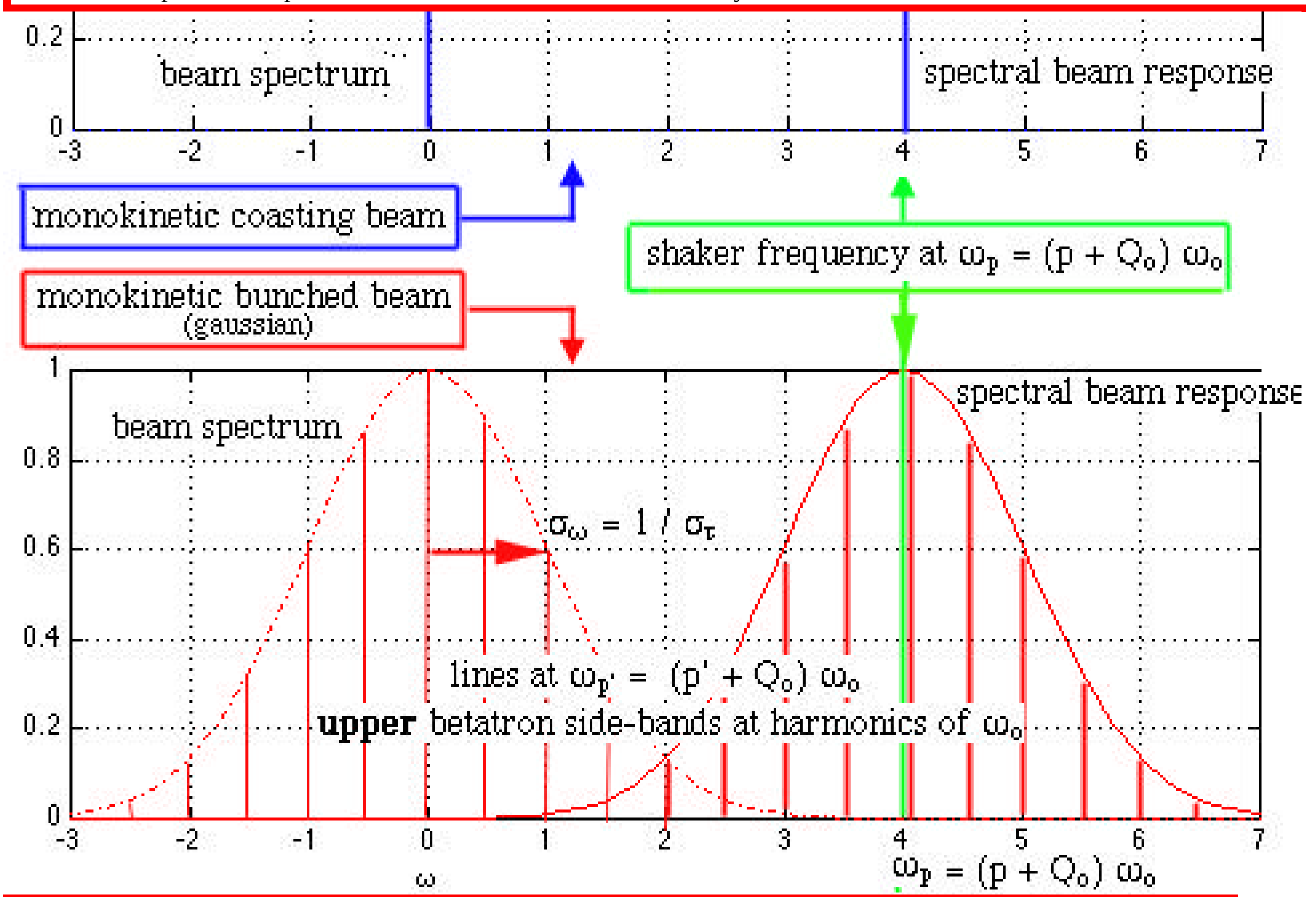
An approximative criterion for the stability of an intense bunched beam at high chromaticity

(G.Besnier, P.Kernel , R.Nagaoka , J.L.Revol)

When the ESRF works with a high chromaticity, the intensity in single bunch mode is suspected to be limited by a fast transverse instability of "post head-tail" type (with growing time shorter than a synchrotron period) . Elements of a theory* in frequency space are presented , using J.L. Laclare 's formalism.

The work on this topic is part of a thesis by Ph.Kernel: some aspects are recent, not fully achieved and possibly need more reflection .

Look at the spectral response of a monokinetic beam driven by a monochromatic shaker



The spectral response is the beam spectrum shifted to the shaker frequency at ω_p

J.L. Laclare's formalism is more easily understood when we define the matrix elements:

$\mathbf{A}_{p',p}$ = the amplitude frequency line $\sigma(\omega_{p'})$ in response to a unit excitation at ω_p
= the spectrum of the longitudinal bunch density shifted at ω_p
(later designed as a "shaker mode" spectrum , centered at ω_p)
$= \delta_{p,p'}$ for a **coasting beam** , $= \exp - [(\omega_p - \omega_{p'})^2 \sigma_\tau^2 / 2]$ for a **gaussian beam** .

such a matrix **A** works like the"impulse response"of a filter in frequency space

* See also a

previous and different analysis by R.D.Ruth and J.M. Wang: Vertical fast blow-up in a single bunch. IEEE Transactions on Nuclear Sciences,NS-28,N°3,june 1982.

