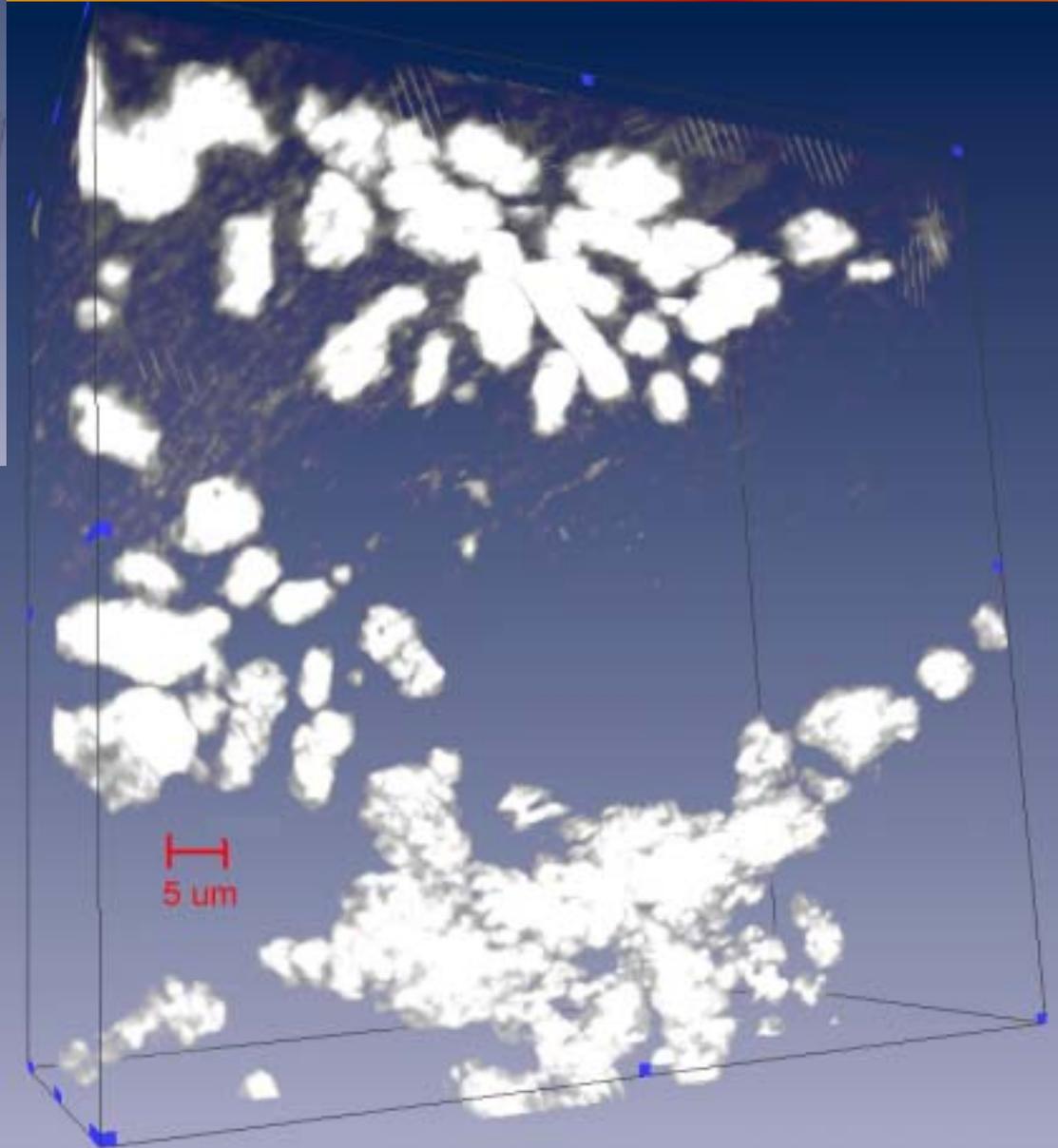
quality of images implies the use of image processing tools in order to obtain 3D volumes that can be segmented

3D visualisation

Paraboloid method



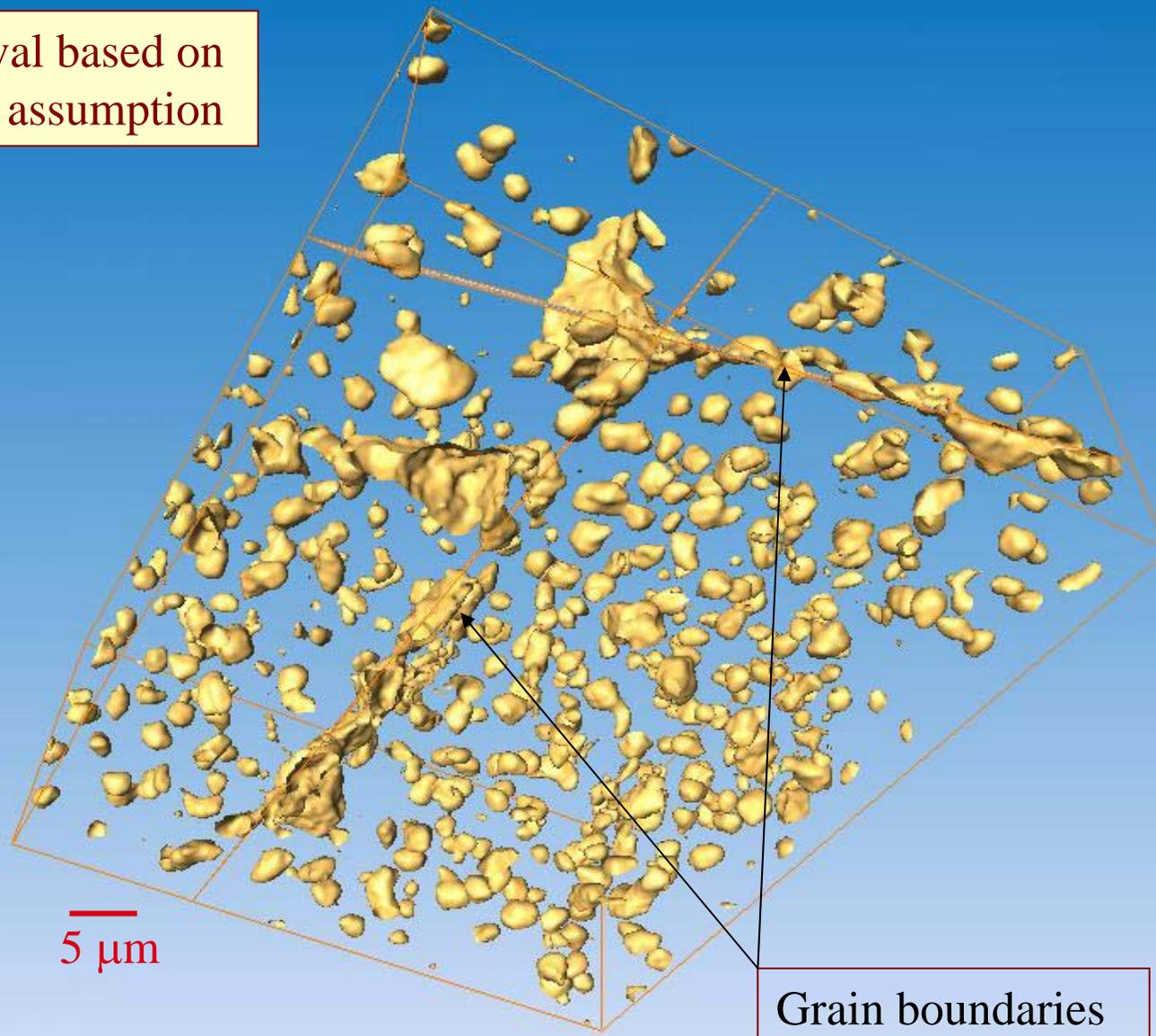
Si located at grain boundaries



AlSi

Al₂Cu alloy in 3D

Phase retrieval based on
weak-object assumption



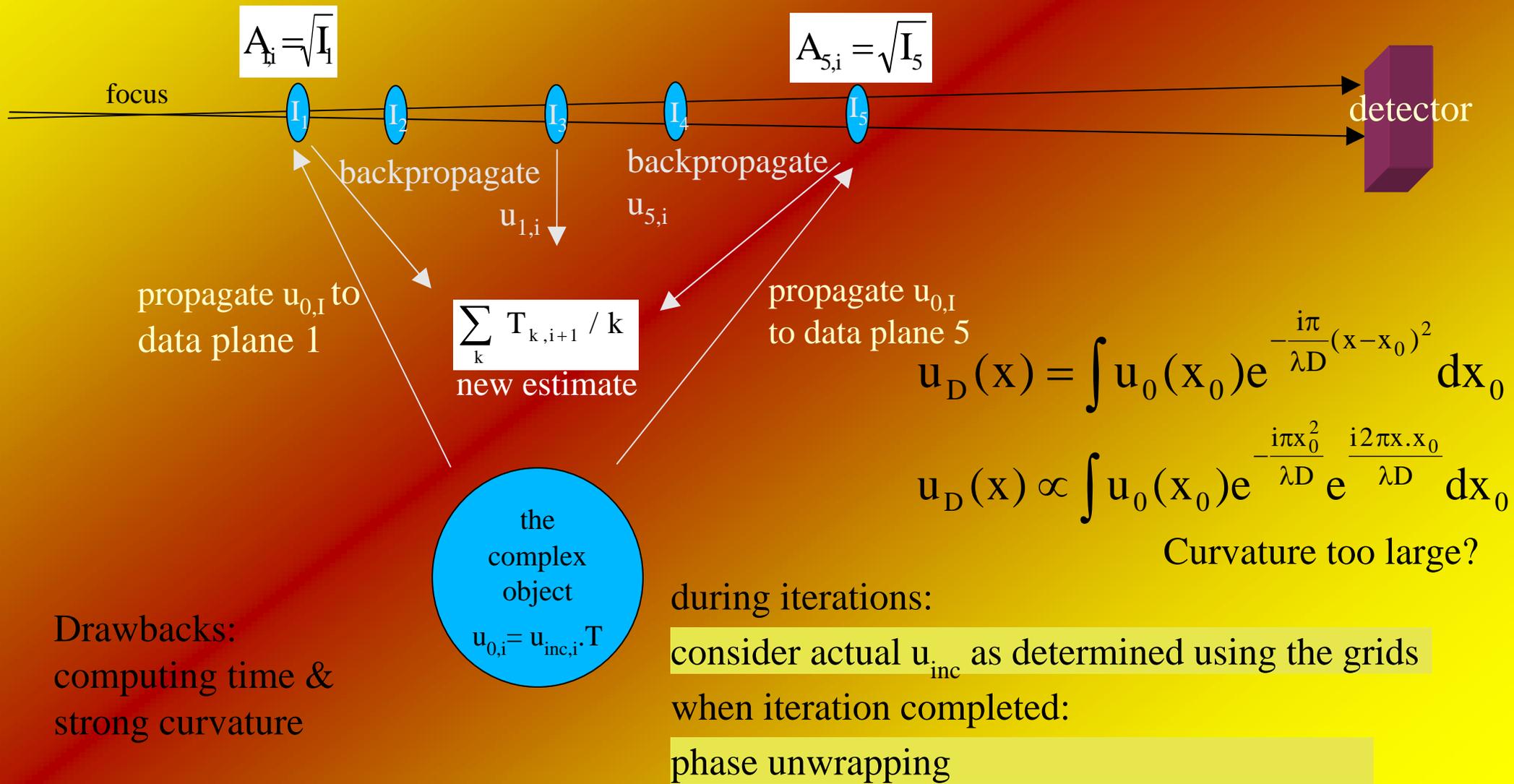
Outlook – iterative method

principle

The Gerchberg-Saxton-Fienup algorithm modified for:

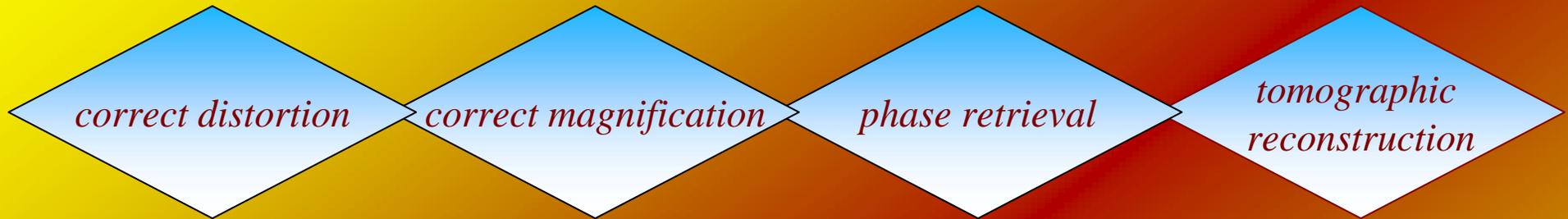
- Fresnel diffraction
- multiple intensity measurements

See also: Curved wavefronts: Nugent, K.A. et. al., (2003). Phys. Rev. Let. 91,203902



summary & perspectives

- Overview of the processing for magnified tomography



- Artefacts: Perturbation of the wavefront introduced by the mirror shape
Flatfield fails in case of strong phase gradients

- Local tomography offers the “easy” solution

- Phase retrieval on alloys with satisfactory results applying the weak-object approximation or the paraboloid method

-Implement different iterative approaches
consider as far-field with curvature
-Implement phase unwrapping

- Spatial resolution in tomography improved in comparison to the highest resolution optics