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part 1(monokinetic)
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Transverse modes of a monokinetic bunch , broad band impedance

the stability of a collective bunch oscillation is given by
the imaginary part of the complex frequency shift $\Delta\omega_c^o$ from $Q_o\,\omega_o$

narrow band impedanc $e^* Z_T(\omega)$

1)admit the **coasting beam** result :

$$\Delta\omega_{cp}^o = \Lambda\, j Z_T(\omega_p) \quad , \quad \Lambda = \frac{c\, I_o}{4\pi\, Q_o\, E_o/e} .$$

[or $\Delta\omega_{cp'}^o\ \sigma(\omega_{p'}) = \Lambda\, \delta_{p',p}\ jZ_T(\omega_p)\ \sigma(\omega_p)$]

2)deduce the **bunched beam** formula :

$$\Delta\omega_c^o\ \sigma(\omega_{p'}) = \Lambda\ \mathbf{A}_{p',p}\ jZ_T(\omega_p)\sigma(\omega_p)$$

(only one input line)

broad band impedance $Z_T(\omega)$

3)same **coasting beam** formula :

$$\Delta\omega_{cp}^o = \Lambda\, j Z_T(\omega_p) \quad , \quad \Lambda = \frac{c\, I_o}{4\pi\, Q_o\, E_o/e}$$

stability : upper side band of mode " **p**": $\omega_{cp}^+ = (\,\mathbf{p} + Q_o + \Delta\omega_{cp}^o)\ \omega_o$

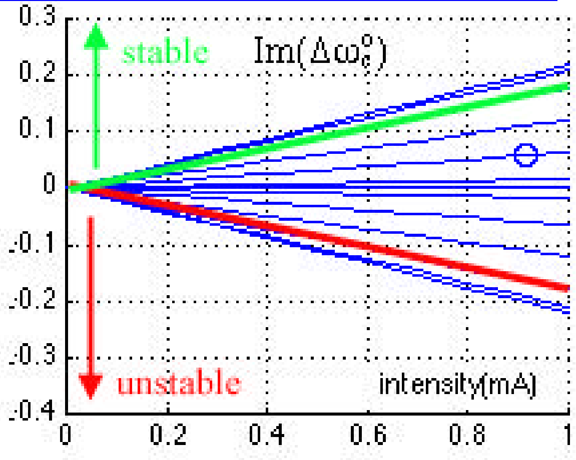
unstable if $\mathbf{Re}\,(\mathbf{Z_T}(\omega_{cp}^+)) < 0$.

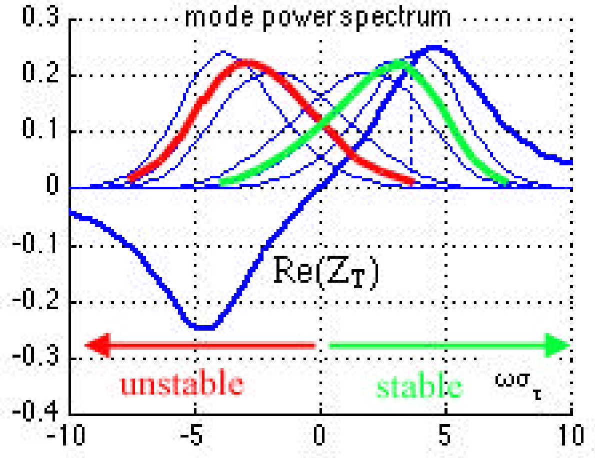
4)**bunched beam** : sum all input lines

$$\Delta\omega_c^o\ \sigma(\omega_{p'}) = \Lambda \sum_p jZ_T(\omega_p)\ A_{p',p}\ \sigma(\omega_p)$$

stability :

1- look for **computed solutions** of this standard eigen-value problem :





stability : 2 - look for **approximative solutions** of the eigen-value problem :

$\sigma_q(\omega_p)$ an eigen-mode spectrum associated with an eigen-value $\Delta\omega_{c\,q}^o$,

C_q the associated eigen-value of matrix **A** , the n** :

$$\Delta\omega_{c\,q}^o = \Lambda\, j[Z_{Tq}]_{eff}\ C_q \text{, with effective impedance : } \quad [Z_{Tq}]_{eff} = \frac{\sum_p Z_T(\omega_p)\ \sigma_q^2(\omega_p)}{\sum_p \sigma_q^2(\omega_p)} .$$

Approximation of σ_q by a "shaker mode" : $\sigma_q(\omega_p) = \exp -[(\omega_p - \omega_q)^2\ \sigma_\tau^2/2]$, then $C_q = \dfrac{2\sqrt{\pi/3}}{\omega_o\,\sigma_\tau}$ (indpt of q) .

Conclusion : using J.L.Laclare 's formalism, we recover rapidly the following property :

Adding the contributions of upper and lower side-bands : monokinetic coasting and bunched beams are unstable with respect to any impedance which has a resistive component. The frequency shift Re($\Delta\omega_{c\,q}$) and growing time Im($\Delta\omega_{c\,q}$)$^{-1}$ of any mode "q" can be computed

* by analogy between a shaker tuned at ω_p and a narrow band impedance $Z_T(\omega)$
which overlaps only one line ω_p of the transverse returned signal.

** F.J. Sacherer derived such approximate but very useful formulas for the stability of head-tail sine modes